# CONSTRUCTION OF FISCHER'S SPORADIC GROUP $Fi'_{24}$ INSIDE $GL_{8671}(13)$

#### HYUN KYU KIM AND GERHARD O. MICHLER

ABSTRACT. In this article we construct an irreducible simple subgroup  $\mathfrak{G} = \langle \mathfrak{q}, \mathfrak{y}, \mathfrak{t}, \mathfrak{w} \rangle$  of  $\mathrm{GL}_{8671}(13)$  from an irreducible subgroup T of  $\mathrm{GL}_{11}(2)$  isomorphic to Mathieu's simple group  $\mathcal{M}_{24}$  by means of Algorithm 2.5 of [13]. We also use the first author's similar construction of Fischer's sporadic simple group  $G_1 = \mathrm{Fi}_{23}$  described in [11]. He starts from an irreducible subgroup  $T_1$  of  $\mathrm{GL}_{11}(2)$  contained in T which is isomorphic to  $\mathcal{M}_{23}$ . In [7] J. Hall and L. S. Soicher published a nice presentation of Fischer's original 3-transposition group  $\mathrm{Fi}_{24}$  [6]. It is used here to show that  $\mathfrak{G}$  is isomorphic to the simple commutator subgroup  $\mathrm{Fi}_{24}'$  of  $\mathrm{Fi}_{24}$ . We also determine a faithful permutation representation of  $\mathfrak{G}$  of degree 306936 with stabilizer  $\mathfrak{G}_1 = \langle \mathfrak{q}, \mathfrak{y}, \mathfrak{w} \rangle \cong \mathrm{Fi}_{23}$ . It enabled MAGMA to calculate the character table of  $\mathfrak{G}$  automatically.

Furthermore, we prove that  $\mathfrak{G}$  has two conjugacy classes of involutions  $\mathfrak{z}$  and  $\mathfrak{u}$  such that  $C_{\mathfrak{G}}(\mathfrak{u}) = \langle \mathfrak{q}, \mathfrak{y}, \mathfrak{t} \rangle \cong 2Aut(\mathrm{Fi}_{22})$ . Moreover, we determine a presentation of  $\mathfrak{H} = C_{\mathfrak{G}}(\mathfrak{z})$  and a faithful permutation representation of degree 258048 for which we document a stabilizer.

### 1. Introduction

In 1971 B. Fischer [6] discovered 3 sporadic simple groups by characterizing all finite groups G which can be generated by a conjugacy class  $D=z^G$  of 3-transpositions. This means that the product of 2 elements of D in G has order 1, 2 or 3. The largest of these 3 sporadic groups turned out to be the commutator subgroup  $\mathrm{Fi}'_{24}$  of the 3-transposition group  $\mathrm{Fi}_{24}$ . In [5] Fischer constructed for each of the 3 groups  $\mathrm{Fi}_k$  a graph  $\mathcal{G}_k$  and showed that  $\mathrm{Fi}_k$  is isomorphic to the automorphism group  $\mathrm{Aut}(\mathcal{G}_k)$ ,  $k \in \{22, 23, 24\}$ . Using the structure of  $\mathcal{G}_{24}$  J. Hall and L. Soicher determined a nice presentation of  $\mathrm{Fi}_{24}$ , see [7] and [15], p. 124.

The results of this article are part of our joint research project Simultaneous construction of the sporadic simple groups of Conway, Fischer and Janko. Its goal is to provide uniform existence proofs for the sporadic sporadic simple groups discovered by Conway, Fischer and Janko by means of Algorithm 2.5 of [13] constructing finite simple groups from irreducible subgroups T of  $GL_n(2)$ . In [10] we constructed Conway's and Fischer's sporadic groups  $Co_2$  and  $Fi_{22}$  simultaneously from the irreducible subgroup  $\mathcal{M}_{22}$  in  $GL_{10}(2)$ . In [11] the first author applied the same methods to the irreducible subgroup  $\mathcal{M}_{23}$  of  $GL_{11}(2)$  and realized  $Fi_{23}$  as an irreducible subgroup of  $GL_{782}(17)$ .

In Lemma 2.1 of Section 2 we construct a presentation of the unique non split extension E of  $\mathcal{M}_{24}$  by the natural GF(2)-vector space V of dimension 11 by means of Holt's Algorithm [8] implemented in MAGMA. By Fischer's work [6] it is known that this extension group has a Sylow 2-subgroup S which is isomorphic to the ones of his sporadic simple group  $\mathrm{Fi}'_{24}$ . Lemma 2.1 also states that E has a unique conjugacy class of 2-central involutions z. Furthermore, it has a subgroup  $N_1$  of index 24 and a non 2-central involution u such that  $N_1/V \cong \mathcal{M}_{23}$  and  $E = \langle N_1, C_E(u) \rangle$ .

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In our first attempt we applied Algorithm 2.5 of [13] to E and constructed a finitely presented group H containing a Sylow 2-subgroup  $S_1$  having a maximal elementary abelian normal subgroup A such that  $N_H(A) \cong D = C_E(z)$  and  $|H:N_H(A)|$  is odd. Furthermore, we calculated the character tables of E, H and D and applied Algorithm 7.4.8 of [12] to show that the free product  $Q = H *_D E$  with amalgamated subgroup D has 939, 080, 024, 064 irreducible representations of minimal dimension 8671 over GF(13). In view of our time constraints we decided not to start Thompson's amalgamation process described in Theorem 7.2.2 of [12] in order to find an irreducible representation of Q of degree 8671 which has a Sylow 2-subgroup isomorphic to  $S_1 \cong S$ . Instead we now construct such a matrix representation  $\mathfrak{G}$  of Q using the first author's work [11].

There he applied Algorithm 2.5 of [13] to  $N_1$  and obtained a simple subgroup  $\mathfrak{G}_1$  of  $\mathrm{GL}_{782}(17)$  which he showed to be isomorphic to Fischer's sporadic group  $\mathrm{Fi}_{23}$ . He also determined its character table, a faithful permutation representation  $PG_1$  of degree 31671 and a presentation of the centralizer  $\mathfrak{H}_1 = C_{\mathfrak{G}_1}(\mathfrak{F}_1)$  of a 2-central involution  $\mathfrak{u}_1$  of  $\mathfrak{G}_1$ . Since  $\mathfrak{H}_1 \cong 2\,\mathrm{Fi}_{22}$  and  $C_{N_1}(u)$  have isomorphic Sylow 2-subgroups by [11], we determine in Lemma 3.1 of Section 2 a presentation for  $A_1 = 2Aut(\mathrm{Fi}_{22})$ , its character table, a system of representatives of its conjugacy classes and a faithful permutation representation  $PA_1$ . Thus we can show that  $A_1$  and  $C_E(u)$  have isomorphic Sylow 2-subgroups where u is the non 2-central involution of E mentioned above.

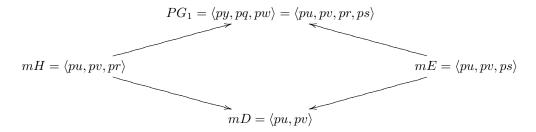
Lemma 3.2 of Section 3 asserts that the amalgam  $\mathfrak{G}_1 \leftarrow \mathfrak{H}_1 \rightarrow A_1$  has 8 distinct compatible pairs of semi-simple characters of  $\mathfrak{G}_1$  and  $A_1$  of degree 8671. All these compatible pairs have the same semi-simple character  $\tau = \tau_3 + \tau_4$  of  $\mathfrak{G}_1$ . Its two irreducible constituents  $\tau_3$  and  $\tau_4$  have respective degrees 3588 and 5083. Each compatible pair  $(\tau, \chi_i)$  determines a subgroup  $\mathfrak{K}_i$  of  $GL_{8671}(13)$ . In the following two sections we construct these 8 matrix groups.

Since  $\mathfrak{G}_1$  does not have any suitable subgroups whose permutation characters of  $\mathfrak{G}_1$  contain  $\tau_3$  or  $\tau_4$  as irreducible constituents we first construct a pair mH and mE of subgroups of the permutation representation  $PG_1$  of  $\mathfrak{G}_1$  such that

$$PG_1 = \langle mH, mE \rangle$$
 and  $mD = mH \cap mE \cong G_2(3) \times Sym(3)$ .

These subgroups enable us to construct the irreducible representations of  $PG_1$  in  $GL_{3588}(13)$  and  $GL_{5083}(13)$  corresponding to  $\tau_3$  or  $\tau_4$ , respectively, by means of Thompson's methods described in Theorem 7.2.2 of [12]. The character tables of mH and mE are Tables B.2 and B.3 of the Appendix, respectively. They were automatically computed by MAGMA using  $PG_1$ .

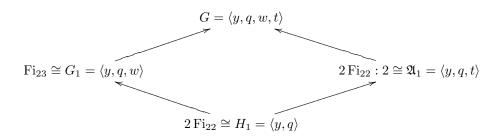
By the main result of [11] we have 3 permutations py, pq and pw and a 2-central involution  $pz_1$  of  $PG_1$  such that  $PG_1 = \langle py, pq, pw \rangle \cong \mathfrak{G}_1$  and  $C_{PG_1}(pz_1) = \langle py, pq \rangle \cong \mathfrak{H}_1$ . We found these two subgroups mH and mE of  $PG_1$  by means of the MAGMA command LowIndexSubgroups(PG\_1: n) for n=137632 and n=148642560. An application of the first author's program GetShortGens(PG\_1,U) provides short words pu, pv, pr and ps in terms of the generators py, pq and pw of  $PG_1$  such that  $mD = \langle pu, pv \rangle$ ,  $mH = \langle mD, pr \rangle$  and  $mE = \langle mD, ps \rangle$ . For the construction of the target matrix group G it is necessary to get short words of the old generators py, pq and pw in terms of the new generators of  $PG_1$ . This is also done in Lemma 4.1 of Section 4. Thus we obtain the following diagram of permutation groups.



The restrictions of the characters  $\tau_3$  and  $\tau_4$  of  $PG_1$  to the two subgroups mH and mE are determined in Lemma 4.1 of Section 4. Each of their irreducible constituents is an irreducible constituent of a permutation character of mH or mE. In Propositions 4.2 and 4.3 we apply the first author's efficient implementation of Algorithm 5.7.1 of [12] and get the various matrix representations corresponding to the irreducible constituents of the permutation characters of mH and mE determined in Lemma 4.1. They enable us to construct the correct blocked matrices u, v, r and s of the generators pu, pv, pr and ps of  $PG_1$  in  $GL_{8671}(13)$  corresponding to the restrictions of the irreducible characters  $\tau_3$  and  $\tau_4$  of  $PG_1$  to mH, mE and mD. Let y, q and w be the matrices of  $GL_{8671}(13)$  obtained by inserting the matrices u, v, r and s into the words of py, pq and pw in terms of pu, pv, pr and ps. Then  $G_1 = \langle y, q, w \rangle \cong PG_1$ , and  $H_1 = \langle y, q \rangle \cong PH_1$ .

Since the restriction of  $\tau_3 + \tau_4$  to mD is not multiplicity-free we have to solve a difficult amalgamation problem in order to get the corresponding irreducible representations of  $PG_1$ . By Lemma 4.1 we know that the generator pr of mH is an involution which commutes in the known group  $PG_1$  with the involution  $pf = (pu^3pvpupspv)^9$ . Using this information and the structure of the blocked matrices r, s, u and v we are able to solve the amalgamation problems in Propositions 4.2 and 4.3 by calculating the solutions of well determined systems of algebraic equations with coefficients in GF(13).

As  $H_1$  is isomorphic to a normal subgroup of index 2 in  $A_1$  we use Clifford's Theorem to determine exactly 8 semi-simple representations of  $A_1$  corresponding to the 8 compatible pairs having the same restriction to  $PH_1$  as  $\tau_3 \oplus \tau_4$ , see Proposition 5.1. In Remark 5.3 we show that for exactly one compatible pair of Lemma 3.2(e) we can construct a matrix  $t \in GL_{8671}(13)$  such that the matrix subgroup  $G = \langle y, q, w, t \rangle$  may have a Sylow 2-subgroup isomorphic to the ones of E. For this matrix t we have the following amalgam of matrix groups:



In Section 6 we show that this matrix group G is isomorphic to the commutator subgroup P' of finitely presented group P of Hall and Soicher [7], see Lemma 6.2. Thus  $G \cong \text{Fi}'_{24}$ . In particular, the subgroup  $G = \langle q, y, w, t \rangle$  of  $\text{GL}_{8671}(13)$  is a simple

group of order  $2^{21} \cdot 3^{16} \cdot 5^2 \cdot 7^3 \cdot 11 \cdot 13 \cdot 17 \cdot 23 \cdot 29$  and it has a faithful permutation representation of degree 306936 with stabilizer  $G_1 = \langle q, y, w \rangle$ , see Theorem 6.3.

In Section 7 we determine generators and a presentation of  $H = C_G(z)$  of a 2-central involution z of G, see Proposition 7.1. Furthermore, we construct a faithful permutation representation of degree 258048 of H with a documented stabilizer. It has been used to calculate a system of representatives of its 167 conjugacy classes and its character table, see Tables A.3 and B.4., respectively.

In Section 8 we show that G has 2 conjugacy classes of involutions. They are represented by  $u = [(y(y^5t)^7)^{14} \text{ and } z = (xyw)^8$ . Their centralizers are  $C_G(u) = \langle q, y, t \rangle = A_1$  and  $C_G(z) = H$ . Using their character tables we calculate the group order of G using Thompson's group order formula and Theorem 6.1.4 of [14], see Proposition 8.1.

In the appendix we collect all the systems of representatives of conjugacy classes in terms of the given generators of the local subgroups of G which have been used to construct this matrix group  $G\cong \mathrm{Fi}'_{24}$ . We also state the character tables of these subgroups. The four generating matrices of the simple subgroup  $\mathfrak{G}=\langle \mathfrak{q},\mathfrak{y},\mathfrak{t},\mathfrak{w}\rangle$  of  $\mathrm{GL}_{8671}(13)$  can be downloaded from the first author's website  $\mathrm{http://www.math.yale.edu/~hk47/Fi24/index.html.}$ 

Concerning our notation and terminology we refer to the books [3] and [12]. The computer algebra system MAGMA is described in Cannon-Playoust [1].

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# 2. An extension of Mathieu's group $\mathcal{M}_{24}$

In [6] B. Fischer stated that his simple group  $Fi'_{24}$  has a subgroup E which is isomorphic to a non split extension of the Mathieu group  $\mathcal{M}_{24}$  by a well determined 11-dimensional irreducible  $\mathcal{M}_{24}$ -module V over GF(2) such that E contains a Sylow 2-subgroup S of his simple group  $Fi'_{24}$ . In this section we construct such an extension by means of Holt's Algorithm [8] implemented in MAGMA. It follows that E is uniquely determined by  $\mathcal{M}_{24}$  up to isomorphism.

A faithful permutation representation of degree 24 of  $\mathcal{M}_{24}$  is stated in Lemma 8.2.2 of [12]. The irreducible 2-modular representations of the Mathieu group  $\mathcal{M}_{24}$  were determined by G. James [9]. Here only the 2 non isomorphic simple modules  $V_i$ , i=1,2, of dimension 11 over F=GF(2) will be used. Todd's permutation representations of the Mathieu groups are stated in Lemma 8.2.2 of [12]. Therefore all conditions of Holt's Algorithm [8] implemented in MAGMA are satisfied. It constructs all split and non split extensions of  $\mathcal{M}_{24}$  by  $V_1$  and  $V_2$  up to isomorphism. Here only the presentation of the non split extension of  $\mathcal{M}_{24}$  by  $V_2$  is stated.

**Lemma 2.1.** Let  $\mathcal{M}_{24} = \langle a, b, c, d, t, g, h, i, j, k \rangle$  be the finitely presented group of Definition 8.2.1 of [12]. Then the following statements hold:

(a) The first irreducible representation  $V_1$  of  $\mathcal{M}_{24}$  is described by the following matrices:

- (b) The second irreducible representation V<sub>2</sub> of M<sub>24</sub> is described by the transpose inverse matrices of the generating matrices of M<sub>24</sub> defining V<sub>1</sub>:

  a<sub>2</sub> = [a<sub>1</sub><sup>-1</sup>]<sup>T</sup>, b<sub>2</sub> = [b<sub>1</sub><sup>-1</sup>]<sup>T</sup>, c<sub>2</sub> = [c<sub>1</sub><sup>-1</sup>]<sup>T</sup>, d<sub>2</sub> = [d<sub>1</sub><sup>-1</sup>]<sup>T</sup>, t<sub>2</sub> = [t<sub>1</sub><sup>-1</sup>]<sup>T</sup>, g<sub>2</sub> = [g<sub>1</sub><sup>-1</sup>]<sup>T</sup>, h<sub>2</sub> = [h<sub>1</sub><sup>-1</sup>]<sup>T</sup>, i<sub>2</sub> = [i<sub>1</sub><sup>-1</sup>]<sup>T</sup>, j<sub>2</sub> = [j<sub>1</sub><sup>-1</sup>]<sup>T</sup>, and k<sub>2</sub> = [k<sub>1</sub><sup>-1</sup>]<sup>T</sup>.
  (c) dim<sub>F</sub>[H<sup>2</sup>(M<sub>24</sub>, V<sub>1</sub>)] = 0 and dim<sub>F</sub>[H<sup>2</sup>(M<sub>24</sub>, V<sub>2</sub>)] = 1.
- (c)  $dim_F[H^2(\mathcal{M}_{24}, V_1)] = 0$  and  $dim_F[H^2(\mathcal{M}_{24}, V_2)] = 1$ . In particular, there is a uniquely determined non split extension E of  $\mathcal{M}_{24}$  by  $V_2$ .
- (d) The non split extension

$$E = E(Fi'_{24}) = \langle a, b, c, d, t, g, h, i, j, k, v_1, v_2, v_3, v_4, v_5, v_6, v_8, v_8, v_9, v_{10}, v_{11} \rangle$$

of  $\mathcal{M}_{24}$  by  $V_2$  has a set  $\mathcal{R}(E)$  of defining relations consisting of  $\mathcal{R}_1(V_2 \rtimes \mathcal{M}_{24})$  and the following relations:

$$\begin{split} v_i^2 &= 1 \quad and \quad v_k v_j = v_j v_k \quad for \ all \quad 1 \leq i, j, k \leq 11. \\ a^{-1} v_1 a v_1^{-1} v_2^{-1} v_5^{-1} v_6^{-1} v_7^{-1} v_8^{-1} v_{10}^{-1} = a^{-1} v_2 a v_{10}^{-1} = a^{-1} v_3 a v_4^{-1} = 1, \\ a^{-1} v_4 a v_3^{-1} &= a^{-1} v_5 a v_6^{-1} = a^{-1} v_6 a v_5^{-1} = a^{-1} v_7 a v_8^{-1} = a^{-1} v_8 a v_7^{-1} = 1, \\ a^{-1} v_9 a v_9^{-1} &= a^{-1} v_{10} a v_2^{-1} = a^{-1} v_{11} a v_{11}^{-1} = b^{-1} v_1 b v_6^{-1} = b^{-1} v_2 b v_8^{-1} = 1, \\ b^{-1} v_3 b v_1^{-1} v_2^{-1} v_4^{-1} v_6^{-1} v_{10}^{-1} v_{11}^{-1} = b^{-1} v_4 b v_1^{-1} v_3^{-1} v_6^{-1} v_7^{-1} v_8^{-1} v_{11}^{-1} = 1, \\ b^{-1} v_5 b v_1^{-1} v_2^{-1} v_5^{-1} v_6^{-1} v_7^{-1} v_8^{-1} v_{10}^{-1} = b^{-1} v_6 b v_1^{-1} = b^{-1} v_7 b v_{10}^{-1} = 1, \\ b^{-1} v_8 b v_2^{-1} &= b^{-1} v_9 b v_9^{-1} = b^{-1} v_{10} b v_7^{-1} = b^{-1} v_{11} b v_{11}^{-1} = c^{-1} v_1 c v_{10}^{-1} = 1, \\ c^{-1} v_2 c v_1^{-1} v_2^{-1} v_5^{-1} v_6^{-1} v_7^{-1} v_8^{-1} v_{10}^{-1} = c^{-1} v_3 c v_2^{-1} v_4^{-1} v_5^{-1} v_8^{-1} v_9^{-1} v_{10}^{-1} = 1, \\ c^{-1} v_4 c v_2^{-1} v_3^{-1} v_6^{-1} v_7^{-1} v_9^{-1} v_{10}^{-1} = c^{-1} v_5 c v_8^{-1} = c^{-1} v_6 c v_7^{-1} = c^{-1} v_7 c v_6^{-1} = 1, \\ c^{-1} v_4 c v_2^{-1} v_3^{-1} v_6^{-1} v_7^{-1} v_9^{-1} v_{10}^{-1} = c^{-1} v_5 c v_8^{-1} = c^{-1} v_6 c v_7^{-1} = c^{-1} v_7 c v_6^{-1} = 1, \\ c^{-1} v_8 c v_5^{-1} &= c^{-1} v_9 c v_9^{-1} = c^{-1} v_{10} c v_1^{-1} = c^{-1} v_{11} c v_{11}^{-1} = 1, \\ d^{-1} v_1 d v_3^{-1} v_5^{-1} v_7^{-1} v_8^{-1} v_9^{-1} v_{10}^{-1} v_1^{-1} = d^{-1} v_2 d v_1^{-1} v_3^{-1} v_6^{-1} v_7^{-1} v_8^{-1} v_{11}^{-1} = 1, \\ d^{-1} v_3 d v_7^{-1} &= d^{-1} v_4 d v_8^{-1} = d^{-1} v_5 d v_2^{-1} v_3^{-1} v_6^{-1} v_7^{-1} v_9^{-1} v_{10}^{-1} = 1, \\ d^{-1} v_6 d v_2^{-1} v_4^{-1} v_5^{-1} v_8^{-1} v_9^{-1} v_{10}^{-1} &= d^{-1} v_7 d v_3^{-1} = d^{-1} v_8 d v_4^{-1} = 1, \\ d^{-1} v_6 d v_2^{-1} v_4^{-1} v_5^{-1} v_8^{-1} v_9^{-1} v_{10}^{-1} &= d^{-1} v_7 d v_3^{-1} = d^{-1} v_8 d v_4^{-1} = 1, \\ d^{-1} v_6 d v_2^{-1} v_4^{-1} v_5^{-1} v_8$$

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d^{-1}v_9dv_9^{-1} = d^{-1}v_{10}dv_1^{-1}v_2^{-1}v_4^{-1}v_6^{-1}v_{10}^{-1}v_{11}^{-1} = d^{-1}v_{11}dv_{11}^{-1} = 1,
t^{-1}v_1tv_6^{-1}v_{11}^{-1} = t^{-1}v_2tv_{10}^{-1}v_{11}^{-1} = t^{-1}v_3tv_7^{-1}v_{11}^{-1} = 1, \\
t^{-1}v_4tv_2^{-1}v_4^{-1}v_5^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1} = t^{-1}v_5tv_8^{-1}v_{11}^{-1} = 1,
t^{-1}v_6tv_2^{-1}v_3^{-1}v_6^{-1}v_7^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1} = 1,
t^{-1}v_7tv_1^{-1}v_2^{-1}v_5^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_{10}^{-1}v_{11}^{-1}=1,\\
t^{-1}v_8tv_1^{-1}v_3^{-1}v_6^{-1}v_7^{-1}v_8^{-1} = t^{-1}v_9tv_{11}^{-1} = 1,
t^{-1}v_{10}tv_3^{-1}v_5^{-1}v_7^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1} = t^{-1}v_{11}tv_9^{-1}v_{11}^{-1} = g^{-1}v_1gv_4^{-1}v_6^{-1} = 1,
g^{-1}v_2gv_4^{-1}v_{10}^{-1} = g^{-1}v_3gv_3^{-1}v_4^{-1} = g^{-1}v_4gv_4^{-1} = g^{-1}v_5gv_4^{-1}v_8^{-1} = 1,
g^{-1}v_6gv_1^{-1}v_4^{-1} = g^{-1}v_7gv_1^{-1}v_2^{-1}v_4^{-1}v_5^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_{10}^{-1} = 1,
g^{-1}v_8gv_4^{-1}v_5^{-1} = g^{-1}v_9gv_2^{-1}v_5^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1} = g^{-1}v_{10}gv_2^{-1}v_4^{-1} = 1,
g^{-1}v_{11}gv_1^{-1}v_2^{-1}v_6^{-1}v_{10}^{-1}v_{11}^{-1} = h^{-1}v_1hv_1^{-1}v_9^{-1} = h^{-1}v_2hv_9^{-1}v_{10}^{-1} = 1,
h^{-1}v_3hv_7^{-1}v_9^{-1} = h^{-1}v_4hv_8^{-1}v_9^{-1} = h^{-1}v_5hv_2^{-1}v_4^{-1}v_5^{-1}v_8^{-1}v_{10}^{-1} = 1,
h^{-1}v_6hv_2^{-1}v_3^{-1}v_6^{-1}v_7^{-1}v_{10}^{-1} = h^{-1}v_7hv_3^{-1}v_9^{-1} = 1,
h^{-1}v_8hv_4^{-1}v_9^{-1} = h^{-1}v_9hv_9^{-1} = h^{-1}v_{10}hv_2^{-1}v_9^{-1} = 1,
h^{-1}v_{11}hv_9^{-1}v_{11}^{-1} = i^{-1}v_1iv_2^{-1}v_3^{-1}v_6^{-1}v_7^{-1}v_{10}^{-1} = 1,
i^{-1}v_2iv_3^{-1}v_5^{-1}v_7^{-1}v_8^{-1}v_{10}^{-1}v_{11}^{-1} = i^{-1}v_3iv_5^{-1}v_9^{-1} = 1,
i^{-1}v_4iv_4^{-1}v_9^{-1}=i^{-1}v_5iv_3^{-1}v_9^{-1}=i^{-1}v_6iv_6^{-1}v_9^{-1}=1,\\
i^{-1}v_7iv_1^{-1}v_3^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_9^{-1}v_{11}^{-1} = i^{-1}v_8iv_1^{-1}v_2^{-1}v_5^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1} = 1,
i^{-1}v_9iv_9^{-1} = i^{-1}v_{10}iv_9^{-1}v_{10}^{-1} = i^{-1}v_{11}iv_9^{-1}v_{11}^{-1} = j^{-1}v_1jv_6^{-1} = 1,
j^{-1}v_2jv_1^{-1}v_2^{-1}v_5^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_{10}^{-1} = 1,
j^{-1}v_3jv_3^{-1}v_5^{-1}v_7^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1} = 1,
j^{-1}v_4jv_4^{-1} = j^{-1}v_5jv_8^{-1} = j^{-1}v_6jv_1^{-1} = j^{-1}v_7jv_{10}^{-1} = 1,
j^{-1}v_8jv_5^{-1} = j^{-1}v_9jv_{11}^{-1} = j^{-1}v_{10}jv_7^{-1} = j^{-1}v_{11}jv_9^{-1} = 1,
k^{-1}v_1kv_3^{-1}v_5^{-1}v_7^{-1}v_8^{-1}v_{10}^{-1}v_{11}^{-1} = 1,
k^{-1}v_2kv_2^{-1}v_3^{-1}v_6^{-1}v_7^{-1}v_{10}^{-1} = k^{-1}v_3kv_7^{-1}v_9^{-1} = 1,
k^{-1}v_4kv_4^{-1}v_9^{-1} = k^{-1}v_5kv_1^{-1}v_3^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_9^{-1}v_{11}^{-1} = 1, \\
k^{-1}v_6kv_9^{-1}v_{10}^{-1} = k^{-1}v_7kv_3^{-1}v_9^{-1} = k^{-1}v_8kv_8^{-1}v_9^{-1} = k^{-1}v_9kv_9^{-1} = 1,
k^{-1}v_{10}kv_6^{-1}v_9^{-1} = k^{-1}v_{11}kv_9^{-1}v_{11}^{-1} = a^2v_5^{-1}v_6^{-1}v_7^{-1}v_8^{-1} = 1,
b^2v_1^{-1}v_3^{-1}v_4^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_{11}^{-1} = c^2v_1^{-1}v_5^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_{10}^{-1} = 1,
d^2v_1^{-1}v_2^{-1}v_3^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_{11}^{-1} = b^{-1}aba^{-1}v_1^{-1}v_3^{-1}v_4^{-1}v_6^{-1}v_7^{-1}v_{10}^{-1} = 1,
c^{-1}aca^{-1}v_2^{-1}v_{10}^{-1}=d^{-1}ada^{-1}v_5^{-1}v_6^{-1}v_9^{-1}=1,\\
c^{-1}bcb^{-1}v_1^{-1}v_3^{-1}v_4^{-1}v_8^{-1}v_9^{-1} = d^{-1}bdb^{-1}v_2^{-1}v_4^{-1} = 1,
d^{-1}cdc^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_{10}^{-1}v_{11}^{-1} = t^3v_1^{-1}v_3^{-1}v_8^{-1} = 1,
t^{-1}atd^{-1}c^{-1}v_2^{-1}v_3^{-1}v_4^{-1}v_5^{-1}v_7^{-1}v_8^{-1}v_9^{-1}v_{11}^{-1} = 1,
t^{-1}btd^{-1}a^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_6^{-1}v_9^{-1}v_{11}^{-1} = 1,
t^{-1}ctd^{-1}b^{-1}v_1^{-1}v_2^{-1}v_4^{-1}v_5^{-1}v_8^{-1}v_9^{-1}v_{11}^{-1} = 1,
t^{-1}dtc^{-1}b^{-1}a^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_7^{-1}v_8^{-1}v_9^{-1}v_{11}^{-1} = 1,
g^{2}v_{5}^{-1}v_{8}^{-1} = gagagav_{2}^{-1}v_{3}^{-1}v_{4}^{-1}v_{7}^{-1}v_{8}^{-1}v_{11}^{-1} = 1,
gbgbgbv_1^{-1}v_3^{-1}v_4^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1} = gcgcgcv_1^{-1}v_2^{-1}v_5^{-1}v_6^{-1}v_9^{-1} = 1,
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gtgtv_4^{-1}v_{11}^{-1} = h^2v_4^{-1}v_8^{-1}v_9^{-1} = h^{-1}aha^{-1}v_2^{-1}v_3^{-1}v_4^{-1}v_9^{-1}v_{10}^{-1} = 1,
h^{-1}bhd^{-1}b^{-1}a^{-1}v_4^{-1}v_5^{-1}v_7^{-1}v_8^{-1}v_{10}^{-1} = 1,
h^{-1}chc^{-1}a^{-1}v_1^{-1}v_3^{-1}v_4^{-1}v_5^{-1}v_7^{-1}v_{10}^{-1}v_{11}^{-1} = 1,
h^{-1}dhd^{-1}v_2^{-1}v_4^{-1}v_8^{-1}v_0^{-1}v_{10}^{-1} = 1.
h^{-1}thtv_1^{-1}v_2^{-1}v_3^{-1}v_4^{-1}v_5^{-1}v_6^{-1}v_7^{-1}v_{10}^{-1}v_{11}^{-1} = 1,
ghghghv_1^{-1}v_3^{-1}v_4^{-1}v_6^{-1}v_8^{-1}v_9^{-1} = 1,
i^2v_1^{-1}v_5^{-1}v_6^{-1}v_8^{-1}v_9^{-1}v_{11}^{-1} = i^{-1}aid^{-1}c^{-1}v_3^{-1}v_5^{-1}v_7^{-1}v_9^{-1}v_{10}^{-1} = 1,
i^{-1}bid^{-1}a^{-1}v_2^{-1}v_3^{-1}v_4^{-1}v_6^{-1}v_9^{-1}v_{11}^{-1} = 1,
i^{-1}cid^{-1}c^{-1}b^{-1}a^{-1}v_3^{-1}v_5^{-1}v_7^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1} = 1,
i^{-1}did^{-1}c^{-1}b^{-1}v_1^{-1}v_3^{-1}v_4^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1} = 1,
i^{-1}titv_1^{-1}v_5^{-1}v_6^{-1}v_8^{-1} = i^{-1}gig^{-1}t^{-1}v_1^{-1}v_2^{-1}v_3^{-1}v_8^{-1}v_{10}^{-1}v_{11}^{-1} = 1,
hihihiv_1^{-1}v_2^{-1}v_4^{-1}v_5^{-1}v_6^{-1}v_7^{-1}v_{10}^{-1}v_{11}^{-1} = j^2v_9^{-1}v_{11}^{-1} = 1,
j^{-1}ajc^{-1}b^{-1}a^{-1}v_1^{-1}v_2^{-1}v_5^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_0^{-1} = 1,
j^{-1}bjb^{-1}v_9^{-1}v_{11}^{-1} = j^{-1}cjc^{-1}v_1^{-1}v_5^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_{10}^{-1} = 1
j^{-1}djd^{-1}c^{-1}v_2^{-1}v_5^{-1}v_8^{-1} = j^{-1}tjtv_3^{-1}v_5^{-1}v_8^{-1}v_{10}^{-1} = 1,
j^{-1}gjg^{-1}v_2^{-1}v_4^{-1}v_5^{-1}v_7^{-1}v_8^{-1}v_9^{-1}v_{11}^{-1} = j^{-1}hjh^{-1}t^{-1}v_3^{-1}v_4^{-1}v_5^{-1}v_6^{-1}v_8^{-1}v_{11}^{-1} = 1,
ijijijv_1^{-1}v_3^{-1}v_4^{-1}v_5^{-1}v_7^{-1}v_8^{-1} = k^2v_1^{-1}v_5^{-1}v_6^{-1}v_8^{-1}v_9^{-1}v_{11}^{-1} = 1,
k^{-1}akd^{-1}a^{-1}v_4^{-1}v_6^{-1}v_8^{-1}v_9^{-1} = k^{-1}bkd^{-1}c^{-1}v_1^{-1}v_2^{-1}v_7^{-1}v_9^{-1} = 1,
k^{-1}ckd^{-1}b^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1} = 1,
k^{-1}dkd^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_7^{-1} = k^{-1}tktv_2^{-1}v_5^{-1}v_6^{-1}v_8^{-1}v_9^{-1}v_{11}^{-1} = 1,
k^{-1}qkq^{-1}t^{-1}v_2^{-1}v_3^{-1}v_4^{-1}v_5^{-1}v_6^{-1}v_0^{-1}v_{10} = k^{-1}hkh^{-1}v_2^{-1}v_4^{-1}v_8^{-1}v_{10}^{-1} = 1,
k^{-1}iki^{-1}v_2^{-1}v_7^{-1}v_8^{-1}v_{10}^{-1}v_{11}^{-1} = jkjkjkv_2^{-1}v_5^{-1}v_8^{-1}v_0^{-1}v_{10}^{-1} = 1.
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- (e) E has a faithful permutation representation PE of degree 1518 with stabilizer  $T = \langle g, h, i, (dg)^5, (dhjk)^3, (ijkj)^2, (dhjidg)^3 \rangle$ .
- (f)  $z = a^2$  is a 2-central involution of E with centralizer  $D = C_E(z) = \langle x, y \rangle$  of order  $2^{21} \cdot 3^3 \cdot 5$  where  $x = a(agik)^3$  and  $y = d(cgihj)^4$  have orders 4 and 6, respectively and  $z = (xy^3)^{12}$ . Furthermore,  $E = \langle x, y, e \rangle$ , where e = g has order 4.
- (g)  $E = \langle x, y, e \rangle$  has 73 conjugacy classes. A system of their representatives is given in Table A.1.
- (h) Table B.1 is the character table of E.
- (i)  $V_2$  is the unique maximal elementary abelian normal subgroup of each Sylow 2-subgroup S of the extension group E.
- (j)  $C_E(V_2) = V_2$ .
- (k)  $N_1 = \langle a, b, c, d, t, g, h, i, j, V_2 \rangle$  is a non split extension of  $M_{23}$  by  $V_2$ .
- (1)  $u = (agt)^5$  is an involution of  $N_1$  generating the center  $Z(R_2)$  of  $C_{R_1}(u)$ .
- (m)  $E = \langle N_1, C_E(u) \rangle$ .

Proof. (a) The 2 irreducible  $F \mathcal{M}_{24}$ -modules  $V_i$ , i = 1, 2, occur as composition factors with multiplicity 1 in the permutation module  $(1_{\mathcal{M}_{23}})^{\mathcal{M}_{24}}$  and can easily be constructed using the faithful permutation representation of  $\mathcal{M}_{24}$  stated in (a) and the Meataxe algorithm implemented in MAGMA. The corresponding matrices of the generators of  $\mathcal{M}_{24}$  with respect to the first irreducible representation of  $\mathcal{M}_{24}$  are stated in (a). The second irreducible representation  $V_2$  of  $\mathcal{M}_{24}$  is dual to  $V_1$  and so it is defined by the equations given in (b).

- (c) The cohomological dimensions  $d_i = \dim_F[H^2(\mathcal{M}_{24}, V_i)]$ , i = 1, 2, have been calculated by means of MAGMA using Holt's Algorithm 7.4.5 of [12], the presentation of  $\mathcal{M}_{24}$  of Definition 8.2.1 of [12] and all the data stated in (a) and (b). It follows that  $d_1 = 0$  and  $d_2 = 1$ .
- (d) The presentation of E has been obtained by means of step 3 of Holt's Algorithm 7.4.5 of [12] and MAGMA.
- (e) Using a stand-alone program due Paul Young we found a faithful permutation representation pE of E of degree 24288 with stabilizer  $\langle (i^{-1}j^{-1}(bg)^2, (i^{-1}bth^{-1})^4 \rangle$ . Applying the MAGMA command DegreeReduction(pE) we obtained the faithful permutation representation PE of degree 1518. The given generators of its stabilizer E were obtained by means of the first author's program GetShortGens(PE,BasicStabilizer(PE,2)).
- (f) Using MAGMA and the faithful permutation representation PE of E the reader easily verifies that the centralizer  $C_E(z)$  of  $z=a^2$  has order  $2^{21} \cdot 3^3 \cdot 5$ . Hence z is a 2-central involution of E by (d). The words of the generators x, y of D were calculated by means of PE, MAGMA and the first author's program GetShortGens(PE,PD). Another check with MAGMA and PE verifies that  $E = \langle D, g \rangle$ .
- (g) Using Kratzer's Algorithm 5.3.18 of [12], the faithful permutation representation PE and MAGMA we observed that E has 73 conjugacy classes. Their representatives are given in Table A.1.
- (h) The character table of E was automatically computed by MAGMA using PE.

The remaining 4 statements can be checked with MAGMA and the faithful permutation representation PE.

#### 3. The 2-fold cover of the automorphism group Aut(Fi<sub>22</sub>)

Applying Algorithm 2.5 of [13] to an extension group isomorphic to the subgroup  $N_1$  of  $E = \langle x, y, e \rangle$  described in Lemma 2.1(k) H. Kim realized Fischer's second sporadic simple group Fi<sub>23</sub> as an irreducible subgroup  $G_1$  of  $GL_{782}(17)$  in his senior thesis [11]. He showed that the centralizer  $H_1 = C_{G_1}(u)$  of a 2-central involution u of  $G_1$  is isomorphic to the 2-fold cover  $2Fi_{22}$  of Fischer's smallest sporadic simple group Fi<sub>22</sub>. Furthermore, he constructed a faithful permutation representation  $PG_1$  of  $G_1$  of degree 31671. In this section we use these results to construct the 2-fold cover  $A_1$  of the automorphism group  $Aut(H_1)$  and show that  $H_1$  is its commutator subgroup. Thus we obtain an amalgam  $A_1 \leftarrow H_1 \rightarrow G_1$  such that  $A_1$  and  $D_1 = C_E(z_1)$  have isomorphic Sylow 2-subgroups where  $z_1$  is the involution  $e^2$  of E. Using the character tables of the 3 groups of the amalgam we also show that it has 8 compatible pairs of semi-simple characters of degree 8671.

**Lemma 3.1.** Let  $A_1$  be the 2-fold cover of the automorphism group  $Aut(Fi_{22})$  of Fischer's simple group  $Fi_{22}$  and let  $H_1 = A'_1$  be its derived subgroup. Let  $E = \langle x, y, e \rangle$  be the non split extension of  $\mathcal{M}_{24}$  by its simple GF(2)-module constructed in Lemma 2.1. Then the following assertions hold:

(a)  $H_1 = \langle a, b, c, d, e, f, g, h, i, z \rangle$  has the following set  $\mathcal{R}(H_1)$  of defining relations:

$$a^{2} = b^{2} = c^{2} = d^{2} = e^{2} = f^{2} = g^{2} = h^{2} = i^{2} = 1,$$

$$(ab)^{3} = 1, (bc)^{3} = z, (cd)^{3} = (de)^{3} = 1, (ef)^{3} = (fg)^{3} = z,$$

$$(ac)^{2} = (ad)^{2} = (ae)^{2} = (af)^{2} = (ag)^{2} = (ah)^{2} = (ai)^{2} = 1,$$

$$(bd)^{2} = (be)^{2} = (bf)^{2} = (bg)^{2} = (bh)^{2} = (bi)^{2} = 1,$$

$$(ce)^{2} = (cf)^{2} = (cg)^{2} = (ch)^{2} = (ci)^{2} = 1,$$

$$(df)^{2} = (dg)^{2} = (eg)^{2} = (eh)^{2} = (ei)^{2} = 1,$$

$$(dh)^{3} = (hi)^{3} = (di)^{2} = (fh)^{2} = (fi)^{2} = (gh)^{2} = (gi)^{2} = 1,$$

$$(dcbdefdhi)^{10} = (abcdefh)^{9} = (bcdefgh)^{9} = 1,$$

$$z^{2} = (z, a) = (z, b) = (z, c) = (z, d) = (z, e) = (z, f) = 1,$$

$$(z, g) = (z, h) = (z, i) = 1.$$

(b)  $A_1 = \langle H_1, t \rangle$  has a set  $\mathcal{R}(A_1)$  of defining relations consisting of  $\mathcal{R}(H_1)$  and the following relations:

$$t^2 = 1$$
,  $(z, t) = 1$ ,  $a^t g = 1$ ,  $b^t f = c^t e = (dt)^2 = (ht)^2 = (it)^2 = z$ .

- (c)  $A_1 = 2Aut(Fi_{22})$  has a faithful permutation representation  $PA_1$  of degree 56320 with stabilizer  $\langle bz, c, d, e, fz, g, h, i \rangle$ .
- (d) A system of representatives  $a_i$  of the 150 conjugacy classes of  $A_1$  and the corresponding centralizers orders  $|C_A(a_i)|$  are given in Table A.2.
- (e) The character table of  $A_1$  is given in Table B.5.
- (f) The group  $A_1$  and the centralizer  $C_E(z_1)$  of the involution  $z_1 = e^2$  of E have isomorphic Sylow 2-subgroups of order  $2^{19}$ .

*Proof.* (a) The given presentation of  $H_1$  is a restatement of Proposition 6.2.3 of [14] due to H. Kim, see [11].

- (b) By that result we also know that  $H_1$  has a faithful permutation representation  $PH_1$  of degree 28160 with stabilizer  $U=\langle bz,c,d,e,fz,g,h,i\rangle$ . Using it and the MAGMA command AutomorphismGroup(H\_1) we see that  $|Aut(H_1)|=|H_1|$ . As  $H_1/\langle z\rangle\cong \mathrm{Fi}_{22}$  has the same presentation as  $\mathrm{Fi}_{22}$  given in [15], p. 110 we can quote the presentation of  $Aut(\mathrm{Fi}_{22})$  given in [15], p. 111, where z is replaced by 1. Now (b) follows from (a) and  $2^6$  MAGMA calculations with  $PH_1$  checking whether 1 or z has to be on the right hand side of the six new relations stated in (b) different from  $t^2=1$  and [z,t]=1. It follows that there is exactly one solution.
- (c) This statement has been verified by means of the MAGMA command CosetAction(A\_1,U).
- (d) The system of representatives of the conjugacy classes of  $A_1$  has been calculated by means of the permutation representations  $PA_1$  of  $A_1 = 2Aut(H_1)$ , MAGMA and Kratzer's Algorithm 5.3.18 of [12].
  - (e) The character table of  $A_1$  has been calculated by means of  $PA_1$  and MAGMA.
- (f) Let PE be the faithful permutation representation of E constructed in Lemma 2.1. Let  $C = C_E(u)$  for the involution  $u = e^2$  of  $E = \langle x, y, e \rangle$ . Now (f) can be verified by using the permutation representations  $PA_1$  and PE together with the Cannon-Holt isomorphism test implemented in MAGMA.

**Lemma 3.2.** Keep the notation of Lemma 3.1. Let  $G_1 = \langle x, y, q, w \rangle \cong \text{Fi}_{23}$  be the simple subgroup of  $\text{GL}_{782}(17)$  of order  $2^{18} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 23$  with centralizer  $H_1 = \langle x, y, q \rangle = C_{G_1}(z)$  of the 2-central involution  $z = (xy^2)^7$  constructed in [11]. Let  $A_1 = 2Aut(H_1)$ . Then the following assertions hold:

- (a)  $H_1 = \langle a, b, c, d, e, f, g, h, i, z \rangle$  where  $a = (xyx)^7, \quad b = [(qy)^2qy^3q^2y^3qy]^7, \quad c = (y^2xyxy^3)^5,$   $d = (qyq^2yqyqyqy^2q^2)^{15}, \quad e = (yxy^5x)^5, \quad f = (yqyq^2yq^2y^2q^2q^2)^5,$   $g = (xy^2xy^3x)^7, \quad h = (y^5xyx)^5, \quad i = (q^2y^2qyq^2)^7.$
- (b)  $H_1 = \langle a, b, c, d, e, f, g, h, i, z \rangle$  satisfies the set  $\mathcal{R}(H_1)$  of defining relations stated in Lemma 3.1(a). Its character table is Table 6.5.2 of [14].
- (c) The character table of  $G_1$  is stated in the Atlas [4], its pp. 178 179.
- (d) The amalgam  $A_1 \leftarrow H_1 \rightarrow G_1$  has Goldschmidt index 1.
- (e) The amalgam  $A_1 \leftarrow H_1 \rightarrow G_1$  has eight compatible pairs

$$(\chi, \tau) \in mfchar_{\mathbb{C}}(A_1) \times mfchar_{\mathbb{C}}(G_1)$$

of degree 8671. All have the same restriction

$$\delta_2 + \delta_6 + \delta_7 + \delta_8 + \delta_9 \in mfchar_{\mathbb{C}}(H_1).$$

They are:

- (1)  $(\chi_3 + \chi_{11} + \chi_{13} + \chi_{17}, \quad \tau_3 + \tau_4),$
- (2)  $(\chi_3 + \chi_{11} + \chi_{14} + \chi_{17}, \quad \tau_3 + \tau_4),$
- (3)  $(\chi_3 + \chi_{12} + \chi_{13} + \chi_{17}, \quad \tau_3 + \tau_4),$
- (4)  $(\chi_3 + \chi_{12} + \chi_{14} + \chi_{17}, \quad \tau_3 + \tau_4),$
- (5)  $(\chi_4 + \chi_{11} + \chi_{13} + \chi_{17}, \quad \tau_3 + \tau_4),$
- (6)  $(\chi_4 + \chi_{11} + \chi_{14} + \chi_{17}, \quad \tau_3 + \tau_4),$
- (7)  $(\chi_4 + \chi_{12} + \chi_{13} + \chi_{17}, \quad \tau_3 + \tau_4),$
- (8)  $(\chi_4 + \chi_{12} + \chi_{14} + \chi_{17}, \quad \tau_3 + \tau_4).$

*Proof.* By Kim's Theorem 6.3.1 of [14] the simple matrix subgroup  $G_1 \cong \text{Fi}_{23}$  of  $\text{GL}_{782}(17)$  has a faithful permutation representation  $PG_1$  of degree 31671 with stabilizer  $H_1$ . It is used throughout this proof.

- (a) The words of the new generators a, b, etc. of  $H_1$  in terms of the given generators x, y and q of  $H_1$  are quoted from Kim's Proposition 6.2.3 of [14].
- (b) Using the faithful permutation representation  $PG_1$  and MAGMA it has been checked that the new generators a, b etc. of  $H_1$  given in statement (a) satisfy all the relations of  $\mathcal{R}(H_1)$  of Lemma 3.1(a).
  - (c) This assertion is a restatement of Theorem 6.3.1 of [14].
- (d) Kratzer's Algorithm 7.1.10 of [12] could not be applied to calculate the Goldschmidt index. When trying to calculate  $Aut(G_1)$  using the Cannon-Holt Algorithm of [2] MAGMA answered: "Sorry, the top factor of order 4089470473293004800 is not currently stored". However, using the faithful permutation representation  $PA_1$  of  $A_1$  stated in Lemma 3.1(c) MAGMA established that the outer automorphism groups  $Out(H_1)$  and  $Out(A_1)$  of  $H_1$  and  $A_1$  are both cyclic of order 2. Hence the Goldschmidt index of the amalgam  $A_1 \leftarrow H_1 \rightarrow G_1$  is 1 by Step 3 of Algorithm 7.1.10 of [12].
- (e) The eight compatible pairs of degree 8671 of the amalgam  $A_1 \leftarrow H_1 \rightarrow G_1$  were determined by means of Kratzer's Algorithm 7.3.10 of [12] and MAGMA.  $\square$

## 4. A semi-simple representation of $Fi_{23}$ over GF(13)

In this section H. Kim's results of his senior thesis [11] are used for the construction of two irreducible representations of degrees 3588 and 5083 of  $G_1 \cong \operatorname{Fi}_{23}$  over the prime field GF(13). They correspond to the 2 irreducible characters  $\tau_3$  and  $\tau_4$  of 13-defect zero of  $G_1$  occurring in the 8 compatible pairs constructed in Lemma 3.2(e). For the construction of these fairly large representations we first determine generators of two large subgroups mH and mE of  $G_1$  and their intersection mD. We also calculate the character tables of these 3 subgroups of  $G_1$ .

**Lemma 4.1.** Keep the notation of Lemmas 3.1 and 3.2. Let  $G_1 = \text{Fi}_{23} = \langle x, y, q, w \rangle$  be the simple subgroup of  $\text{GL}_{782}(17)$  of order  $2^{18} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 23$  with faithful permutation representation  $PG_1$  of degree 31671 and stabilizer  $H_1 = \langle x, y, q \rangle$  constructed in Kim's Theorem 6.3.1 of [14]. Let  $r = s_1 = (yqy^2q)^7$ ,  $s_2 = (yqyqy)^7$ ,  $s_3 = (yeyeq)^{21}$ ,  $s_4 = (qyqey^2)^7$ ,  $u = s_4^2(s_1s_2s_4)^3$ ,  $s = qyqey^2$  and  $v = (s_1s_3s_1s_4)^2(s_1s_3s_1s_3s_1s_4)^3s_3s_4s_3s_4^2s_3$ . Then the following assertions hold:

- (a)  $mH = \langle s_1, s_2, s_3, s_4 \rangle = \langle r, u, v \rangle \cong O_8^+(3) : S_3$ .
- (b) The character table of mH is given in Table B.2.
- (c)  $mE = \langle u, v, s \rangle \cong O_7(3) \times S_3$ .
- (d) The character table of mE is Table B.3.
- (e)  $mD = mH \cap mE = \langle u, v \rangle \cong G_2(3) \times S_3$ .
- (f)  $G_1 = \langle u, v, r, s \rangle$  and the original generators q, y, w and x of  $G_1$  are equal to the following words in its generators u, v, r and s:

```
\begin{split} q &= [(us^2vsr)^9[(svs^3rsr)^{12}(vrsrsvu)^{12}]^2]^{24},\\ w &= [(w_2w_4w_2w_4w_3w_2w_4)^3(w_1w_2w_3w_1w_4w_3w_1w_4)^7]^3,\\ y &= (m_1m_2m_1m_2m_3)^7[(m_1m_2)^5(n_3n_1n_2n_3n_1n_3^2n_1)^5]^4,\\ x &= [(yq^2yqyq^2)^{11}(q^2y^2qyqy)^{11}(qy^2qyqyqyq)^4]^{12},\quad where\\ w_1 &= (vuvs^2)^9,\quad w_2 &= (v^2uruvr)^{18},\quad w_3 &= (v^2s^2urv)^{11},\\ w_4 &= (uvrv^4r)^{10},\quad n_1 &= m_1m_2m_3m_2^2m_1,\quad n_2 &= m_2m_3m_1m_3m_2^3,\\ n_3 &= (m_3m_1m_2m_3m_2m_1m_2)^2,\quad m_1 &= (t_1t_3t_1t_3t_1^2t_3t_1)^2,\\ m_2 &= (t_1t_3t_1t_3t_1^2t_2t_3t_2)^2,\quad m_3 &= (t_2t_1t_3t_2t_3t_1t_2t_3t_2)^9,\\ t_1 &= (srsrsrs)^5,\quad t_2 &= (s^2rs^4)^{11},\quad and\quad t_3 &= (rs^7)^3. \end{split}
```

- (g) The restrictions of the irreducible character  $\tau_3$  of degree 3588 of  $G_1$  to mH and mE are  $\pi_8 + \pi_{16} \in mf \operatorname{char}_{\mathbb{C}}(mH)$  and  $\psi_4 + \psi_{10} + \psi_{19} + \psi_{36} + \psi_{61} \in mf \operatorname{char}_{\mathbb{C}}(mE)$ , respectively.
- (h) The restrictions of the irreducible character  $\tau_4$  of degree 5083 of  $G_1$  to mH and mE are  $\pi_{12} + \pi_{15} \in mfchar_{\mathbb{C}}(mH)$  and  $\psi_{14} + \psi_{22} + \psi_{45} + \psi_{74}) \in mfchar_{\mathbb{C}}(mE)$ , respectively.
- (i) The irreducible characters  $\pi_9$ ,  $\pi_{11}$  and  $\pi_{15}$  of mH are constituents of the permutation characters  $1^{mH}_{mH_9}$ ,  $1^{mH}_{mH_{11}}$  and  $1^{mH}_{mH_{15}}$  of the subgroups

$$mH_9 = \langle (vru^2)^{13}, (rvur)^2, (uvrv^2)^4 \rangle,$$
  

$$mH_{11} = \langle (uvurv)^4, (v^5ur)^9, (ururv^2u^2)^{13} \rangle \quad and$$
  

$$mH_{15} = (uvuru)^{13}, (uv^2uv)^2, (u^5v)^3 \rangle$$

of mH with indices 3240, 72800 and 2274480, respectively.

- (j) The linear character  $\pi_2$  of mH has values -1 and 1 at v and u, r, respectively. Furthermore,  $\pi_8 = \pi_2 \otimes \pi_9$ ,  $\pi_{12} = \pi_2 \otimes \pi_{11}$ , and  $\pi_{16} = \pi_2 \otimes \pi_{15}$ .
- (k) The irreducible characters  $\psi_5$  and  $\psi_{10}$  of mE are constituents of the permutation character  $1_{mE_1}^{mE}$  of the subgroup  $mE_1 = \langle (s^2vs)^9, (svsv^2s^2)^2 \rangle$  of index 2106.
- (1) The irreducible characters  $\psi_{14}$ ,  $\psi_{22}$ ,  $\psi_{36}$ ,  $\psi_{45}$ ,  $\psi_{61}$  and  $\psi_{74}$  of mE are constituents of the permutation characters  $1_{mE_{14}}^{mE}$ ,  $1_{mE_{22}}^{mE}$ ,  $1_{mE_{36}}^{mE}$ ,  $1_{mE_{45}}^{mE}$ ,  $1_{mE_{45}}^{mE}$ ,  $1_{mE_{74}}^{mE}$  of the subgroups

$$\begin{split} mE_{14} &= \langle (s^2us)^6, (su^2s^2)^4, (susu^2)^2, (us^3us)^7 \rangle, \\ mE_{22} &= \langle (us^4u^2)^6, usu^2susus \rangle, \\ mE_{36} &= \langle (s^2vs)^9, (v^2svsv^2)^5, (vsv^2svs)^2 \rangle, \\ mE_{45} &= \langle (s^2v^2)^7, (v^5s)^3, (vsv^2sv^3sv)^6 \rangle, \\ mE_{61} &= \langle (vs^2)^7, (v^4s^2v^2)^{12}, (v^2svsv^4)^{10}, (vs^2v^3s^3)^{30} \rangle \quad \text{and} \\ mE_{74} &= \langle (s^2v)^7, (v^2s^2)^{21}, (s^2v^2)^{21}, (svsv^2sv)^3 \rangle \end{split}$$

of mE with indices 702, 2160, 19656, 7280, 85293 and 29484, respectively.

- (m) The linear character  $\psi_2$  of mE has values -1 and 1 at v and u, s, respectively. Furthermore,  $\psi_4 = \psi_2 \otimes \psi_5$ , and  $\psi_{19} = \psi_2 \otimes \psi_{22}$ .
- (n) Both r and  $f = (u^3vsv)^9$  are involutions of  $G_1$  such that (r, f) = 1,  $rf \notin mH$  and  $rf \notin mE$ .
- *Proof.* (a) The subgroup mH of  $G_1$  has been constructed by means of the faithful permutation representation  $PG_1$  of  $G_1$  of degree 31671 and the MAGMA command LowIndexSubgroups (PG\_1, 137632). The four generators  $s_i$  of mH,  $1 \le i \le 4$ , were calculated with Kim's program GetShortGens (PG\_1, mH). Another application of MAGMA determined the composition factors of mH.
  - (b) The character table of mH was calculated by MAGMA using  $PG_1$ .
- (c) and (e) By Table 6.5.4 of [14]  $|C_{G_1}((qw)^4)| = 2^9 \cdot 3^{10} \cdot 5 \cdot 7 \cdot 13$ . Let  $mX = N_{G_1}(\langle (qw)^4 \rangle)$ . Using  $PG_1$  and MAGMA we searched for an element  $x \in mX$  of order 3 such that  $|N_{mH}(\langle x \rangle)| = 2^7 \cdot 3^7 \cdot 7 \cdot 13$ . MAGMA found such an element and stated that  $mD = N_{mH}(\langle x \rangle) = \langle u, v \rangle$ . Furthermore,  $mE = N_{G_1}(\langle x \rangle) = \langle mD, s \rangle$  where u, v and s are defined in the statement of this lemma. The composition factors of mD and mE have been determined by means of MAGMA.
  - (d) The character table of mE was calculated by means of  $PG_1$  and MAGMA.
- (f) Using the faithful permutation representation  $PG_1$  of  $G_1$  and MAGMA one verifies that  $G_1 = \langle mH, mE \rangle$ . Hence  $G_1 = \langle u, v, r, s \rangle$  by (a) and (c). The words for q, y, w and x can easily be checked computationally.
- (g)  $G_1$  has a unique character  $\tau_3$  of degree 3588 by the character table of  $G_1 \cong Fi_{23}$ , see [4], p. 178. Its restrictions to mH and mE given in the statements have been determined by means of  $PG_1$ , the character tables of the subgroups mH and mE of  $G_1$ , and MAGMA.
  - (h) This assertion is proved as (g).
- (i) Using the MAGMA command LowIndexSubgroups (mH, k) we searched for conjugacy classes of subgroups  $H_k$  of index  $|mH:mH_k|=m_k$  such that  $\pi_k$  is an irreducible constituent of the permutation character  $1_{mH_k}^{mH}$  for  $k \in \{9,11,15\}$ . Thus we found 3 subgroups  $mH_k$  of respective indices  $m_9=3240, m_{11}=72800$

and  $m_{15} = 2274480$ . Their given generators have been obtained by means of Kim's program GetShortGens (mH, mH\_k).

(j)  $\pi_2$  is the unique non trivial linear character of mH by its character table. The character equations of the statement are easily verified by means of Table B.2.

The statements (k) and (l) are proved similarly as (i).

- (m) mE has a unique non trivial linear character  $\psi_2$ , see Table B.3. The character equations of the statement are easily verified by means of Table B.2.
- (n) Using  $PG_1$  and MAGMA we checked that r and f are commuting involutions such that  $rf \notin mH$  and  $rf \notin mE$ .

**Proposition 4.2.** Keep the notation of Lemmas 3.1, 3.2 and 4.1. Let  $PG_1$  be the faithful permutation representation of the simple group  $G_1 = \langle x, y, q, w \rangle = \langle u, v, r, s \rangle$  of degree 31671 with stabilizer  $H_1 = \langle x, y, q \rangle$ . Let  $mH = \langle u, v, r \rangle$ ,  $mD = \langle u, v \rangle$  and  $mE = \langle u, v, s \rangle$ . Let  $F^*$  be the multiplicative group of the prime field F = GF(13). Let  $Y = GL_{3588}(13)$ .

Let  $\mathfrak{V}$  and  $\mathfrak{W}$  be the up to isomorphism uniquely determined faithful semi-simple 3588-dimensional modules of mH and mE over F corresponding to the restrictions  $\tau_{\mathbf{3}|mH}$  and  $\tau_{\mathbf{3}|mE}$  of the irreducible character  $\tau_3$  of  $G_1$ , respectively.

Let  $\kappa_{\mathfrak{V}}: mH \to \mathrm{GL}_{3588}(13)$  and  $\kappa_{\mathfrak{W}}: mE \to \mathrm{GL}_{3588}(13)$  be the representations of mH and mE afforded by the modules  $\mathfrak{V}$  and  $\mathfrak{W}$ , respectively.

Let 
$$\mathfrak{r} = \kappa_{\mathfrak{Y}}(r)$$
,  $\mathfrak{u} = \kappa_{\mathfrak{Y}}(u)$ ,  $\mathfrak{v} = \kappa_{\mathfrak{Y}}(v)$  in  $\kappa_{\mathfrak{Y}}(mH) \leq \mathrm{GL}_{3588}(13)$ .

Then  $\mathfrak{V}_{|mD} \cong \mathfrak{W}_{|mD}$ , and there is a transformation matrix  $\mathcal{T}_1 \in GL_{3588}(13)$  such that

$$\mathfrak{u} = \mathcal{T}_1^{-1} \kappa_{\mathfrak{W}}(u) \mathcal{T}_1, \mathfrak{v} = \mathcal{T}_1^{-1} \kappa_{\mathfrak{W}}(v) \mathcal{T}_1.$$

Let  $\mathfrak{mD} = \langle \mathfrak{u}, \mathfrak{v} \rangle$ ,  $\mathfrak{mS} = \langle \mathfrak{u}, \mathfrak{v}, \mathfrak{r} \rangle$ . Let  $\mathcal{D} = C_Y(\mathfrak{mD})$  and  $\mathcal{H} = C_Y(\mathfrak{mS})$ . Let  $\mathfrak{s}_1 = \mathcal{T}_1^{-1} \kappa_{\mathfrak{W}}(s) \mathcal{T}_1$ . Let  $\mathfrak{mE} = \langle \mathfrak{mD}, \mathfrak{s}_1 \rangle$  and  $\mathcal{E} = C_Y(\mathfrak{mE})$ . Then the following statements hold:

(a) There is an isomorphism

$$\alpha: \mathcal{D} \to \mathcal{D}_1 = \operatorname{GL}_2(13) \times \operatorname{GL}_2(13) \times F^{*5} \leq \operatorname{GL}_9(13).$$

(b)  $\mathcal{H}_1 = \alpha(\mathcal{H})$  is generated by the two blocked diagonal matrices

(c)  $\mathcal{E}_1 = \alpha(\mathcal{E})$  is generated by the five blocked diagonal matrices

$$b_3 = diag(\left(\begin{smallmatrix} 1 & 0 \\ 0 & 2 \end{smallmatrix}\right), \left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right), 1, 2, 1, 1, 1), \quad b_4 = diag(\left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right), \left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right), 2, 1, 1, 2, 1),$$

and 
$$b_5 = diag(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, 1, 1, 2, 1, 2).$$

- (d)  $\mathcal{D}$  has  $2184^2 \times 12 = 57238272 \,\mathcal{H}\text{-}\mathcal{E}$  double cosets.
- (e) The free product  $mH *_{mD} mE$  of mH and mE with amalgamated subgroup mD has an irreducible 3588-dimensional representation over F which induces an irreducible representation of  $G_1$ . It corresponds to the  $\mathcal{H}$ - $\mathcal{E}$  double coset representative

$$\mathcal{F} = diag(w, z, 1^{182}, 1^{182}, 1^{364}, 1^{728}, 1^{1664}) \in GL_{3588}(13), \quad where$$

$$w = \left( \begin{array}{c} 1^{78} \ 4^{78} \\ 12^{78} \ 2^{78} \end{array} \right), \quad z = \left( \begin{array}{c} 1^{156} \ 6^{156} \\ 5^{156} \ 10^{156} \end{array} \right),$$

and  $a^n$  denotes a diagonal  $n \times n$  matrix with unique diagonal non zero entry  $a \in GF(13)$ .

Let  $\mathfrak{s}=\mathcal{F}^{-1}\mathfrak{s}_1\mathcal{F}$  and  $\mathfrak{G}_1=\langle\mathfrak{u},\mathfrak{v},\mathfrak{r},\mathfrak{s}\rangle$ . Inserting these four generating matrices of  $\mathfrak{G}_1$  into the formulas of Lemma 4.1(f) one obtains the matrices  $\mathfrak{x}_{3588}$ ,  $\mathfrak{y}_{3588}$ ,  $\mathfrak{q}_{3588}$ , and  $\mathfrak{w}_{3588}$  of the original generators x, y, q and w of  $G_1=\mathrm{Fi}_{23}$  as words in the generators u, v, r and s. The matrices  $\mathfrak{x}_{3588}$ ,  $\mathfrak{q}_{3588}$ ,  $\mathfrak{q}_{3588}$ , and  $\mathfrak{w}_{3588}$  can be downloaded from the first author's website http://www.math.yale.edu/~hk47/Fi24/index.html.

*Proof.* Let  $\mathfrak{V}$  be the up to isomorphism uniquely determined faithful semi-simple 3588-dimensional module of mH over F = GF(13) corresponding to the restriction  $\tau_{\mathbf{3}|mH}$ . By Lemma 4.1(g)  $\tau_{\mathbf{3}|mH} = \pi_8 + \pi_{16}$ . Lemma 4.1(j) states that  $\pi_8 = \pi_2 \otimes \pi_9$ .

The irreducible characters  $\pi_9$  and  $\pi_{15}$  of mH are constituents of the permutation characters  $1_{mH_9}^{mH}$  and  $1_{mH_{15}}^{mH}$  of the respective subgroups  $mH_9$  and  $mH_{15}$  of mH determined in Lemma 4.1(i). Using a stand-alone program of the first author which is based on Algorithm 5.7.1 of [12] we calculated the primitive idempotents of the endomorphism rings of these permutation modules. Thus we obtained the corresponding irreducible representations  $M(\pi_9)$  and  $M(\pi_{15})$  of the respective dimensions 780 and 2808 over F. The irreducible FmH-modules  $M(\pi_8)$  and  $M(\pi_{16})$  are the tensor products of  $M(\pi_9)$  and  $M(\pi_{15})$  with the linear character  $\pi_2$  of mH over F. Thus  $\mathfrak{V} = M(\pi_8) \oplus M(\pi_{16})$ .

Let  $\mathfrak{W}$  be the up to isomorphism uniquely determined faithful semi-simple 3588-dimensional module of mE over F corresponding to the restriction  $\tau_{\mathbf{3}|mE}$ . By Lemma 4.1(g)  $\tau_{\mathbf{3}|mE} = \psi_4 + \psi_{10} + \psi_{19} + \psi_{36} + \psi_{61}$ . Lemma 4.1(m) states that  $\psi_4 = \psi_2 \otimes \pi_5$  and  $\psi_{19} = \psi_2 \otimes \pi_{22}$ .

The irreducible characters  $\psi_5$  and  $\psi_{10}$  of mE are constituents of the permutation character  $1_{mE_1}^{mE}$  by 4.1(k). The irreducible characters  $\psi_{22}$ ,  $\psi_{36}$  and  $\psi_{61}$  of mE are constituents of the permutation characters  $1_{mE_{22}}^{mE}$ ,  $1_{mE_{36}}^{mE}$  and  $1_{mE_{61}}^{mE}$ , respectively, see Lemma 4.1(m). Using a standalone program of the first author which is based on Algorithm 5.7.1 of [12] we calculated the primitive idempotents of the endomorphism rings of these four permutation modules. Thus we obtained the corresponding irreducible representations  $N(\psi_5)$ ,  $N(\psi_{10})$ ,  $N(\psi_{22})$ ,  $N(\psi_{36})$  and  $N(\psi_{61})$  of the respective dimensions 78, 156, 260, 910 and 2184 over F. The irreducible FmE-modules  $N(\psi_4)$  and  $N(\psi_{19})$  are the tensor products of  $N(\psi_5)$  and  $N(\psi_{22})$  with the linear character  $\psi_2$  of mE over F. Thus

$$\mathfrak{W} = N(\psi_4) \oplus N(\psi_{10}) \oplus N(\psi_{19}) \oplus N(\psi_{36}) \oplus N(\psi_{61}).$$

Fixing a basis in each irreducible constituent  $M\pi_k$  of  $\mathfrak{V}$  we get a basis  $\mathcal{B}_V$  of  $\mathfrak{V}$ . It induces a representation  $\kappa_{\mathfrak{V}}: mH \to \mathrm{GL}_{3588}(13)$  of mH. Let  $\mathfrak{r} = \kappa_{\mathfrak{V}}(r)$ ,  $\mathfrak{u} = \kappa_{\mathfrak{V}}(u)$ ,  $\mathfrak{v} = \kappa_{\mathfrak{V}}(v)$  in  $\kappa_{\mathfrak{V}}(mH) \leq \mathrm{GL}_{3588}(13)$ .

Fixing a basis in each irreducible constituent  $N\psi_j$  of  $\mathfrak{W}$  we get a basis  $\mathcal{B}_W$  of  $\mathfrak{W}$ . It induces a representation  $\kappa_{\mathfrak{W}}: mE \to \mathrm{GL}_{3588}(13)$  of mE. By Lemma 4.1(g)  $\mathfrak{V}_{|mD} \cong \mathfrak{W}_{|mD}$ . Let  $Y = \mathrm{GL}_{3588}(13)$ . Applying now Parker's isomorphism test of Proposition 6.1.6 of [12] by means of the MAGMA command

IsIsomorphic(GModule(sub<Y|V(u),V(v)>),GModule(sub<Y|W(u),W(v)>)) one obtains the transformation matrix  $\mathcal{T}_1$  satisfying  $\mathfrak{u} = \kappa_{\mathfrak{W}}(u)^{\mathcal{T}_1}$  and  $\mathfrak{v} = \kappa_{\mathfrak{W}}(v)^{\mathcal{T}_1}$ .

(a) Let  $\mathfrak{mD} = \langle \mathfrak{u}, \mathfrak{v} \rangle$ ,  $\mathfrak{mS} = \langle \mathfrak{u}, \mathfrak{v}, \mathfrak{r} \rangle$ . Let  $\mathcal{D} = C_Y(\mathfrak{mD})$  and  $\mathcal{H} = C_Y(\mathfrak{mS})$ . Let  $\delta_{12a}$ ,  $\delta_{12b}$  and  $\delta_{23a}$ ,  $\delta_{23b}$  be two distinct copies of the irreducible characters  $\delta_{12}$  and  $\delta_{23}$  of mD, respectively. Using  $PG_1$  and MAGMA we checked that the irreducible characters  $\pi_8$  and  $\pi_{16}$  of mH have the following restrictions to  $mD = \langle u, v \rangle$ :

$$\pi_{8|mD} = \delta_{12a} + \delta_{23a} + \delta_{27} + \delta_{39}, \quad \pi_{16|mD} = \delta_{12b} + \delta_{23b} + \delta_{29} + \delta_{54} + \delta_{69},$$

where the irreducible characters  $\delta_{12}$ ,  $\delta_{23}$ ,  $\delta_{27}$ ,  $\delta_{29}$ ,  $\delta_{39}$ ,  $\delta_{54}$ , and  $\delta_{69}$  of  $mD \cong G_2(3) \times S_3$  have degrees 78, 156, 182, 182, 364, 728 and 1664, respectively.

- (b) Furthermore, Schur's Lemma asserts that  $\mathcal{H}_1 = \alpha(\mathcal{H})$  is generated by the two blocked diagonal matrices given in the statement because 2 is a primitive element of the multiplicative group  $F^*$  of F = GF(13).
- (c) Let  $\mathfrak{s}_1 = \mathcal{T}_1^{-1} \kappa_{\mathfrak{W}}(s) \mathcal{T}_1$ . Let  $\mathfrak{mE} = \langle \mathfrak{mD}, \mathfrak{s}_1 \rangle$  and  $\mathcal{E} = C_Y(\mathfrak{mE})$ . Let  $\delta_{12a}$ ,  $\delta_{12b}$  and  $\delta_{23a}$ ,  $\delta_{23b}$  are two distinct copies of the irreducible characters  $\delta_{12}$  and  $\delta_{23}$  of mD, respectively. Using  $PG_1$  and MAGMA we checked that the irreducible characters  $\psi_4$ ,  $\psi_{10}$ ,  $\psi_{19}$ ,  $\psi_{36}$  and  $\pi_{61}$  of mE have the following restrictions to  $mD = \langle u, v \rangle$ :

$$\begin{split} & \psi_{4|_{m}D} = \delta_{12a}, \quad \psi_{10|_{m}D} = \delta_{23a}, \quad \psi_{19|_{m}D} = \delta_{12b} + \delta_{29}, \\ & \psi_{36|_{m}D} = \delta_{27} + \delta_{54}, \quad \psi_{61|_{m}D} = \delta_{23b} + \delta_{39} + \delta_{69}. \end{split}$$

Now Schur's Lemma implies that  $\mathcal{E}_1 = \alpha(\mathcal{E})$  is generated by the five blocked diagonal matrices  $b_j$  given in the statement.

- (d) Every  $\mathcal{H}$ - $\mathcal{E}$  double coset representative is of the form diag(A,B,1,1,1,1,v) for some  $A,B\in Y$  and  $v\in F^*$ . By multiplying from left and right, we observe that diag(A,B,1,1,1,1,v) and diag(A',B',1,1,1,1,v) represent the same double coset if and only if the first columns of A and B are each a scalar multiple of the first columns of A' and B', respectively. So, we have 12 choices for v, and  $|\operatorname{GL}(2,13)|/12\rangle = 2184$  choices for A and B. Thus there are  $2184^2 \cdot 12 = 57238272$   $\mathcal{H}$ - $\mathcal{E}$  double cosets.
- (e) By Theorem 7.2.2 of [12] the irreducible representations of the free product  $mH *_{mD} mE$  of the groups mH and mE with amalgamated subgroup mD are described by the  $\mathcal{H}\text{-}\mathcal{E}$  double coset representatives T of  $\mathcal{D}$ . The elements r and  $f = (u^3vsv)^9$  are two commuting involutions of  $G_1 \cong \mathrm{Fi}_{23}$  by Lemma 4.1(o). Let  $\mathfrak{u}$  and  $\mathfrak{f}$  be their matrices in  $\mathfrak{m}\mathfrak{H}$  and  $\mathfrak{m}\mathfrak{E}$ , respectively. If  $T = diag(\left(\begin{smallmatrix} a & c \\ b & d \end{smallmatrix}\right), \left(\begin{smallmatrix} p & t \\ q & u \end{smallmatrix}\right), 1, 1, 1, 1, 1, v)$  describes a 3588-dimensional representation of  $G_1$  over F then (\*)  $(\mathfrak{r}, \mathcal{T}^{-1}\mathfrak{f}\mathcal{T}) = 1$  holds, where  $\mathcal{T} \in \mathrm{GL}_{3588}(13)$  corresponds to T.

Since  $\mathfrak{V}_{|mD} \cong \mathfrak{W}_{|mD}$  is a direct sum of 9 irreducible FmD-modules both matrices  $\mathfrak{r}$  and  $\mathfrak{f}$  consist of 81 blocks  $R_{i,j}$  and  $F_{i,j}$ ,  $1 \leq i,j \leq 9$ , respectively, such that all diagonal blocks  $R_{i,i}$  and  $F_{i,i}$  are non zero. Furthermore a non diagonal block  $R_{i,j}$  of  $\mathfrak{r}$  is non zero if and only if the i-th irreducible and the j-th irreducible representations of mD appear in the restriction of an irreducible representation of mH to mD. A similar description holds for the blocks of  $\mathfrak{f}$ . Hence the system of equations in the proofs of (a) and (c) imply that

Let  $e = (ad - bc)^{-1}$  and  $g = (pu - tq)^{-1}$ . Then  $e \neq 0 \neq g$ . For each integer k let  $I_k$  denote the  $k \times k$  identity matrix over F. Then

Hence  $\mathfrak{f}' = \mathcal{T}^{-1}\mathfrak{f}\mathcal{T}$  equals the matrix

$$\begin{pmatrix} G_{1,1}G_{1,2} & \cdot & \cdot & -ecF_{2,6} & \cdot & \cdot & \cdot \\ G_{2,1}G_{2,2} & \cdot & \cdot & eaF_{2,6} & \cdot & \cdot & \cdot \\ \cdot & \cdot & G_{3,3}G_{3,4} & \cdot & -gtF_{4,7} & \cdot & -vgtF_{4,9} \\ \cdot & \cdot & G_{4,3}G_{4,4} & \cdot & gpF_{4,7} & vgpF_{4,9} \\ \cdot & \cdot & \cdot & F_{5,5} & \cdot & F_{5,8} & \cdot \\ \cdot & \cdot & \cdot & F_{5,5} & \cdot & F_{5,8} & \cdot \\ \cdot & \cdot & G_{7,3}G_{7,4} & \cdot & F_{7,7} & vF_{7,9} \\ \cdot & \cdot & \cdot & F_{8,5} & \cdot & F_{8,8} & \cdot \\ \cdot & \cdot & G_{9,3}G_{9,4} & \cdot & v^{-1}F_{9,7} & F_{9,9} \end{pmatrix},$$

where

$$\begin{split} G_{1,1} &= e(adF_{1,1} - bcF_{2,2}), \quad G_{1,2} = ecd(F_{1,1} - F_{2,2}), \\ G_{2,1} &= -eab(F_{1,1} - F_{2,2}), \quad G_{2,2} = -e(bcF_{1,1} - adF_{2,2}), \\ G_{6,1} &= bF_{6,2}, \quad G_{6,2} = dF_{6,2}, \\ G_{3,3} &= g(puF_{3,3} - qtF_{4,4}), \\ G_{3,4} &= gut(F_{3,3} - F_{4,4}), \\ G_{4,3} &= -gpq(F_{3,3} - F_{4,4}), \\ G_{4,4} &= -g(tqF_{3,3} - puF_{4,4}), \\ G_{7,3} &= qF_{7,4}, \quad G_{7,4} = uF_{7,4}, \\ G_{9,3} &= qv^{-1}F_{9,4}, \quad G_{9,4} = uv^{-1}F_{9,4}. \end{split}$$

Now (\*) implies the following equations

$$(1,1): G_{1,1}R_{1,1} = R_{1,1}G_{1,1},$$

$$(1,2): G_{1,2}R_{2,2} + (-ec)F_{2,6}R_{6,2} = R_{1,1}G_{1,2},$$

$$(6,1): G_{6,1}R_{1,1} = R_{6,2}G_{2,1} + R_{6,6}G_{6,1},$$

$$(8,1): F_{8,5}R_{5,1} = R_{8,2}G_{2,1} + R_{8,6}G_{6,1}.$$

Inserting the previous equations into these 4 equations yields:

$$\begin{split} &(1,1): e(adF_{1,1}-bcF_{2,2})R_{1,1} = eR_{1,1}(adF_{1,1}-bcF_{2,2}),\\ &(1,2): ec(d(F_{1,1}-F_{2,2})R_{2,2}-F_{2,6}R_{6,2}) = ecdR_{1,1}(F_{1,1}-F_{2,2}),\\ &(6,1): bF_{6,2}R_{1,1} = b(-eaR_{6,2}(F_{1,1}-F_{2,2}) + R_{6,6}F_{6,2}),\\ &(8,1): F_{8,5}R_{5,1} = b(-eaR_{8,2}(F_{1,1}-F_{2,2}) + R_{8,6}F_{6,2}). \end{split}$$

Since  $e^{-1} = ad - bc \neq 0$ , at least one of ad or bc is nonzero. Suppose ad is zero, and bc is nonzero. Then (1,1) yields  $F_{1,1}R_{1,1} = R_{1,1}F_{1,1}$ . A MAGMA calculation disproves this equation. Hence  $ad \neq 0$ . If bc is zero and ad is nonzero, then (1,1) equation implies  $F_{2,2}R_{1,1} = R_{1,1}F_{2,2}$  which is also wrong by MAGMA. Therefore all a, b, c, d are nonzero. We modify T so that a = 1 by multiplying some power of  $diag(\left( \begin{smallmatrix} 2 & 0 \\ 0 & 1 \end{smallmatrix}\right), \left( \begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right), 1, 1, 1, 1, 1, 1$  from the right.

Since ec is nonzero, it can be cancelled on both sides of equation (1,2). Hence

$$(1,2): d(F_{1,1}-F_{2,2})R_{2,2}-F_{2,6}R_{6,2}=dR_{1,1}(F_{1,1}-F_{2,2}).$$

Using MAGMA it can be verified that this equation holds only for d=2. As  $b\neq 0$  and a=1 equation (6,1) implies

$$F_{6,2}R_{1,1} = -eR_{6,2}(F_{1,1} - F_{2,2}) + R_{6,6}F_{6,2}.$$

By MAGMA it has the solution e = 11. Now equation (8,1) implies that

$$F_{8.5}R_{5.1} = b(-11R_{8.2}(F_{1.1} - F_{2.2}) + R_{8.6}F_{6.2}).$$

This equation has the solution b = 4 by MAGMA. From the equation  $ad - bc = e^{-1}$  we now deduce that c = 12.

In order to determine the remaining coefficients of the matrix T we use the following matrix equations derived from (\*).

$$(9,8): G_{9,4}R_{4,8} + F_{9,9}R_{9,8} = R_{9,8}F_{8,8},$$

$$(9,9): G_{9,4}R_{4,9} + F_{9,9}R_{9,9} = vgpR_{9,4}F_{4,9} + R_{9,9}F_{9,9},$$

$$(4,5): G_{4,3}R_{3,5} + gpF_{4,7}R_{7,5} = R_{4,8}F_{8,5}.$$

Inserting the first set of equations yields:

$$(9,8): uv^{-1}F_{9,4}R_{4,8} + F_{9,9}R_{9,8} = R_{9,8}F_{8,8},$$

$$(9,9): uv^{-1}F_{9,4}R_{4,9} + F_{9,9}R_{9,9} = vgpR_{9,4}F_{4,9} + R_{9,9}F_{9,9},$$

$$(4,5): -gpq(F_{3,3} - F_{4,4})R_{3,5} + gpF_{4,7}R_{7,5} = R_{4,8}F_{8,5}.$$

By MAGMA the equation (9,8) has the solution  $uv^{-1} = 10$ . Now MAGMA asserts that the equation (9,9) has the solution vgp = 11. Hence  $p \neq 0$ . By multiplying some power of  $diag(\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, 1, 1, 1, 1, 1)$  from the right we can modify T so that p = 1. Thus vg = 11 and the third equation (4,5) implies that

$$(4,5): g(-q(F_{3,3}-F_{4,4})R_{3,5}+F_{4,7}R_{7,5})=R_{4,8}F_{8,5}.$$

This is non linear equation in the unknowns q and g has a unique solution q = 6 and g = 11 as has been checked by running with MAGMA through all  $13^2$  cases. From  $6 = g^{-1} = pu - tq = 10 - 6t$  we now deduce that t = 5. This completes the determination of the coefficients of the two matrices w and z of statement (e). The remaining assertions are now clear.

**Proposition 4.3.** Keep the notation of Lemmas 3.1, 3.2 and 4.1. Let  $PG_1$  be the faithful permutation representation of the simple group  $G_1 = \langle x, y, q, w \rangle = \langle u, v, r, s \rangle$  of degree 31671 with stabilizer  $H_1 = \langle x, y, q \rangle$ . Let  $mH = \langle u, v, r \rangle$ ,  $mD = \langle u, v, r \rangle$  and  $mE = \langle u, v, s \rangle$ . Let  $F^*$  be the multiplicative group of the prime field F = GF(13). Let  $Y = GL_{5083}(13)$ .

Let  $\mathfrak{V}$  and  $\mathfrak{W}$  be the up to isomorphism uniquely determined faithful semi-simple 5083-dimensional modules of mH and mE over F corresponding to the restrictions  $\tau_{\mathbf{4}|mH}$  and  $\tau_{\mathbf{4}|mE}$  of the irreducible character  $\tau_{\mathbf{4}}$  of  $G_1$ , respectively.

Let  $\kappa_{\mathfrak{V}}: mH \to \mathrm{GL}_{5083}(13)$  and  $\kappa_{\mathfrak{W}}: mE \to \mathrm{GL}_{5083}(13)$  be the representations of mH and mE afforded by the modules  $\mathfrak{V}$  and  $\mathfrak{W}$ , respectively.

Let 
$$\mathfrak{r} = \kappa_{\mathfrak{V}}(r)$$
,  $\mathfrak{u} = \kappa_{\mathfrak{V}}(u)$ ,  $\mathfrak{v} = \kappa_{\mathfrak{V}}(v)$  in  $\kappa_{\mathfrak{V}}(mH) \leq \mathrm{GL}_{5083}(13)$ .

Then  $\mathfrak{V}_{|mD} \cong \mathfrak{W}_{|mD}$ , and there is a transformation matrix  $\mathcal{T} \in GL_{5083}(13)$  such that

$$\mathfrak{u} = \mathcal{T}^{-1} \kappa_{\mathfrak{W}}(u) \mathcal{T}, \mathfrak{v} = \mathcal{T}^{-1} \kappa_{\mathfrak{W}}(v) \mathcal{T}.$$

Let  $\mathfrak{mD} = \langle \mathfrak{u}, \mathfrak{v} \rangle$ ,  $\mathfrak{mS} = \langle \mathfrak{u}, \mathfrak{v}, \mathfrak{r} \rangle$ . Let  $\mathcal{D} = C_Y(\mathfrak{mD})$  and  $\mathcal{H} = C_Y(\mathfrak{mS})$ . Let  $\mathfrak{s}_1 = \mathcal{T}^{-1}\kappa_{\mathfrak{W}}(s)\mathcal{T}$ . Let  $\mathfrak{mE} = \langle \mathfrak{mD}, \mathfrak{s}_1 \rangle$  and  $\mathcal{E} = C_Y(\mathfrak{mE})$ . Then the following statements hold:

(a) There is an isomorphism

$$\alpha: \mathcal{D} \to \mathcal{D}_1 = \mathrm{GL}_2(13) \times F^{*8} \le \mathrm{GL}_{10}(13).$$

(b)  $\mathcal{H}_1 = \alpha(\mathcal{H})$  is generated by the two blocked diagonal matrices

$$a_1 = diag(\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}, 1, 2, 1, 2, 2, 1, 2, 1)$$
 and  $a_2 = diag(\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, 2, 1, 2, 1, 1, 2, 1, 2)$ ,

- (d)  $\mathcal{D}$  has  $2184 \times 12^4 = 4587424 \,\mathcal{H}\text{-}\mathcal{E}$  double cosets.
- (e) The free product  $mH *_{mD} mE$  of mH and mE with amalgamated subgroup mD has an irreducible 5083-dimensional representation over F which induces an irreducible representation of  $G_1$ . It corresponds to the  $\mathcal{H}$ - $\mathcal{E}$  double coset representative

$$\mathcal{F} = diag(w, 1^{78}, 1^{91}, 11^{156}, 6^{273}, 11^{273}, 1^{728}, 1^{1456}, 1^{1664}) \in \mathrm{GL}_{5083}(13), \quad where$$

$$w = \left(\begin{array}{c} 1^{182} \ 9^{182} \\ 11^{182} \ 9^{182} \end{array}\right),$$

and  $a^n$  denotes a diagonal  $n \times n$  matrix with unique diagonal non zero entry  $a \in GF(13)$ .

Let  $\mathfrak{s}=\mathcal{F}^{-1}\mathfrak{s}_1\mathcal{F}$  and  $\mathfrak{G}_1=\langle\mathfrak{u},\mathfrak{v},\mathfrak{r},\mathfrak{s}\rangle$ . Inserting these four generating matrices of  $\mathfrak{G}_1$  into the formulas of Lemma 4.1(f) one obtains the matrices  $\mathfrak{x}_{5083}$ ,  $\mathfrak{y}_{5083}$ ,  $\mathfrak{q}_{5083}$ , and  $\mathfrak{w}_{5083}$  of the original generators x, y, q and w of  $G_1=\mathrm{Fi}_{23}$  as words in the generators u, v, r and s. The matrices  $\mathfrak{x}_{5083}$ ,  $\mathfrak{y}_{5083}$ ,  $\mathfrak{q}_{5083}$ , and  $\mathfrak{w}_{5083}$  can be downloaded from the first author's website http://www.math.yale.edu/~hk47/Fi24/index.html.

*Proof.* Let  $\mathfrak{V}$  be the up to isomorphism uniquely determined faithful semi-simple 5083-dimensional module of mH over F = GF(13) corresponding to the restriction  $\tau_{4|mH}$ . By Lemma 4.1(g)  $\tau_{4|mH} = \pi_{12} + \pi_{15}$ .

The irreducible characters  $\pi_{11}$  and  $\pi_{15}$  of mH are constituents of the permutation characters  $1_{mH_{11}}^{mH}$  and  $1_{mH_{15}}^{mH}$  of the respective subgroups  $mH_{11}$  and  $mH_{15}$  of mH determined in Lemma 4.1(i). Using a stand-alone program of the first author which is based on Algorithm 5.7.1 of [12] we calculated the primitive idempotents of the endomorphism rings of these permutation modules. Thus we obtained the corresponding irreducible representations  $M(\pi_{11})$  and  $M(\pi_{15})$  of the respective dimensions 2275 and 2808 over F = GF(13). Hence the irreducible FmH-module  $M(\pi_{12})$  is the tensor product of  $M(\pi_{11})$  with the linear character  $\pi_2$  of mH over F. Thus  $\mathfrak{V} = M(\pi_{12}) \oplus M(\pi_{15})$ .

Let  $\mathfrak{W}$  be the up to isomorphism uniquely determined faithful semi-simple 5083-dimensional module of mE over F corresponding to the restriction  $\tau_{\mathbf{4}|mE} = \psi_{14} + \psi_{22} + \psi_{45} + \psi_{74}$ , see Lemma 4.1(i).

The irreducible characters  $\psi_{14}$ ,  $\psi_{22}$ ,  $\psi_{45}$  and  $\psi_{74}$  of mE are constituents of the permutation characters  $1_{mE_{14}}^{mE}$ ,  $1_{mE_{22}}^{mE}$ ,  $1_{mE_{45}}^{mE}$  and  $1_{mE_{74}}^{mE}$ , respectively, by Lemma 4.1(l). Using a stand alone program of the first author which is based on Algorithm 5.7.1 of [12] we calculated the primitive idempotents of the endomorphism rings of these four permutation modules. Thus we obtained the corresponding irreducible representations  $N(\psi_{14})$ ,  $N(\psi_{22})$ ,  $N(\psi_{45})$  and  $N(\psi_{74})$  of the respective dimensions 182, 260, 1365 and 3276 over F. Hence

$$\mathfrak{W} = N(\psi_{14}) \oplus N(\psi_{22}) \oplus N(\psi_{45}) \oplus N(\psi_{74}).$$

Fixing a basis in each irreducible constituent  $M\pi_k$  of  $\mathfrak{V}$  we get a basis  $\mathcal{B}_V$  of  $\mathfrak{V}$ . It induces a representation  $\kappa_{\mathfrak{V}}: mH \to \mathrm{GL}_{5083}(13)$  of mH. Let  $\mathfrak{r} = \kappa_{\mathfrak{V}}(r)$ ,  $\mathfrak{u} = \kappa_{\mathfrak{V}}(u)$ ,  $\mathfrak{v} = \kappa_{\mathfrak{V}}(v)$  in  $\kappa_{\mathfrak{V}}(mH) \leq \mathrm{GL}_{5083}(13)$ .

Fixing a basis in each irreducible constituent  $N\psi_j$  of  $\mathfrak{W}$  we get a basis  $\mathcal{B}_W$  of  $\mathfrak{W}$ . It induces a representation  $\kappa_{\mathfrak{W}}: mE \to \mathrm{GL}_{5083}(13)$  of mE. By Lemma 4.1(h)  $\mathfrak{V}_{|mD} \cong \mathfrak{W}_{|mD}$ . Let  $Y = \mathrm{GL}_{5083}(13)$ . Applying now Parker's isomorphism test of Proposition 6.1.6 of [12] by means of the MAGMA command

Is Isomorphic (GModule(sub < Y | V(u), V(v) >), GModule(sub < Y | W(u), W(v) >))

one obtains the transformation matrix  $\mathcal{T}_1$  satisfying  $\mathfrak{u} = \kappa_{\mathfrak{M}}(u)^{\mathcal{T}_1}$  and  $\mathfrak{v} = \kappa_{\mathfrak{M}}(v)^{\mathcal{T}_1}$ .

(a) Let  $\mathfrak{mD} = \langle \mathfrak{u}, \mathfrak{v} \rangle$ ,  $\mathfrak{mH} = \langle \mathfrak{u}, \mathfrak{v}, \mathfrak{r} \rangle$ . Let  $\mathcal{D} = C_Y(\mathfrak{mD})$  and  $\mathcal{H} = C_Y(\mathfrak{mH})$ . Let  $\delta_{32a}$  and  $\delta_{32b}$  be two distinct copies of the irreducible character  $\delta_{32}$  of mD. Using  $PG_1$  and MAGMA it can be checked that the irreducible characters  $\pi_{12}$  and  $\pi_{15}$  of mH have the following restrictions to  $mD = \langle u, v \rangle$ :

$$\pi_{12|mD} = \delta_{16} + \delta_{32a} + \delta_{34} + \delta_{37} + \delta_{66},$$
  

$$\pi_{15|mD} = \delta_{11} + \delta_{23} + \delta_{32b} + \delta_{53} + \delta_{69},$$

where the irreducible characters  $\delta_{11}$ ,  $\delta_{16}$ ,  $\delta_{23}$ ,  $\delta_{32}$ ,  $\delta_{34}$ ,  $\delta_{37}$ ,  $\delta_{53}$ ,  $\delta_{66}$ , and  $\delta_{69}$  of  $mD \cong G_2(3) \times S_3$  have degrees 78, 91, 156, 182, 273, 273, 728, 1456 and 1664, respectively.

Since  $\mathfrak{V}_{|mD}$  is a semi-simple FmD-module the Theorem 2.1.27 of [12] implies that there is an isomorphism

$$\alpha: \mathcal{D} \to \mathcal{D}_1 = \operatorname{GL}_2(13) \times F^{*8} \le \operatorname{GL}_{10}(13).$$

- (b) Furthermore, Schur's Lemma asserts that  $\mathcal{H}_1 = \alpha(\mathcal{H})$  is generated by the two blocked diagonal matrices  $a_1$  and  $a_2$  given in the statement because 2 is a primitive element in the multiplicative group  $F^*$  of F = GF(13).
- (c) Let  $\mathfrak{s}_1 = \mathcal{T}_1^{-1} \kappa_{\mathfrak{W}}(s) \mathcal{T}_1$ . Let  $\mathfrak{mE} = \langle \mathfrak{mD}, \mathfrak{s}_1 \rangle$  and  $\mathcal{E} = C_Y(\mathfrak{mE})$ . Let  $\delta_{32a}$  and  $\delta_{32b}$  be two distinct copies of the irreducible character  $\delta_{32}$  of mD. Using  $PG_1$  and MAGMA it can be checked that the irreducible characters  $\psi_{14}$ ,  $\psi_{22}$ ,  $\psi_{45}$  and  $\psi_{74}$  of mE have the following restrictions to  $mD = \langle u, v \rangle$ :

$$\begin{split} &\psi_{14}{}_{|mD} = \delta_{32a}, \quad \psi_{22}{}_{|mD} = \delta_{11} + \delta_{32b}, \\ &\psi_{45}{}_{|mD} = \delta_{16} + \delta_{34} + \delta_{37} + \delta_{53}, \quad \psi_{74}{}_{|mD} = \delta_{23} + \delta_{66} + \delta_{69}. \end{split}$$

Now Schur's Lemma implies that  $\mathcal{E}_1 = \alpha(\mathcal{E})$  is generated by the four blocked diagonal matrices  $b_i$  given in the statement.

(d) Every  $\mathcal{H}$ - $\mathcal{E}$  double coset representative is of the form

$$diag(A, 1, v_2, v_3, v_4, v_5, 1, 1, 1)$$

for some  $A \in Y$  and  $v \in F^*$ . By multiplying from left and right, we can find out that  $diag(A, 1, v_2, v_3, v_4, v_5, 1, 1, 1)$  and  $diag(A', 1, v_2, v_3, v_4, v_5, 1, 1, 1)$  represent the same double coset if and only if the first column of A is a scalar multiple of that of A'. So, we have  $12^4$  choices for  $v_i$ , and  $|\operatorname{GL}(2, 13)|/12 = 2184$  choices for A. Thus there are  $2184 \cdot 12^4 = 4587424 \,\mathcal{H}\text{-}\mathcal{E}$  double cosets.

(e) By Theorem 7.2.2 of [12] the irreducible representations of the free product  $mH *_{mD} mE$  of the groups mH and mE with amalgamated subgroup mD are described by the  $\mathcal{H}$ - $\mathcal{E}$  double coset representatives T of  $\mathcal{D}$ . The elements r and  $f = (u^3vsv)^9$  are two commuting involutions of  $G_1 \cong \text{Fi}_{23}$  by Lemma 4.1(o). Let  $\mathfrak{r}$  and  $\mathfrak{f}$  be their matrices in  $\mathfrak{m}\mathfrak{H}$  and  $\mathfrak{m}\mathfrak{E}$ , respectively. If  $T_1 =$ 

 $diag(\begin{pmatrix} a & c \\ b & d \end{pmatrix}, 1, v_2, v_3, v_4, v_5, 1, 1, 1)$  describes a 5083-dimensional irreducible representation of  $G_1$  over F then (\*\*)  $(\mathfrak{r}, \mathcal{T}_1^{-1}\mathfrak{f}\mathcal{T}_1) = 1$  holds.

Since  $\mathfrak{V}_{|mD} \cong \mathfrak{W}_{|mD}$  is a direct sum of 10 irreducible FmD-modules both matrices  $\mathfrak{r}$  and  $\mathfrak{f}$  consist of 100 blocks  $R_{i,j}$  and  $F_{i,j}$ ,  $1 \leq i,j \leq 10$  such that all diagonal blocks  $R_{i,i}$  and  $F_{i,i}$  are nontrivial. Furthermore, a non diagonal block  $R_{i,j}$  of  $\mathfrak{r}$  is non zero if and only if the i-th irreducible and the j-th irreducible representations of mD appear in the restriction of an irreducible representation of mH to mD. A similar description holds for the blocks of  $\mathfrak{f}$ . Hence the system of equations in the proofs of (a) and (c) imply that

Let  $e = (ad - bc)^{-1}$  and  $g = (pu - tq)^{-1}$ . Then  $e \neq 0 \neq g$ . For each integer k let  $I_k$  denote the  $k \times k$  identity matrix over F. Then

Hence  $f' = T^{-1}fT$  equals the matrix

where

$$\begin{split} G_{1,1} &= e(adF_{1,1} - bcF_{2,2}), \quad G_{1,2} = ecd(F_{1,1} - F_{2,2}), \quad G_{1,3} = -ecF_{2,3}, \\ G_{2,1} &= -eab(F_{1,1} - F_{2,2}), \quad G_{2,2} = -e(bcF_{1,1} - adF_{2,2}), \quad G_{2,3} = eaF_{2,3}, \\ G_{4,6} &= v_2^{-1}v_4F_{4,6}, \quad G_{4,7} = v_2^{-1}v_5F_{4,7}, \quad G_{6,4} = v_4^{-1}v_2F_{6,4}, \\ G_{6,7} &= v_4^{-1}v_5F_{6,7}, \quad G_{7,4} = v_5^{-1}v_2F_{7,4}, \quad G_{7,6} = v_5^{-1}v_4F_{7,6}. \end{split}$$

Now (\*\*) implies the following equations

$$(10,1): F_{10,9}R_{9,1} = R_{10,2}G_{2,1} + R_{10,3}(bF_{3,2}),$$

$$(9,1): F_{9,9}R_{9,1} = R_{9,1}G_{1,1},$$

$$(9,2): v_3F_{9,5}R_{5,2} + F_{9,10}R_{10,2} = R_{9,1}G_{1,2},$$

$$(8,1): v_2F_{8,4}R_{4,1} + v_4F_{8,6}R_{6,1} + v_5F_{8,7}R_{7,1} = R_{8,2}G_{2,1} + R_{8,3}(bF_{3,2}).$$

Inserting the first set of equations yields:

$$(10,1): F_{10,9}R_{9,1} = b[-eaR_{10,2}(F_{1,1} - F_{2,2}) + R_{10,3}F_{3,2}],$$

$$(9,1): F_{9,9}R_{9,1} = eR_{9,1}(adF_{1,1} - bcF_{2,2}),$$

$$(9,2): v_3F_{9,5}R_{5,2} + F_{9,10}R_{10,2} = ecdR_{9,1}(F_{1,1} - F_{2,2}).$$

$$(8,1): v_2F_{8,4}R_{4,1} + v_4F_{8,6}R_{6,1} + v_5F_{8,7}R_{7,1} = b[-eaR_{8,2}(F_{1,1} - F_{2,2}) + R_{8,3}F_{3,2}].$$

Equation (10,1) has only one solution b=9, ea=1 in F=GF(13) as has been checked by means of MAGMA. Hence  $a\neq 0$ . By multiplying some power of  $diag(\left(\begin{smallmatrix} 2&0\\0&1\end{smallmatrix}\right),1,1,1,1,1,1,1,1)$  from the right we can modify the matrix T so that a=1. Thus also e=1. Now equation (9,1) becomes

$$(9,1): F_{9,9}R_{9,1} = R_{9,1}(dF_{1,1} - 9cF_{2,2}).$$

Another MAGMA calculation running through  $13^2$  pairs (c, d) of elements of F shows that (9, 1) has the unique solution c = 11 and d = 9.

Inserting the known values for c, d and e into equation (9,2) yields that  $v_3 = 11$  as has been checked by means of MAGMA. Finally, inserting the known values for a, b and e into equation (8,1) yields

$$(8,1): v_2F_{8,4}R_{4,1} + v_4F_{8,6}R_{6,1} + v_5F_{8,7}R_{7,1} = 9[-R_{8,2}(F_{1,1} - F_{2,2}) + R_{8,3}F_{3,2}].$$

A MAGMA calculation shows that this equation has the unique solution  $v_2 = 1$ ,  $v_4 = 6$ ,  $v_5 = 11$ . This completes the proof because all remaining statements of (e) are straightforward.

### 5. Construction of the irreducible subgroup $\mathfrak{G}$ of $\mathrm{GL}_{8671}(13)$

In this section we construct the 8 semi-simple representations of the 2-fold cover  $A_1$  of the automorphism group  $\operatorname{Aut}(\operatorname{Fi} 22)$  of  $\operatorname{Fi}_{22}$  corresponding to the 8 compatible pairs of Lemma 3.2(e). Since  $H_1 = \langle q, y \rangle = C_{G_1}(u) \cong A'_1$  for some involution u of  $G_1 = \langle q, y, w \rangle$  and  $A'_1$  has index 2 in  $A_1$  it is not difficult to construct the irreducible constituents of these representations of  $A_1$  from the 3588-dimensional irreducible representation of  $G_1 \cong \operatorname{Fi}_{23} = \langle q, y, w \rangle$  by means of Clifford's Theorem. In fact, we construct 8 new matrices  $t_i$  of order 2 such that  $K_i = \langle G_1, t_i \rangle$  is an irreducible subgroup of  $\operatorname{GL}_{8671}(13)$ . It turns out that only  $K_3$  corresponding to the compatible pair (3) of Lemma 3.2(e) may have a Sylow 2-subgroup which is isomorphic to a Sylow 2-subgroup of the extension group E of Lemma 2.1.

**Proposition 5.1.** Let  $\mathfrak{G}_1 = \langle \mathfrak{y}_{3588}, \mathfrak{q}_{3588}, \mathfrak{w}_{3588} \rangle$  be the simple subgroup of  $Y = \operatorname{GL}_{3588}(13)$  of order  $2^{18} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 23$  constructed in Proposition 4.2. Let  $y_1 = \mathfrak{y}_{3588}$ ,  $q_1 = \mathfrak{q}_{3588}$ . Let  $\mathfrak{H}_1 = \langle y_1, q_1 \rangle$ .

Let  $A_1 = 2Aut(Fi_{22}) = \langle a, b, c, d, e, f, g, h, i, z, t \rangle$  be the finitely presented group defined in Lemma 3.1. Then the following assertions hold:

- (a) There is an isomorphism  $\phi$  from the subgroup  $H_1 = \langle a, b, c, d, e, f, g, h, i, z \rangle$  of  $A_1$  to  $\mathfrak{H}_1$ .
- (b)  $A_1 = \langle y, q, t \rangle$ , where  $y = \phi^{-1}(y_1)$  and  $q = \phi^{-1}(q_1)$ .
- (c) There is a transformation matrix  $T \in Y$  such that

$$\mathcal{T}^{-1}\mathfrak{g}\mathcal{T} = diag(\mathfrak{g}_{78}, \mathfrak{g}_{1430}, \mathfrak{g}_{2080}) \in Y,$$
  
 $\mathcal{T}^{-1}\mathfrak{g}\mathcal{T} = diag(\mathfrak{g}_{78}, \mathfrak{g}_{1430}, \mathfrak{g}_{2080}) \in Y,$ 

where  $y_k, q_k \in GL_k(13)$  for  $k \in \{78, 1430, 2080\}$ .

(d)  $A_1$  has a faithful irreducible representation  $\lambda: A_1 \to \operatorname{GL}_{4160}(13)$  such that

$$\lambda(y) = diag(\mathfrak{y}_{2080}, \phi(y^t)_{2080}) \in GL_{4160}(13)),$$
  
$$\lambda(q) = diag(\mathfrak{y}_{2080}, \phi(q^t)_{2080}) \in GL_{4160}(13)),$$
  
$$\lambda(t) = \begin{pmatrix} 0 & I_{2080} \\ I_{2080} & 0 \end{pmatrix}.$$

where  $I_{2080}$  denotes the identity matrix of  $GL_{2080}(13)$ .

(e) The irreducible characters  $\chi_3$ ,  $\chi_4$ ,  $\chi_{11}$  and  $\chi_{12}$  of Table B.5 of respective degrees 78, 78, 1430 and 1430 are constituents of the permutation character  $1_{U}^{A_1}$  of the subgroup

$$U = \langle (q_1^2 y_1^3 q_1 y_1^3)^4, (y_1^2 q_1 y_1^3 q_1^2 y_1 q_1)^6, (y_1^4 q_1 y_1 q_1 y_1 q_1 y_1^2)^2 \rangle$$

of  $A_1$  of index 2358720.

(f) The tensor product  $\chi_3 \otimes \chi_3$  contains a copy of the irreducible character  $\chi_{13}$ . Furthermore,  $\chi_{14} = \chi_{13} \otimes \chi_2$ , where  $\chi_2$  is the non trivial linear character of  $A_1$ .

*Proof.* (a) In the simple subgroup  $\mathfrak{G}_1$  of  $Y = GL_{3588}(13)$  let

```
\begin{aligned} x_1 &= [(y_1q_1^2y_1q_1y_1q_1^2)^{11}(q_1^2y_1^2q_1y_1q_1y_1)^{11}(q_1y_1^2q_1y_1q_1y_1q_1y_1q_1)^4]^{12}, \\ a_1 &= (x_1y_1x_1)^7, \quad b_1 = [(q_1y_1)^2q_1y_1^3q_1^2y_1^3q_1y_1]^7, \quad c_1 = (y_1^2x_1y_1x_1y_1^3)^5, \\ d_1 &= (q_1y_1q_1^2y_1q_1y_1q_1y_1^2q_1^2)^{15}, \quad e_1 = (y_1x_1y_1^5x_1)^5, \\ f_1 &= (y_1q_1y_1q_1^2y_1^2q_1y_1^4q_1^2)^5, \quad g_1 = (x_1y_1^2x_1y_1^3x_1)^7, \\ h_1 &= (y_1^5x_1y_1x_1)^5, \quad i_1 = (q_1^2y_1^2q_1y_1q_1^2)^7. \end{aligned}
```

By Lemma 3.2 and Proposition 4.2 the matrix subgroup  $G_1 = \langle x, y, q, w \rangle$  of  $GL_{782}(17)$  and the matrix subgroup  $\mathfrak{G}_1 = \langle \mathfrak{y}_{3588}, \mathfrak{q}_{3588}, \mathfrak{w}_{3588} \rangle$  are isomorphic under the map  $\theta$  sending x, y, q and w to  $x_1, y_1, q_1$  and  $w_{3588}$ , respectively. Thus Lemma 3.2 (b) implies that the normal subgroup  $H_1 = \langle a, b, c, d, e, f, g, h, i, z \rangle$  of  $A_1$  is isomorphic to  $\mathfrak{H}_1 = \langle a_1, b_1, c_1, d_1, e_1, f_1, g_1, h_1, i_1, z_1 \rangle$ , where  $z_1 = (x_1 y_1^2)^7$ . In particular, the map  $\phi: H_1 \to \mathfrak{H}_1$  sending  $a, b, \ldots, z$  to  $a_1, b_1, \ldots, z_1$  is an isomorphism such that  $x = \phi^{-1}(x_1), y = \phi^{-1}(y_1), q = \phi^{-1}(q_1)$ .

Statement (b) is an immediate consequence of (a) and Lemma 3.1(b).

- (c) The natural F-vector space  $V=F^{3588}$  is an  $F\mathfrak{H}_1$ -module because  $\mathfrak{H}_1=\langle y_1,q_1\rangle$ . Applying the Meataxe algorithm it follows that V has three composition factors  $V_{78}$ ,  $V_{1430}$  and  $V_{2080}$  of dimensions 78, 1430 and 2080, respectively. Since all three dimensions are divisible by 13 and 13 divides  $|\mathfrak{H}_1|=|2\operatorname{Fi}_{22}|$  to the first power only all three simple composition factors of V are projective  $F\mathfrak{H}$ -modules by Theorems 3.12.2 and 3.12.4 of [12]. Hence V is isomorphic to their direct sum. Thus (c) holds.
- (d) The group  $A_1$  of Lemma 3.1 has a unique irreducible character  $\chi_{17}$  of degree 4160 Table B.5. Clifford's Theorem 2.6.15 of [12] asserts that its restriction to  $H_1$  is a sum of two inequivalent irreducible characters  $\nu$  and  $\nu^t$  of degree 2080. In particular, the induced  $FA_1$ -module  $V_{2080}^{A_1}$  is the reduction modulo 13 of a lattice which affords the irreducible character  $\chi_{17}$  of  $A_1$ . It corresponds to the irreducible representation  $\lambda: A_1 \to \mathrm{GL}_{4160}(13)$  of 13-defect zero defined in statement (d). It is well defined because Lemma 3.1(b) implies that  $a^t = g^{-1}$ ,  $b^t = f^{-1}$ ,  $c^t = e^{-1}$ ,  $d^t = d^{-1}$ ,  $h^t = h^{-1}$  and  $i^t = i^{-1}$ . Therefore  $\phi(y)_{2080}$  and  $\phi(q)_{2080}$  are well defined by (a) and (c).
- (e) Using the MAGMA command LowIndexSubgroups (A\_1, m) we searched for conjugacy classes of subgroups U of index  $|A_1:U|=m$  such that  $\chi_k$  is an irreducible constituent of the permutation character  $1_U^{A_1}$  for  $k\in\{3,4,11,12\}$ . Thus we found a subgroup U of index m=2358720 such that its permutation character contains all four irreducible characters  $\chi_k$ . Its generators have been obtained by means of Kim's program GetShortGens(A\_1, U).

Statement (f) can be verified by means of the character table of  $A_1$ .

**Proposition 5.2.** Keep the notation of Lemma 3.1 and Propositions 4.2, 4.3. Let  $A_1 \leftarrow H_1 \rightarrow G_1$  be the amalgam constructed in Lemma 3.2, where  $G_1 \cong \text{Fi}_{23}$ . Let  $\sigma: H_1 \rightarrow A_1$  denote its monomorphism of  $H_1$  into  $G_1$ . Let  $Y = \text{GL}_{8671}(13)$ . Let  $\sigma(y) = diag(\mathfrak{y}_{3588}, \mathfrak{y}_{5083})$ ,  $\sigma(q) = diag(\mathfrak{q}_{3588}, \mathfrak{q}_{5083})$ ,  $\mathfrak{w}_1 = diag(\mathfrak{w}_{3588}, \mathfrak{w}_{5083})$  in Y. Let  $\mathfrak{H}_1 = \langle \sigma(y), \sigma(q) \rangle$  and  $\mathfrak{H}_1 = \langle \mathfrak{H}_1, \mathfrak{w}_1 \rangle$ . Keep the notation of Table B.5, Table 6.6.3 of [14] and of the character table of Fi<sub>23</sub>, see [4], its p. 178 - 179.

Then the following statements hold:

- (a) There is a compatible pair of characters
- $(\chi, \quad \tau) = (\chi_3 + \chi_{12} + \chi_{13} + \chi_{17}, \quad \tau_3 + \tau_4) \in mf \, char_{\mathbb{C}}(A_1) \times mf \, char_{\mathbb{C}}(G_1)$ of degree 8671 of the groups  $A_1 = \langle H_1, t \rangle$  and  $G_1 = \langle H_1, e_1 \rangle$  with common restriction

$$\tau_{|H_1} = \chi_{|H_1} = \delta_2 + \delta_6 + \delta_7 + \delta_8 + \delta_9 \in mfchar_{\mathbb{C}}(H_1),$$

where irreducible characters with bold face indices denote faithful irreducible characters.

(b) Let  $\mathfrak{V}$  and  $\mathfrak{W}$  be the up to isomorphism uniquely determined faithful semisimple multiplicity-free 8671-dimensional modules of  $A_1$  and  $G_1$  over F = GF(13) corresponding to the compatible pair  $\chi, \tau$ , respectively.

Let  $\kappa_{\mathfrak{V}}: H \to \mathrm{GL}_{8671}(13)$  and  $\kappa_{\mathfrak{W}}: E \to \mathrm{GL}_{8671}(13)$  be the representations of  $A_1$  and  $G_1$  afforded by the modules  $\mathfrak{V}$  and  $\mathfrak{W}$ , respectively.

Let  $\mathfrak{q} = \kappa_{\mathfrak{V}}(q)$ ,  $\mathfrak{y} = \kappa_{\mathfrak{V}}(y)$ ,  $\mathfrak{t} = \kappa_{\mathfrak{V}}(t)$  in  $\kappa_{\mathfrak{V}}(A_1) \leq \mathrm{GL}_{8671}(13)$ . Then the following assertions hold:

(1)  $\mathfrak{V}_{|\mathfrak{H}_1} \cong \mathfrak{W}_{|\mathfrak{H}_1}$ , and there is a transformation matrix  $\mathcal{T} \in GL_{8671}(13)$  such that

$$\mathfrak{q} = \mathcal{T}^{-1} \kappa_{\mathfrak{W}}(\sigma(q)) \mathcal{T}, \mathfrak{y} = \mathcal{T}^{-1} \kappa_{\mathfrak{W}}(\sigma(y))) \mathcal{T}.$$

Let  $\mathfrak{w} = \mathcal{T}^{-1} \kappa_{\mathfrak{W}}(w_1) \mathcal{T} \in GL_{8671}(13)$ .

(2) The subgroup  $\mathfrak{G} = \langle \mathfrak{q}, \mathfrak{p}, \mathfrak{t}, \mathfrak{w} \rangle$  of Y is the uniquely determined irreducible representation of the free product  $P = G_1 *_{H_1} A_1$  of  $G_1$  and  $A_1$  with amalgamated subgroup  $H_1$  corresponding to the compatible pair (3) of Lemma 3.2(e). Its four generating matrices can be downloaded from the first author's website

http://www.math.yale.edu/~hk47/Fi24/index.html.

- *Proof.* (a) By Lemma 3.2(e) the amalgam  $A_1 \leftarrow H_1 \rightarrow G_1$  has 8 compatible pairs of degree 8671. We constructed the corresponding semi-simple representations of  $P = G_1 *_{H_1} A_1$  for each of them. But we give a proof only for that pair (3) of Lemma 3.2(e). It belongs to a group of a suitable order.
- (b) By Propositions 4.2 and 4.3 the semi-simple  $FG_1$ -module  $\mathfrak{W}=\mathfrak{W}_3\oplus\mathfrak{W}_4$  of dimension 8671 corresponding to the multiplicity free character  $\tau_3+\tau_4$  is described by the three matrices  $\mathfrak{y}=\sigma(y),\ \mathfrak{q}=\sigma(q),\ \mathfrak{w}$  of Y. The semi-simple  $FA_1$ -module  $\mathfrak{V}$  of the same dimension corresponding to the multiplicity free character  $\chi_3+\chi_{12}+\chi_{14}+\chi_{17}$  of  $A_1$  is defined by three blocked diagonal matrices  $\mathfrak{q}=diag(\mathfrak{q}_3,\mathfrak{q}_{12},\mathfrak{q}_{13},\mathfrak{q}_{17}),\ \mathfrak{y}=diag(\mathfrak{y}_3,\mathfrak{y}_{12},\mathfrak{y}_{13},\mathfrak{y}_{17})$  and  $\mathfrak{t}=diag(\mathfrak{t}_3,\mathfrak{t}_{12},\mathfrak{t}_{13},\mathfrak{t}_{17})$  whose entries can be calculated by means of Proposition 5.1 as follows.

The tensor product  $\chi_3 \otimes \chi_3$  contains a copy of the irreducible character  $\chi_{13}$  by Proposition 5.1(f). Since  $\mathfrak{V}_3$  has dimension 78 the Meataxe algorithm implemented

in MAGMA can be applied to the tensor product  $\mathfrak{V}_3 \otimes \mathfrak{V}_3$ . This application provides the three  $3003 \times 3003$  matrices  $\mathfrak{q}_{13}$ ,  $\mathfrak{y}_{13}$  and  $\mathfrak{t}_{13}$  corresponding to the irreducible  $FA_1$ -module  $\mathfrak{V}_{13}$ . Hence  $\mathfrak{V} = \mathfrak{V}_3 \oplus \mathfrak{V}_{12} \oplus \mathfrak{V}_{13} \oplus \mathfrak{V}_{17}$ .

By (a) the restrictions  $\mathfrak{V}_{3|\mathfrak{H}_1}$ ,  $\mathfrak{V}_{12|\mathfrak{H}_1}$  and  $\mathfrak{V}_{13|\mathfrak{H}_1}$  to  $\mathfrak{H}_1$  are irreducible. By the proof of Proposition 5.1(d) we know that

$$\begin{split} \mathfrak{V}_{17\mid\mathfrak{H}_{1}} &= \mathfrak{V}_{2080} \oplus \mathfrak{V}_{2080} \otimes t, \\ \mathfrak{V}_{3\mid\mathfrak{H}_{1}} \oplus \mathfrak{V}_{12\mid\mathfrak{H}_{1}} \oplus \mathfrak{V}_{2080} &\cong \mathfrak{W}_{3\mid\mathfrak{H}_{1}}, \\ \mathfrak{V}_{13\mid\mathfrak{H}_{1}} \oplus \mathfrak{V}_{2080} \otimes t &\cong \mathfrak{W}_{4\mid\mathfrak{H}_{1}}. \end{split}$$

Let  $X_3 = \operatorname{GL}_{3588}(13)$ ,  $X_4 = \operatorname{GL}_{5083}(13)$ ,  $V_3 = \mathfrak{V}_{3|\mathfrak{H}_1} \oplus \mathfrak{V}_{12|\mathfrak{H}_1}$ ,  $V_4 = \mathfrak{V}_{13|\mathfrak{H}_1} \oplus \mathfrak{V}_{2080} \otimes t$ ,  $W_3 = \mathfrak{W}_{3|\mathfrak{H}_1}$  and  $W_4 = \mathfrak{W}_{4|\mathfrak{H}_1}$ . By the proof of Proposition 5.1(d)  $V_3 \cong W_3$  and  $V_4 \cong W_4$  as  $F\mathfrak{H}_1$ -modules of respective dimensions 3588 and 5083. Applying now Parker's isomorphism test of Proposition 6.1.6 of [12] by means of the MAGMA command

Is Isomorphic (GModule (sub<X\_i|V\_i(h),V\_i(y)>), GModule (sub<X\_i|W\_i(h),W\_i(y)>)),  $i \in \{3,4\}$ , one obtains the transformation matrices  $\mathcal{T}_3 \in X_3$  and  $\mathcal{T}_4 \in X_4$  such that

 $\mathcal{T} = diag(\mathcal{T}_3, \mathcal{T}_4) \in Y$  satisfies  $\mathfrak{q} = \kappa_{\mathfrak{W}}(\sigma(q))^{\mathcal{T}}$  and  $\mathfrak{y} = \kappa_{\mathfrak{W}}(\sigma(y))^{\mathcal{T}}$ . Let  $\mathfrak{w} = \mathcal{T}^{-1}\kappa_{\mathfrak{W}}(w_1)\mathcal{T} \in GL_{8671}(13)$ . Corollary 7.2.4 of [12] asserts that the matrix group  $\mathfrak{G} = \langle \mathfrak{q}, \mathfrak{y}, \mathfrak{t}, \mathfrak{w} \rangle$  is uniquely determined by the compatible given in

**Remark 5.3.** Using Proposition 5.1 we constructed for each of the eight compatible pairs (k) of Lemma 3.2(e) a matrix  $\mathfrak{t}_k$  for the new generator t of  $A_1 = \langle q, y, w, t \rangle$  using the methods of the proof of Proposition 5.2. Thus we obtained 8 subgroups  $\mathfrak{G}_k = \langle \mathfrak{q}, \mathfrak{y}, \mathfrak{t}_k, \mathfrak{w} \rangle$  of  $GL_{8671}(13)$ . In each of them we tried to calculate the orders of

the following products of the generators:

Group Name	$\mathfrak{wt}_k$	$\mathfrak{ywt}_k$	$\mathfrak{qwt}_k$	$\mathfrak{ywt}_k\mathfrak{q}$	$\mathfrak{gt}_k\mathfrak{wqt}_k$
$\mathfrak{G}_1$	12	fail	ı	_	
$\mathfrak{G}_2$	24	fail		_	
$\mathfrak{G}_3$	4	24	24	21	33
$\mathfrak{G}_4$	8	24	fail	_	_
$\mathfrak{G}_5$	8	24	fail	_	_
$\mathfrak{G}_6$	4	24	24	42	66
$\mathfrak{G}_7$	24	fail	_	_	_
$\mathfrak{G}_8$	12	fail	_	_	_

where "fail" means the the product has an order which is greater than 100. The group  $\mathfrak{G}$  of Proposition 5.2 is  $\mathfrak{G}_3$ . Looking at the orders of many random elements we saw that all such orders were bounded by 60. In particular,  $\mathfrak{p} = \mathfrak{y}^2 \cdot \mathfrak{t}_3 \cdot \mathfrak{w} \cdot \mathfrak{q}$  has order 29.

Therefore we prove in the remainder of the article that  $\mathfrak{G}$  is isomorphic to Fischer's simple group  $\mathrm{Fi}_{24}^{\prime}$ . Most likely,  $\mathfrak{G}_6$  is isomorphic to Fischer's non simple group  $\mathrm{Fi}_{24}$ . In the other cases we were not able to calculate the orders of non trivial words of the generators in reasonable time.

6. Isomorphism between  ${\mathfrak G}$  and Fischer's group  ${\rm Fi}_{24}'$ 

In this section we construct an isomorphism between the matrix group  $\mathfrak{G} = \langle \mathfrak{q}, \mathfrak{y}, \mathfrak{t}, \mathfrak{w} \rangle$  of Proposition 5.2 and the commutator subgroup of the finitely presented group G of Hall and Soicher, see [15], p.111. Hence  $\mathfrak{G}$  is isomorphic to Fischer's simple group  $\mathrm{Fi}_{24}'$ .

**Proposition 6.1.** Let  $\mathfrak{G} = \langle \mathfrak{q}, \mathfrak{y}, \mathfrak{t}, \mathfrak{w} \rangle$  be the subgroup of  $GL_{8671}(13)$  constructed in Proposition 5.2. Let  $\mathfrak{H}_1 = \langle \mathfrak{p}, \mathfrak{q} \rangle$ ,  $\mathfrak{A}_1 = \langle \mathfrak{H}_1, \mathfrak{t} \rangle$  and  $\mathfrak{G}_1 = \langle \mathfrak{H}_1, \mathfrak{w} \rangle$ .

Let  $E = \langle a, b, c, d, t, g, h, i, j, k, v_i \mid 1 \leq i \leq 11 \rangle$  be the non-split extension of the Mathieu group  $\mathcal{M}_{24}$  by its simple GF(2)-module  $V_2$  constructed in Lemma 2.1, and let  $E_{23} = \langle a, b, c, d, t, g, h, i, j \rangle$ .

 $\begin{array}{l} \mathit{Let}\ \mathfrak{x} = [(\mathfrak{yq}^2\mathfrak{nqnq}^2)^{11}(\mathfrak{q}^2\mathfrak{n}^2\mathfrak{qnqn})^{11}(\mathfrak{qn}^2\mathfrak{qnqnqnq})^4]^{12},\ \mathfrak{u}_1 = (\mathfrak{xyx})^7,\ \mathfrak{u}_2 = (\mathfrak{xyxnxnx})^4, \\ \mathfrak{u}_3 = (\mathfrak{xyxn}^2\mathfrak{xn}^2\mathfrak{xnxn})^2,\ \mathfrak{u}_4 = (\mathfrak{xyxn}^5\mathfrak{xn}^4)^2,\ \mathit{and}\ \mathfrak{u}_5 = [\mathfrak{y(sm)}^2\mathfrak{nm}]^7. \end{array}$ 

Then the following assertions hold:

- (a) The subgroup  $\mathfrak{T}_1 = \mathfrak{u}_i \mid 1 \leq i \leq 4$  of  $\mathfrak{D} = \langle \mathfrak{x}, \mathfrak{y} \rangle$  is a Sylow 2-subgroup of  $\mathfrak{G}_1 = \langle \mathfrak{q}, \mathfrak{y}, \mathfrak{w} \rangle$  of order  $2^{18}$ .
- (b)  $\mathfrak{T}_1$  has a unique maximal elementary abelian normal subgroup  $\mathfrak{B}$  of order  $2^{11}$ . It is generated by the 11 involutions:

$$\begin{array}{llll} \mathfrak{u}_1, & \mathfrak{u}_2^2, & (\mathfrak{u}_1\mathfrak{u}_2)^2, & (\mathfrak{u}_1\mathfrak{u}_3)^2, & (\mathfrak{u}_1\mathfrak{u}_4)^2, & (\mathfrak{u}_2\mathfrak{u}_4)^4, \\ (\mathfrak{u}_1\mathfrak{u}_2\mathfrak{u}_3)^4, & (\mathfrak{u}_1\mathfrak{u}_2\mathfrak{u}_4)^4, & (\mathfrak{u}_1\mathfrak{u}_3\mathfrak{u}_4)^4, & (\mathfrak{u}_2^2\mathfrak{u}_3)^2, & (\mathfrak{u}_1\mathfrak{u}_2\mathfrak{u}_4\mathfrak{u}_2)^2. \end{array}$$

- (c)  $\mathfrak{s} = (\mathfrak{y}^5\mathfrak{t})^7$  is an involution of  $\mathfrak{A}_1$  such that  $\mathfrak{A}_1 = \langle \mathfrak{H}_1, \mathfrak{s} \rangle$ ,  $\mathfrak{T}_1^{\mathfrak{s}} = \mathfrak{T}_1$ ,  $\mathfrak{B}^{\mathfrak{s}} = \mathfrak{B}$ , and  $\mathfrak{S} = \langle \mathfrak{T}_1, \mathfrak{s} \rangle$  is a Sylow 2-subgroup of  $\mathfrak{A}_1$  of order  $2^{19}$ .
- (d)  $\mathfrak{N}_1 = N_{\mathfrak{G}_1}(\mathfrak{B}) = \langle \mathfrak{x}, \mathfrak{y}, \mathfrak{w} \rangle$  is isomorphic to a non-split extension of  $\mathcal{M}_{23}$  by  $V_2 | \mathcal{M}_{23}$  and  $\mathfrak{D}_1 = N_{\mathfrak{A}_1}(\mathfrak{B}) = \langle \mathfrak{x}, \mathfrak{y}, \mathfrak{s} \rangle$
- (e) There is an isomorphism  $\rho$  between  $\mathfrak{N}_1$  and the subgroup  $E_{23}$  of E such that  $\rho(\mathfrak{y}) = (y_2^5 y_3 y_2 y_3)^3 (w_3 w_1 w_2 w_1 w_2 w_1 w_3 w_1^2 w_3 w_2 w_3 w_2 w_1 w_3)^{20}$ ,  $\rho(\mathfrak{x}) = (x_1 x_2 x_4 x_5 x_4 x_2 x_5)^3$ ,  $\rho(\mathfrak{w}) = (e_2 e_3 e_2 e_3^2)^7$ , where

$$\begin{split} x_1 &= (ij)^3, \quad x_2 = (gahigai)^2, \quad x_3 = (aghijagh)^4, \\ x_4 &= (jhighaji)^4, \quad x_5 = (aighjigai)^4, \\ y_1 &= i, \quad y_2 = ag, \quad y_3 = (ahj)^3, \\ w_1 &= (y_2y_3^2)^2, \quad w_2 = (y_1y_2y_1y_2y_3)^3, \quad w_3 = (y_1y_2y_3y_2^2)^3, \\ e_1 &= (agijih)^4, \quad e_2 = (ag^3ihj)^7, \quad e_3 = ghghiai. \end{split}$$

- (f) There is an isomorphism  $\mu$  between  $\mathfrak{D}_1 = N_{\mathfrak{A}_1}(\mathfrak{B})$  and the centralizer  $C_E(u)$  of the involution  $u = (\rho(\mathfrak{x})\rho(\mathfrak{y})^2)^7$  of E such that  $\mu(\mathfrak{x}) = \rho(\mathfrak{x})$ ,  $\mu(\mathfrak{y}) = \rho(\mathfrak{y})$  and  $\mu(\mathfrak{s}) = (m_1^4 m_2 m_1 m_2)^2$ , where  $m_1 = agahj$ ,  $m_2 = (ijhkj)^2$ ,  $m_3 = (ahjagk)^5$ .
- (g) The subgroup  $\mathfrak{E} = \langle \mathfrak{x}, \mathfrak{y}, \mathfrak{w}, \mathfrak{s} \rangle$  of  $\mathfrak{G}$  has a faithful permutation representation  $P\mathfrak{E}$  of degree 1518 with stabilizer  $\langle (\mathfrak{ys})^7, (\mathfrak{wysy})^3, (\mathfrak{sy}^3)^2, (\mathfrak{y}^2\mathfrak{wy}^2)^3 \rangle$ .
- (h) The groups  $\mathfrak{E}$  and E are isomorphic.
- (i)  $\mathfrak{z} = (\mathfrak{r}\mathfrak{y}\mathfrak{w})^8$  is a 2-central involution of  $\mathfrak{E}$  with centralizer  $C_{\mathfrak{E}}(\mathfrak{z})$  of order  $2^{21} \cdot 3^3 \cdot 5$  generated by the elements  $\mathfrak{r}_1 = (\mathfrak{s}\mathfrak{y}^3)^3$ ,  $\mathfrak{r}_2 = (\mathfrak{y}^2\mathfrak{w}\mathfrak{y}\mathfrak{s})^6$ ,  $\mathfrak{r}_3 = (\mathfrak{s}\mathfrak{y}\mathfrak{w}\mathfrak{y}\mathfrak{s})^2$ , and  $\mathfrak{r}_4 = (\mathfrak{s}\mathfrak{w}\mathfrak{y}\mathfrak{s}\mathfrak{w})^6$  with respective orders 2, 4, 4, and 2.
- (j)  $C_{\mathfrak{G}_1}(\mathfrak{z})$  has order  $2^{18} \cdot 3^5 \cdot 5$ . It is generated by  $\mathfrak{f}_1 = \mathfrak{x}\mathfrak{y}\mathfrak{w}$ ,  $\mathfrak{f}_2 = (\mathfrak{x}\mathfrak{y}\mathfrak{x}\mathfrak{w})^7$ ,  $\mathfrak{f}_3 = (\mathfrak{x}\mathfrak{w}\mathfrak{y}\mathfrak{w}\mathfrak{y}^2)^7$ , and  $\mathfrak{v} = (\mathfrak{w}\mathfrak{q}\mathfrak{w}\mathfrak{y}\mathfrak{q}\mathfrak{y})^7$ .

- (k) The subgroup  $\langle \mathfrak{vf}_2\mathfrak{v}, (\mathfrak{f}_1\mathfrak{f}_2\mathfrak{f}_1\mathfrak{v}\mathfrak{f}_1)^4, (\mathfrak{f}_2\mathfrak{f}_1\mathfrak{v}\mathfrak{f}_1\mathfrak{v})^4 \rangle$  of  $C_{\mathfrak{G}_1}(\mathfrak{z})$  has index 512. Furthermore, it does not contain  $\mathfrak{z}$ .
- (1)  $\mathfrak{B}$  is the Fitting subgroup of  $\mathfrak{E}$ . It is also the unique maximal elementary abelian normal subgroup of the Sylow 2-subgroup  $\mathfrak{S} = \langle \mathfrak{u}_i \mid 1 \leq i \leq 5 \rangle$  of  $\mathfrak{E}$  contained in  $C_{\mathfrak{E}}(\mathfrak{z})$ .

*Proof.* In order to simplify the notation of the proof we replace the German letters by Roman letters. In particular, we let  $ME = \langle x, y, w, s \rangle$  be the subgroup  $\mathfrak{E}$ .

- (a) Let  $PA_1$  be the faithful permutation representation of  $A_1$  of degree 56320 constructed in Lemma 3.1(c). By Lemma 3.2 and Lemma 4.1(f) we know that  $H_1 = \langle y, q \rangle = \langle x, y, q \rangle$ . Now [11] asserts that  $D = \langle x, y \rangle$  has odd index in  $H_1$  and therefore in  $G_1$ . Thus D contains a Sylow 2-subgroup of  $G_1$ . The given Sylow 2-subgroup  $T_1$  of  $G_1$  and its generators  $t_i$  have been found by using MAGMA, the permutation representation  $PA_1$ , and the program GetShortGens(H\_1,T\_1).
  - (b) Applying the MAGMA command

```
Subgroups(T_1: Al:=Normal, IsElementaryAbelian := true)
```

we observed that  $T_1$  has 44 elementary abelian normal subgroups. Exactly one of them is maximal and has order  $2^{11}$ . It is denoted by B. Its given generators have been calculated by means of the first author's program GetShortGens(T\_1, B).

(c) Since  $|A_1: H_1| = 2$  a Sylow 2-subgroup of  $A_1$  has order  $2^{19}$ . Let  $W_1 = N_{A_1}(T_1)$ . Applying  $PA_1$  and the MAGMA command

```
exists(r)\{x: x \in A_1 \mid T_1 = T_1 \text{ and } x^2 \in 1 \text{ and } x \in H_1\}
```

we found the involution  $s \in A_1$  of the statement satisfying  $s \notin H_1$ . It satisfies the equation  $T_1^s = T_1$ . Hence  $B^s = B$  holds trivially by (b).

- (d) By another application of  $PA_1$  and MAGMA we verified that  $N_{A_1}(B) = \langle x, y, s \rangle$ . Using the faithful permutation representation  $PG_1$  of degree 31671 with stabilizer  $H_1$  of Kim's Theorem 6.3.1 of [14] one establishes that  $N_1 = N_{G_1}(B) = \langle x, y, w \rangle$ . Hence  $N_1$  is a non split extension of  $\mathcal{M}_{23}$  by B, see Lemma 6.1.2 and Theorem 6.3.1 of [14].
- (e) By Lemma 2.1(e)  $E = \langle a, b, c, d, t, g, h, i, j, k \rangle$  has a the faithful permutation representation PE with stabilizer  $U_3 = \langle g, h, i, (dg)^5, (dhjk)^3, (ijkj)^2, (dhjidg)^3 \rangle$ . Lemma 8.2.2 of [12] states that its subgroup  $E_{23} = \langle a, b, c, d, t, g, h, i, j \rangle$  has index  $|E:E_{23}|=24$ . Applying the command IsIsomorphic (N\_1,E\_{23}) MAGMA establishes an isomorphism  $\rho:N_1\to E_{23}$ . The words of the images  $\rho(x)$ ,  $\rho(y)$  and  $\rho(w)$  of the generators x,y and x of x of the images x follows. Let x definitions x because x is a subgraph of the generators x is generators x in x and x of x in x in x is generators x in x in
- (f) By Kim's Theorem 6.3.1 of [14] we know that  $z_1 = (xy^2)^7$  is a 2-central involution of  $G_1$ . Clearly  $u = (\rho(x)\rho(y)^2)^7$  is an involution of  $E_{23}$ . Applying MAGMA and PE the reader can verify that  $C_u = C_{E_{23}}(u)$  has order  $2^{19} \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$ . Similarly one observes that  $D_1 = N_{A_1}(B) = \langle x, y, s \rangle$  has the same order. Using  $PA_1$ , PE and the command IsIsomorphic (D\_1,C\_u) MAGMA establishes an isomorphism  $\mu$ :

 $D_1 \to C_u$ . Applying the MAGMA command IsConjugate(C\_u,\mu(y),\rh(y)) we found an element  $c_1 \in C_u$  such that  $\mu(y)^{c_1} = \rho(y)$ . Using the command

```
exists(c){q: q in C_{C_u}(\rho(y)) | (\mu(x)^{c_1})^q = \rho(x)
```

one gets an element  $c_2 \in C_u$  such that  $\mu(x)^{c_1c_2} = \rho(x)$ . Hence  $\mu': D_1 \to C_u$  defined by  $\mu'(d) = \mu(d)^{c_1c_2}$ ,  $d \in D_1$ , is an isomorphism between  $D_1$  and  $C_u$  such that  $\mu'(x) = \rho(x)$  and  $\mu'(y) = \rho(y)$ . It has been checked that  $C_E(\langle \rho(y), \rho(x) \rangle) = \langle u \rangle$ . Furthermore,  $\mu'(s)$  has a centralizer  $C_{M_{23}}(\mu'(s))$  of order  $2^{17}$  which is generated by the three elements  $m_1$ ,  $m_2$  and  $m_3$  of  $M_{23}$  given in the statement. Another application of the Lookup command yields the word  $\mu'(s) = (m_1^4 m_2 m_1 m_2)^2$ . Hence the map  $\mu': D_1 \to C_E(u)$  satisfies all conditions of (f).

- (g) Using (e), (f), PE and MAGMA it has been verified that  $E = \langle \rho(x), \rho(y), \rho(w), \mu(s) \rangle$ . As  $U_3 = \langle g, h, i, (dg)^5, (dhjk)^3, (ijkj)^2, (dhjidg)^3 \rangle$  is a stabilizer of PE we apply the program GetShortGens(E,U\_3) w.r.t. the given generators of E. MAGMA returns  $U_3 = \langle (\rho(y)\mu(s))^7, (\rho(w)\rho(y)\mu(s)\rho(y))^3, (\mu(s)\rho(y)^3)^2, (\rho(y)^2\rho(w)\rho(y)^2)^3 \rangle$ .
- Thus  $MU = \langle (ys)^7, (wysy)^3, (sy^3)^2, (y^2wy^2)^3 \rangle$  is a subgroup of  $ME = \langle x, y, w, s \rangle$  which is isomorphic to  $U_3$ . Let V be the 8671-dimensional vector space over F = GF(13). Using the Meataxe Algorithm implemented in MAGMA we see that the restriction  $V_{MU}$  of V to the subgroup MU has a 7-dimensional FMU-submodule W which has a complement of dimension 8664. Applying now the algorithm described in Theorem 6.2.1 of [12] we obtain a faithful permutation representation PME of the matrix group ME of degree 1518 with stabilizer MU.
- (h) Using PE, PME and the isomorphism test IsIsomorphic (PE, PME) MAGMA established that  $ME \cong E$ .
- (i) By (d) and Table 6.5.1 of [14] we know that  $z=(xyw)^8$  is an involution of  $N_1=N_{G_1}(B)\cong E_{23}$  with centralizer  $C_{N_1}(z)$  of order  $2^{18}\cdot 3^2\cdot 5$ . Therefore we calculate  $C_E(z)$  by means of PE and MAGMA. It follows that  $|C_E(z)|=2^{21}\cdot 3^3\cdot 5$ . Hence E has a Sylow 2-subgroup  $S_3$  of order  $2^{21}$  with center  $Z(S_3)=\langle z\rangle$  by Table A.1. The given generators  $r_i$  of  $C_z=C_E(z)$  have been determined by means of MAGMA and the program GetShortGens(E, C\_z).
- (j) Table 6.5.6 of [14] implies that  $|C_{G_1}(z)| = 2^{18} \cdot 3^5 \cdot 5$  because  $z = (xyw)^8 \in G_1 = \langle x, y, w, q \rangle$ . Using MAGMA and the faithful permutation representation  $PG_1$  of  $G_1$  we found the involution  $v = (wqwyqy)^7$  such that (z, v) = 1, and  $C_{G_1}(z) = \langle f_1, f_2, f_3, v \rangle$  for the elements  $f_i \in G_1$  given in the statement.
- (k) All assertions of the statement are easily checked by means of MAGMA and the faithful permutation representation  $PG_1$  of  $G_1$ .
- (l) By (b) and (c) the elementary abelian subgroup B is normal in ME. Hence it is the Fitting subgroup ME by (h) and Lemma 2.1. Using the faithful permutation representation of ME given in (g) the remaining assertions can be verified by means of MAGMA.

The following presentation of the 3-transposition Fischer group  $P = \text{Fi}_{24}$  is taken from [15], its p. 124. It is due to J. Hall and L. S. Soicher [7].

**Lemma 6.2.** Let  $\mathfrak{G} = \langle \mathfrak{y}, \mathfrak{q}, \mathfrak{t}, \mathfrak{w} \rangle$  be the subgroup of  $GL_{8671}(13)$  constructed in Proposition 5.2. Let  $\mathfrak{H}_1 = \langle \mathfrak{y}, \mathfrak{q} \rangle$ ,  $\mathfrak{G}_1 = \langle \mathfrak{H}_1, \mathfrak{w} \rangle$  and  $\mathfrak{A}_1 = \langle \mathfrak{H}_1, \mathfrak{t} \rangle$ . Let  $\mathfrak{s} = (\mathfrak{y}^5 \mathfrak{t})^7$  and  $\mathfrak{x} = [(\mathfrak{y}\mathfrak{q}^2\mathfrak{y}\mathfrak{q}\mathfrak{q}\mathfrak{q}\mathfrak{q}^2)^{11}(\mathfrak{q}^2\mathfrak{p}^2\mathfrak{q}\mathfrak{y}\mathfrak{q}\mathfrak{y})^{11}(\mathfrak{q}\mathfrak{p}^2\mathfrak{q}\mathfrak{y}\mathfrak{q}\mathfrak{q}\mathfrak{q}\mathfrak{q})^4]^{12}$ . Let  $\mathfrak{E} = \langle \mathfrak{x}, \mathfrak{y}, \mathfrak{w}, \mathfrak{s} \rangle$ 

Let  $P = \langle a, b, c, d, e, f, g, h, i, j, k, l \rangle$  be the finitely generated group with the following set  $\mathcal{R}(P)$  of defining relations:

$$l^{2} = k^{2} = a^{2} = b^{2} = c^{2} = d^{2} = e^{2} = f^{2} = g^{2} = j^{2} = h^{2} = i^{2} = 1,$$

$$(lk)^{3} = (ka)^{3} = (ab)^{3} = (bc)^{3} = (cd)^{3} = (de)^{3} = (ef)^{3} = (fg)^{3} = (gj)^{3} = 1,$$

$$(la)^{2} = (lb)^{2} = (lc)^{2} = (ld)^{2} = (le)^{2} = (lf)^{2} = (lg)^{2} = (lj)^{2} = (lh)^{2} = (li)^{2} = 1,$$

$$(kb)^{2} = (kc)^{2} = (kd)^{2} = (ke)^{2} = (kf)^{2} = (kg)^{2} = (kj)^{2} = (kh)^{2} = (ki)^{2} = 1,$$

$$(ac)^{2} = (ad)^{2} = (ae)^{2} = (af)^{2} = (ag)^{2} = (aj)^{2} = (ah)^{2} = (ai)^{2} = 1,$$

$$(bd)^{2} = (be)^{2} = (bf)^{2} = (bg)^{2} = (bj)^{2} = (bh)^{2} = (bi)^{2} = 1,$$

$$(ce)^{2} = (cf)^{2} = (cg)^{2} = (cj)^{2} = (ch)^{2} = (ci)^{2} = (df)^{2} = (dg)^{2} = (dj)^{2} = 1,$$

$$(eg)^{2} = (ej)^{2} = (eh)^{2} = (ei)^{2} = (fj)^{2} = (fh)^{2} = (fi)^{2} = (gh)^{2} = (gi)^{2} = 1,$$

$$(jh)^{2} = (ji)^{2} = (dh)^{3} = (hi)^{3} = (di)^{2} = 1,$$

$$l = (abcde fh)^{9}, (dcbaklde fgjdhi)^{17} = 1.$$

Then the following statements hold:

- (a) P has a faithful permutation representation PP of degree 306936 with stabilizer  $M = \langle a, b, c, d, e, f, g, h, i, j, l \rangle$ .
- (b) The commutator subgroup G = P' is a finite simple group of order  $2^{21} \cdot 3^{16} \cdot 5^2 \cdot 7^3 \cdot 11 \cdot 13 \cdot 17 \cdot 23 \cdot 29$ .
- (c)  $G = \langle b_1, c_1, d_1, e_1, f_1, g_1, h_1, i_1, j_1, k_1 \rangle$ , where  $b_1 = ab$ ,  $c_1 = ac$ ,  $d_1 = ad$ ,  $e_1 = ae$ ,  $f_1 = af$ ,  $g_1 = ag$ ,  $h_1 = ah$ ,  $i_1 = ai$ ,  $j_1 = aj$ , and  $k_1 = ak$ .

Furthermore, G has a faithful permutation representation PG of degree 306936 with stabilizer  $M_1 = \langle b_1, c_1, d_1, e_1, f_1, g_1, h_1, i_1, j_1 \rangle$  and  $M_1$  is a simple group of order  $2^{18} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 23$ .

(d) The centralizer  $C_1 = C_G(c_1)$  of the involution  $c_1$  of G has order  $2^{19} \cdot 3^9 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13$ , and  $C_1 = \langle e_1, g_1, i_1, j_1, m_1, n_1 \rangle$ , where  $m_1 = (c_1 d_1 e_1 h_1)^3$  and  $n_1 = (b_1 c_1 d_1 k_1 f_1)^3$  are involutions.

Furthermore,  $C_1$  has a faithful permutation representation  $PC_1$  of degree 56320 with stabilizer  $\langle i_1, j_1, (e_1g_1n_1)^2, n_1m_1n_1 \rangle$ .

(e) There is an isomorphism  $\phi$  between  $\mathfrak{A}_1 = \langle \mathfrak{y}, \mathfrak{q}, \mathfrak{t} \rangle$  and the finitely presented group  $A_1 = \langle a, b, c, d, e, f, g, h, i, z, t \rangle$  constructed in Lemma 3.1(b) such that

$$y = \phi(\mathfrak{y}) = (S_1 S_2 S_4 S_1 S_4 S_2 S_4)^5 (T_1 T_3^2 T_1 T_3 T_1 T_2 T_1 T_3)^{20},$$

$$q = \phi(\mathfrak{q}) = [(p \cdot o \cdot j \cdot o \cdot k)^4 \cdot (j \cdot k \cdot o \cdot k^2 p)^4]^4,$$

$$t = \phi(\mathfrak{t}), \quad where$$

$$S_1 = (b \cdot a \cdot c \cdot b) \cdot (d \cdot e \cdot h \cdot d) \cdot (b \cdot a \cdot c \cdot b),$$

$$S_2 = [(b \cdot a \cdot c \cdot b) \cdot (d \cdot e \cdot h \cdot d) \cdot (f \cdot e \cdot g \cdot f)]^2,$$

$$S_3 = [(b \cdot a \cdot c \cdot b) \cdot (d \cdot e \cdot h \cdot d) \cdot (c \cdot d \cdot e \cdot h \cdot i)^4 \cdot (b \cdot a \cdot c \cdot b)]^2,$$

$$S_4 = [(b \cdot a \cdot c \cdot b) \cdot (c \cdot d \cdot e \cdot h \cdot i)^4 \cdot (d \cdot e \cdot h \cdot d) \cdot (f \cdot e \cdot g \cdot f)]^4,$$

$$T_1 = S_2 S_4 S_2^2, \quad T_2 = (S_4 S_2 S_1 S_3 S_4)^2, \quad T_3 = (S_4 S_2 S_4 S_2 S_1 S_3)^2,$$

$$j = (c \cdot d \cdot e \cdot h)^3,$$

$$k = (c \cdot d \cdot e \cdot f \cdot g)^2,$$

$$l = (a \cdot b \cdot c \cdot d \cdot e \cdot h)^5,$$

$$o = (l \cdot b \cdot k \cdot j \cdot b \cdot i)^6,$$

$$p = (j \cdot k \cdot j \cdot l \cdot b \cdot i \cdot j \cdot i)^5.$$

(f) There is an isomorphism

$$\rho: A_1 = \langle a, b, c, d, e, f, q, h, i, z, t \rangle \rightarrow C_1 = \langle e_1, q_1, i_1, j_1, m_1, n_1 \rangle$$

such that  $\rho(t) = (m_1 n_1)^3$  and:

$$\rho(a) = [(i_1 m_1 n_1)^6 \cdot (j_1 n_1 m_1 n_1) \cdot (g_1 n_1 e_1 g_1 n_1)^2 \cdot (i_1 m_1 n_1)^6 \cdot (e_1 g_1 n_1 m_1 n_1 e_1)^2 \cdot (g_1 n_1 e_1 g_1 n_1)^2]^7,$$

$$\rho(b) = [(i_1 n_1 g_1 j_1 m_1)^{12} \cdot (e_1 g_1 j_1 m_1 i_1 n_1)^4 \cdot (i_1 n_1 g_1 j_1 m_1)^{12} \cdot (e_1 g_1 j_1 m_1 i_1 n_1)^{12} \cdot (e_1 g_1 j_1 m_1 i_1 n_1)^8]^{11},$$

$$\rho(c) = [(j_1 n_1 m_1 n_1) \cdot (e_1 n_1 q_1 n_1 e_1) \cdot (e_1 q_1 i_1 j_1 m_1 n_1)^{18}]^{11},$$

$$\rho(d) = [(j_1 m_1) \cdot (n_1 g_1 n_1) \cdot (e_1 i_1 j_1 n_1 g_1 m_1)^{18} \cdot (j_1 m_1) \cdot (e_1 i_1 j_1 n_1 g_1 m_1)^{18} \cdot (j_1 m_1) \cdot (e_1 i_1 j_1 n_1 g_1 m_1)^{18}]^{11},$$

$$\rho(e) = (bc)^3 \cdot c^t$$
,  $\rho(f) = (bc)^3 \cdot b^t$ ,  $\rho(g) = a^t$ ,

$$\rho(h) = [e_1 \cdot i_1 \cdot (m_1 i_1 n_1 m_1 n_1)^3 \cdot (e_1 g_1 n_1 g_1 j_1 m_1)^6 \cdot i_1 \cdot (e_1 g_1 n_1 g_1 j_1 m_1)^6]^{11},$$

$$\rho(i) = [(m_1 i_1 m_1) \cdot (e_1 g_1 j_1 n_1 m_1)^{12}]^9.$$

In particular,  $\psi = \rho \circ \phi : \mathfrak{A}_1 \to C_1$  is an isomorphism such that  $y_1 = \psi(\mathfrak{y}) = \rho(y)$ ,  $q_1 = \psi(\mathfrak{q}) = \rho(q)$ , and  $t_1 = \psi(\mathfrak{t}) = \rho(t)$  generate  $C_1$ .

(g) In  $C_1$  let  $x_1 = \psi(\mathfrak{x})$  and  $s_1 = \psi(\mathfrak{s}) = (y_1^5 t_1)^7$ . Then  $S_1 = \langle u_1, u_2, u_3, u_4, s_1 \rangle$  is a Sylow 2-subgroup of  $C_1$ , where  $u_1 = (x_1 y_1 x_1)^7$ ,  $u_2 = (x_1 y_1 x_1 y_1 x_1 y_1 x_1 y_1 x_1)^4$ ,  $u_3 = (x_1 y_1 x_1 y_1^2 x_1 y_1^2 x_1 y_1 x_1 y_1)^2$ ,  $u_4 = (x_1 y_1 x_1 y_1^5 x_1 y_1^4)^2$ .

Moreover,  $S_1$  has a unique maximal elementary abelian normal subgroup  $B_1$ . It is generated by the eleven involutions  $u_1$ ,  $u_2^2$ ,  $(u_1u_2)^2$ ,  $(u_1u_3)^2$ ,  $(u_1u_4)^2$ ,  $(u_2u_4)^4$ ,  $(u_1u_2u_3)^4$ ,  $(u_1u_2u_4)^4$ ,  $(u_1u_3u_4)^4$ ,  $(u_2^2u_3)^2$ ,  $(u_1u_2u_4u_2)^2$ .

- $(u_1u_4)^2, (u_2u_4)^4, (u_1u_2u_3)^4, (u_1u_2u_4)^4, (u_1u_3u_4)^4, (u_2^2u_3)^2, (u_1u_2u_4u_2)^2.$ (h)  $N_1 = N_G(B_1) = \langle x_1, y_1, s_1, o_1 \rangle$ , where  $o_1 = [d_1 \cdot y_1 \cdot s_1 \cdot (y_1)^2 \cdot s_1)]^{10}$  and  $|N_1| = 2^{21} \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 23.$
- (i) There is an isomorphism  $\mu : \mathfrak{E} \to N_1$  such that  $\mu(\mathfrak{x}) = \psi(\mathfrak{x}) = x_1$ ,  $\mu(\mathfrak{y}) = \psi(\mathfrak{y}) = y_1$ ,  $\mu(\mathfrak{s}) = \psi(\mathfrak{t}) = s_1$  and

$$\mu(w) = w_1 = [(s_1 y_1 x_1 y_1)^2 \cdot (x_1 y_1 o_1 y_1 s_1 y_1^2)^5]^7.$$

- (j)  $G = \langle C_1, N_1 \rangle = \langle q_1, y_1, w_1, t_1 \rangle$ .
- (k) The map  $\kappa : G \to \mathfrak{G}$  given by  $\kappa(q_1) = \mathfrak{q}$ ,  $\kappa(y_1) = \mathfrak{y}$ ,  $\kappa(w_1) = \mathfrak{w}$  and  $\kappa(s_1) = \mathfrak{s}$  is a group isomorphism.

In particular,  $\mathfrak{G}$  is a simple group of order  $2^{21} \cdot 3^{16} \cdot 5^2 \cdot 7^3 \cdot 11 \cdot 13 \cdot 17 \cdot 23 \cdot 29$ .

- *Proof.* (a) This statement has been verified by running the Todd-Coxeter Algorithm CosetAction(P,M) built into MAGMA.
- (b) Using (a) and MAGMA it has been checked that G=P' is a simple group of the stated order.
- (c) Using then the program GetShortGens(P,G) we found the given generators of G. Using the faithful permutation representation PP of (a) and MAGMA it has been checked that  $M_1 = \langle b_1, c_1, d_1, e_1, f_1, g_1, h_1, i_1, j_1 \rangle$  is a simple group of order  $2^{18} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 23$ . It is the stabilizer of the faithful permutation representation PG of degree 306936 as has been checked by means of MAGMA and the command CosetAction(G,M\_1).
- (d) The centralizer  $C_1 = C_G(c_1)$  of the involution  $c_1$  and  $|C_1|$  have been determined by means of the permutation representation PG of G and MAGMA. Its given generators have been found by MAGMA using the program GetShortGens(G,C\_1). Applying the MAGMA command DegreeReduction we get a faithful permutation representation  $PC_1$  of  $C_1$  having degree 56320. Its stabilizer  $U_1$  has been found

by means of the MAGMA command BasicStabilizer(~,2). Its given generators were gotten by another application of GetShortGens(C\_1,U\_1).

- (e) This statement follows immediately from Lemma 3.1(b), Proposition 6.2.3 of [14], and Proposition 5.2.
- (f) By Lemma 3.1(c) the finitely presented group  $A_1=\langle a,b,c,d,e,f,g,h,i,z,t\rangle$  has a faithful permutation representation  $PA_1$  of degree 56320. Let  $PC_1$  be the faithful permutation representation of  $C_1$  constructed in (d). A successful application of the MAGMA command IsIsomorphic (PA\_1,PC\_1) provides an isomorphism  $\eta:\mathfrak{A}_1\to C_1$ . Now  $C_{C_1}(\eta(t))$  and  $C_{C_1}((m_1n_1)^3)$  have the same order. Using the MAGMA command exists (w) {x: x in C\_1| \eta(t)^x eq (m\_1n\_1)^3} we found an element  $a\in C_1$  such that  $\eta(t)^a=(m_1n_1)^3$ . Let  $\alpha$  be the inner automorphism of  $C_1$  induced by a. Then the map  $\rho=\alpha\circ\eta$  is an isomorphism from  $A_1$  onto  $C_1$  such that  $\rho(t)=(m_1n_1)^3$ . The centralizers of the images of  $\rho(a),\rho(b),\ldots,\rho(i)$  in  $C_1$  all have the same order  $2^{17}\cdot 3^6\cdot 5\cdot 7\cdot 11$ . Their generators and their words in them are obtained computationally using the programs GetShortGens and LookupWord, respectively, as follows:
  - (1)  $C_{C_1}(\rho(a))$  is generated by  $a_1 = (i_1 m_1 n_1)^6$ ,  $a_2 = j_1 n_1 m_1 n_1$ ,  $a_3 = (g_1 n_1 e_1 g_1 n_1)^2$ ,  $a_4 = (e_1 g_1 n_1 m_1 n_1 e_1)^2$ , and  $\rho(a) = (a_1 a_2 a_3 a_1 a_4 a_3)^7$ .
  - (2)  $C_{C_1}(\rho(b))$  is generated by  $b_1 = (i_1 n_1 g_1 j_1 m_1)^{12}$ ,  $b_2 = (e_1 g_1 j_1 m_1 i_1 n_1)^4$ , and  $\rho(b) = (b_1 b_2 b_1 b_2^3 b_1 b_2^2)^{11}$ .
  - (3)  $C_{C_1}(\rho(c))$  is generated by  $c_1 = j_1 n_1 m_1 n_1$ ,  $c_2 = e_1 n_1 g_1 n_1 e_1$ ,  $c_3 = (e_1 g_1 i_1 j_1 m_1 n_1)^{18}$ , and  $\rho(c) = (c_1 c_2 c_3)^{11}$ .
  - (4)  $C_{C_1}(\rho(d))$  is generated by  $d_1 = j_1 m_1$ ,  $d_2 = n_1 g_1 n_1$ ,  $d_3 = (e_1 i_1 j_1 n_1 g_1 m_1)^{18}$ , and  $\rho(d) = (d_1 d_2 d_3 d_1 d_3 d_1 d_3)^{11}$ .
  - (5)  $C_{C_1}(\rho(h))$  is generated by  $h_1 = e_1$ ,  $h_2 = i_1$ ,  $h_3 = (m_1 i_1 n_1 m_1 n_1)^3$ ,  $h_4 = (e_1 g_1 n_1 g_1 j_1 m_1)^6$ , and  $\rho(h) = (h_1 h_2 h_3 h_4 h_2 h_4)^{11}$ .
  - (6)  $C_{C_1}(\rho(i))$  is generated by  $i_1 = m_1 i_1 m_1$ ,  $i_2 = j_1 n_1 m_1 n_1$ ,  $i_3 = (e_1 g_1 j_1 n_1 m_1)^6$ , and  $\rho(i) = (i_1 i_3^2)^9$ .

The given words for the images  $\rho(e)$ ,  $\rho(f)$  and  $\rho(g)$  can now be calculated from these images and the relations of Lemma 3.1(b).

Statement (e) implies that the composition of the isomorphisms  $\phi: \mathfrak{A}_1 \to A_1$  and  $\rho: A_1 \to C_1$  is an isomorphism  $\psi: \mathfrak{A}_1 \to C_1$ . Furthermore, the given images  $\psi(\mathfrak{q})$ ,  $\psi(\mathfrak{q})$  and  $\psi(\mathfrak{q})$  in  $C_1$  of the 3 generators of  $\mathfrak{A}_1$  are well defined.

(g) Since  $\mathfrak{x} \in \mathfrak{A}_1$  its image

$$\psi(\mathfrak{x}) = \left[ (y_1 q_1^2 y_1 q_1 y_1 q_1^2)^{11} (q_1^2 y_1^2 q_1 y_1 q_1 y_1)^{11} (q_1 y_1^2 q_1 y_1 q_1 y_1 q_1 y_1 q_1)^4 \right]^{12} = x_1$$

is a well defined element of  $C_1$ .

Let  $\mathfrak{u}_i$  be the generators of the Sylow 2-subgroup  $\mathfrak{T}_1$  of the subgroup  $\mathfrak{G}_1 = \langle \mathfrak{q}, \mathfrak{y}, \mathfrak{w} \rangle$  of  $\mathfrak{G}$  given in Proposition 6.1(a). Hence  $\mathfrak{T}_1$  is a subgroup of  $\mathfrak{A}_1$  because its generators  $\mathfrak{u}_i$  are words in  $\mathfrak{x}$  and  $\mathfrak{y}$  by Proposition 6.1. Therefore their images  $u_i = \psi(\mathfrak{u}_i), 1 \leq i \leq 4$ , are well defined. So is  $s_1 = \psi(\mathfrak{s}) = (y_1^5 t_1)^7$ . Hence  $S_1 = \psi(\mathfrak{S}) = \langle u_1, u_2, u_3, u_4, s_1 \rangle$  is a Sylow 2-subgroup of  $C_1$  by Proposition 6.1(c) and (f).

Let  $B_1\psi(\mathfrak{B}$  where  $\mathfrak{B}$  is the unique maximal elementary abelian normal subgroup of  $\mathfrak{S}$  defined in Proposition 6.1(b and (d)). Then  $B_1$  is generated by the 11 involutions given in the statement.

- (h) Let  $N_1 = N_G(B_1)$ . Using the faithful permutation representation PG of G and MAGMA the reader can easily check that  $|N_1| = 2^{21} \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 23$  and that  $N_1$  is generated by  $x_1, y_1, s_1$  and the element  $o_1 \in G$  given in the statement.
- (i) By Proposition 6.1(g) the subgroup  $\mathfrak E$  of  $\mathfrak G$  has a faithful permutation representation  $P\mathfrak E$  of degree 1518 with stabilizer  $\langle (\mathfrak{ps})^7, (\mathfrak{wnsn})^3, (\mathfrak{sn}^3)^2, (\mathfrak{p}^2\mathfrak{wn}^2)^3 \rangle$ . Let  $PN_1$  be the reduction of PG to  $N_1 = N_G(B_1)$ . A successful application of the MAGMA command IsIsomorphic (PE,PN\_1) provides an isomorphism  $\tau : \mathfrak E \to N_1$ . As in the proof of (f) we find an element  $a \in N_1$  such that  $(\tau(\mathfrak{p}))^a = y_1 = \psi(\mathfrak{p})$ . Using MAGMA again we verified that  $C_{N_1}(y_1)$  has order 56. Searching through its elements we find an element  $b \in C_{N_1}(y_1)$  such that  $(\tau(\mathfrak{p}))^{ab} = \psi(\mathfrak{p}) = x_1$  and  $(\tau(\mathfrak{s}))^{ab} = \psi(\mathfrak{s}) = s_1$ . Let  $\beta$  denote the inner automorphism of  $N_1$  induced by conjugation with ab. Then the map  $\mu = \beta \circ \nu$  is an isomorphism from  $\mathfrak E$  onto  $N_1 = N_G(B_1)$  such that  $\mu(\mathfrak{p}) = x_1$ ,  $\mu(\mathfrak{p}) = y_1$  and  $\mu(\mathfrak{s}) = s_1$ . Let  $w_1 = \mu(\mathfrak{w})$ . Another application of MAGMA and  $PN_1$  yields that  $N_1 = \langle x_1, y_1, s_1, w_1 \rangle$ .

The word for  $w_1$  in the generators of G is obtained as follows. Let  $C_2 = C_{N_1}(w_1)$ . Using the generators of  $N_1$  given in (h), MAGMA, and the program GetShortGens (N\_1,C\_2) we see that  $C_2 = \langle v_i \mid 1 \leq i \leq 3 \rangle$ , where  $v_1 = (s_1y_1x_1y_1)^2$ ,  $v_2 = (y_1o_1x_1s_1o_1)^4$ ,  $v_3 = (x_1y_1o_1y_1s_1y_1^2)^5$ . Applying then the command LookupWord(C\_2, w\_1) MAGMA provides the solution  $w_1 = (v_1v_3)^7$ .

- (j) Using the faithful permutation representation PG of G and MAGMA the reader easily verifies that  $G = \langle C_1, N_1 \rangle$ . Hence (f) and (i) imply that  $G = \langle q_1, y_1, t_1, x_1, s_1, w_1 \rangle = \langle q_1, y_1, t_1, w_1 \rangle$ .
- (k) In  $\mathfrak{G}$  let  $\mathfrak{E}_{23} = \langle \mathfrak{x}, \mathfrak{y}, \mathfrak{w} \rangle$  and  $\mathfrak{H}_1 = \langle \mathfrak{q}, \mathfrak{y} \rangle$ . Then Proposition 5.2 and Kim's Theorem 6.3.1 of [14] imply that  $\mathfrak{G}_1 = \langle \mathfrak{H}_1, \mathfrak{E}_{23} \rangle$  is a simple subgroup of  $\mathfrak{G}$  such that  $\mathfrak{D}_1 = \mathfrak{E}_{23} \cap \mathfrak{H}_1 = \langle \mathfrak{x}, \mathfrak{y} \rangle$  and  $\mathfrak{H}_1 = C_{\mathfrak{G}_1}(\mathfrak{z}_1)$ , where  $\mathfrak{z}_1 = (\mathfrak{x}\mathfrak{y}^2)^7$  is a 2-central involution of  $\mathfrak{G}_1$ .

Let  $E_{23} = \mu(\mathfrak{E}_{23})$ ,  $H_1 = \psi(\mathfrak{H}_1)$  and  $G_1 = \langle E_{23}, H_1 \rangle$  in G. As  $\mu$  and  $\psi$  agree on  $\mathfrak{H}_1$  by (j) we have  $D_1 = \langle x_1, y_1 \rangle = E_{23} \cap H_1$ . Using PG and MAGMA it has been checked that  $G_1$  is a simple group of order  $2^{18} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 23$ . Furthermore,  $C_{G_1}(z_1) = H_1$ , where  $z_1 = (x_1 y_1^2)^7 = \psi(\mathfrak{z}_1)$  is a 2-central involution of  $G_1$ . Thus Kim's Theorem 6.3.1 of [14] implies that  $G_1 \cong \mathrm{Fi}_{23}$ . In particular, the map  $\kappa_1 : G_1 \to \mathfrak{G}_1$  defined by  $\kappa_1(q_1) = \mathfrak{q}$ ,  $\kappa_1(y_1) = \mathfrak{x}$ , and  $\kappa_1(w_1) = \mathfrak{w}$  is an isomorphism.

By Proposition 5.2 and Lemma 3.2(e) the amalgam  $\mathfrak{A}_1 \leftarrow \mathfrak{H}_1 \rightarrow \mathfrak{G}_1$  has Goldschmidt index 1. Therefore by Corollary 7.1.9 of [12], (f) and (i) imply that the free products  $\mathfrak{A}_1 *_{\mathfrak{H}_1} \mathfrak{G}_1$  and  $C_1 *_{H_1} G_1$  with respective amalgamated subgroups  $\mathfrak{H}_1$  and  $H_1$  are isomorphic. Thus (j) and Proposition 5.2 imply that the map  $\kappa : G \rightarrow \mathfrak{G}$  defined by  $\kappa(q_1) = \mathfrak{q}$ ,  $\kappa(y_1) = \mathfrak{y}$ ,  $\kappa(w_1) = \mathfrak{w}$  and  $\kappa(t_1) = \mathfrak{t}$  is an irreducible 8671-dimensional representation of the group G over GF(13). Hence  $\kappa : G \rightarrow \mathfrak{G}$  is an isomorphism because  $\kappa$  is surjective and G is simple by (b). This completes the proof.

**Theorem 6.3.** Let  $\mathfrak{G} = \langle \mathfrak{q}, \mathfrak{h}, \mathfrak{t}, \mathfrak{w} \rangle$  be the subgroup of  $GL_{8671}(13)$  constructed in Proposition 5.2. Then the following statements hold:

- (a)  $\mathfrak{G} = \langle \mathfrak{q}, \mathfrak{y}, \mathfrak{t}, \mathfrak{w} \rangle$  has a faithful permutation representation of degree 306936 with stabilizer  $\mathfrak{G}_1 = \langle \mathfrak{q}, \mathfrak{y}, \mathfrak{w} \rangle$ .
- (b) S is a finite simple group of order

$$|\mathfrak{G}| = 2^{21} \cdot 3^{16} \cdot 5^2 \cdot 7^3 \cdot 11 \cdot 13 \cdot 17 \cdot 23 \cdot 29.$$

- (c)  $\mathfrak{G}$  is isomorphic to the commutator subgroup P' of the finitely presented group  $P = \langle a, b, c, d, e, f, g, h, i, j, k, l \rangle$  with set of defining relations  $\mathcal{R}(P)$  stated in Lemma 6.2.
- (d) The character table of  $\mathfrak{G}$  is equivalent to that of  $\mathrm{Fi}_{24}'$  in the Atlas [4], its pp. 200 -202.

*Proof.* The first three statements hold by Lemma 6.2(j) and (k).

(d) Let G = P' be the commutator subgroup of the finitely presented group P. Then  $\mathfrak{G} \cong G$  by (c). The character table of G has been calculated by means of MAGMA and the faithful permutation representation PG of G with stabilizer  $M = \langle b_1, c_1, d_1, e_1, f_1, g_1, h_1, i_1, j_1 \rangle$  given in Lemma 6.2(c).

## 7. Presentation of 2-central involution centralizer

In this section we determine generators and a presentation of the centralizer  $\mathfrak{H}=C_{\mathfrak{G}}(\mathfrak{z})$  of a 2-central involution  $\mathfrak{z}$  of the simple subgroup  $\mathfrak{G}=\langle \mathfrak{q},\mathfrak{y},\mathfrak{w},\mathfrak{t}\rangle$  of  $\mathrm{GL}_{8671}(13)$ . Thus we are able to show that  $\mathfrak{G}$  satisfies all conditions of Algorithm 2.5 of [13]. In particular,  $\mathfrak{G}=\langle \mathfrak{H},\mathfrak{E}\rangle$  and  $\mathfrak{D}=N_{\mathfrak{H}}(\mathfrak{B})=C_{\mathfrak{E}}(\mathfrak{z})$  where  $\mathfrak{B}$  is the unique maximal elementary abelian normal subgroup of a well defined Sylow 2-subgroup  $\mathfrak{S}$  of  $\mathfrak{G}$  and  $\mathfrak{E}=\mathfrak{E}$  with amalgamated subgroup  $D=\mathfrak{D}$  has 939, 080, 024, 064 irreducible representations of dimension 8671 over GF(13). Therefore we have not tried to find one satisfying the Sylow 2-subgroup test of Step 5 c) of Algorithm 7.4.8 of [12].

**Proposition 7.1.** Let  $\mathfrak{G} = \langle \mathfrak{q}, \mathfrak{y}, \mathfrak{t}, \mathfrak{w} \rangle$  be the subgroup of  $Y = \mathrm{GL}_{8671}(13)$  constructed in Proposition 5.2. Let  $\mathfrak{x} = [(\mathfrak{yq}^2\mathfrak{yq}\mathfrak{yq}\mathfrak{q}^2)^{11}(\mathfrak{q}^2\mathfrak{y}^2\mathfrak{q}\mathfrak{yq}\mathfrak{y})^{11}(\mathfrak{q}\mathfrak{p}^2\mathfrak{q}\mathfrak{yq}\mathfrak{p}\mathfrak{q}\mathfrak{p})^4]^{12}$  and  $\mathfrak{s} = (\mathfrak{yt})^7$ . Let  $\mathfrak{r}_1 = (\mathfrak{s}\mathfrak{y}^3)^3$ ,  $\mathfrak{r}_2 = (\mathfrak{p}^2\mathfrak{w}\mathfrak{ys})^6$ ,  $\mathfrak{r}_3 = (\mathfrak{s}\mathfrak{y}\mathfrak{w}\mathfrak{ys})^2$ ,  $\mathfrak{r}_4 = (\mathfrak{s}\mathfrak{w}\mathfrak{ys}\mathfrak{w})^6$ , and  $\mathfrak{v} = (\mathfrak{w}\mathfrak{q}\mathfrak{w}\mathfrak{yq}\mathfrak{y})^7$ . Let  $\mathfrak{H} = \langle \mathfrak{r}_1, \mathfrak{r}_2, \mathfrak{r}_3, \mathfrak{r}_4, \mathfrak{v} \rangle$ .

Then the following statements hold:

- (a) The element  $\mathfrak{z} = (\mathfrak{xyw})^8$  is a 2-central involution  $\mathfrak{z} = (\mathfrak{xyw})^8$  of  $\mathfrak{G}$  such that  $C_{\mathfrak{G}}(\mathfrak{z}) = \mathfrak{H}$  has order  $2^{21} \cdot 3^7 \cdot 5 \cdot 7$ .
- (b)  $\mathfrak{H}$  has a faithful permutation representation  $P\mathfrak{H}$  of degree 258048 with stabilizer  $\mathfrak{U}_1 = \langle \mathfrak{v}\mathfrak{f}_2\mathfrak{v}, (\mathfrak{f}_1\mathfrak{f}_2\mathfrak{f}_1\mathfrak{v}\mathfrak{f}_1)^4, (\mathfrak{f}_2\mathfrak{f}_1\mathfrak{v}\mathfrak{f}_1\mathfrak{v})^4 \rangle$ , where  $\mathfrak{f}_1 = \mathfrak{x}\mathfrak{y}\mathfrak{w}$ ,  $\mathfrak{f}_2 = (\mathfrak{x}\mathfrak{y}\mathfrak{x}\mathfrak{w})^7$ ,  $\mathfrak{f}_3 = (\mathfrak{x}\mathfrak{w}\mathfrak{y}\mathfrak{w})^2$ .
- (c)  $\mathfrak{S} = \langle \mathfrak{s}, \mathfrak{u}_i \mid 1 \leq i \leq 5 \rangle$  is a Sylow 2-subgroup of  $\mathfrak{E}$  contained in  $\mathfrak{H}$  with center  $Z(\mathfrak{H}) = \langle \mathfrak{z} \rangle$ , where  $\mathfrak{u}_1 = (\mathfrak{rhr})^7$ ,  $\mathfrak{u}_2 = (\mathfrak{rhrhrhrh})^4$ ,  $\mathfrak{u}_3 = (\mathfrak{rhrh}^2\mathfrak{rh}^2\mathfrak{rhrh})^2$ ,  $\mathfrak{u}_4 = (\mathfrak{rhrh}^5\mathfrak{rh}^4)^2$  and  $\mathfrak{u}_5 = (\mathfrak{hswshrh})^7$ .
- (d)  $\mathfrak{B} = \langle \mathfrak{b}_i \mid 1 \leq i \leq 11 \rangle$  is the unique maximal elementary abelian normal subgroup of  $\mathfrak{S}$  where  $\mathfrak{b}_1 = \mathfrak{u}_1$ ,  $\mathfrak{b}_2 = \mathfrak{u}_2^2$ ,  $\mathfrak{b}_3 = (\mathfrak{u}_1\mathfrak{u}_2)^2$ ,  $\mathfrak{b}_4 = (\mathfrak{u}_1\mathfrak{u}_3)^2$ ,  $\mathfrak{b}_5 = (\mathfrak{u}_1\mathfrak{u}_4)^2$ ,  $\mathfrak{b}_6 = (\mathfrak{u}_2\mathfrak{u}_4)^4$ ,  $\mathfrak{b}_7 = (\mathfrak{u}_1\mathfrak{u}_2\mathfrak{u}_3)^4$ ,  $\mathfrak{b}_8 = (\mathfrak{u}_1\mathfrak{u}_2\mathfrak{u}_4)^4$ ,  $\mathfrak{b}_9 = (\mathfrak{u}_1\mathfrak{u}_3\mathfrak{u}_4)^4$ ,  $\mathfrak{b}_{10} = (\mathfrak{u}_2^2\mathfrak{u}_3)^2$ ,  $\mathfrak{b}_{11} = (\mathfrak{u}_1\mathfrak{u}_2\mathfrak{u}_4\mathfrak{u}_2)^2$ .
- (e)  $N_{\mathfrak{G}}(\mathfrak{B}) = \langle \mathfrak{x}, \mathfrak{y}, \mathfrak{w}, \mathfrak{s} \rangle = \mathfrak{E}.$
- (f)  $\mathfrak{D} = \langle \mathfrak{r}_i \mid 1 \leq i \leq 4 \rangle = N_{\mathfrak{H}}(\mathfrak{B}) = C_{\mathfrak{E}}(\mathfrak{z}).$
- (g) The Fitting subgroup  $\mathfrak O$  of  $\mathfrak H$  is extra-special of order  $2^{13}$  and center  $Z(\mathfrak O) = \langle \mathfrak z \rangle$ . It is generated by the twelve involutions

$$\begin{array}{lll} \mathfrak{p}_{1}=(\mathfrak{r}_{2})^{2}, & \mathfrak{p}_{2}=(\mathfrak{r}_{1}\mathfrak{r}_{2})^{4}, & \mathfrak{p}_{3}=(\mathfrak{r}_{3}\mathfrak{r}_{4})^{3}, & \mathfrak{p}_{4}=(\mathfrak{r}_{1}\mathfrak{r}_{2}\mathfrak{r}_{1})^{2}, \\ \mathfrak{p}_{5}=(\mathfrak{r}_{1}\mathfrak{r}_{3}\mathfrak{r}_{4})^{6}, & \mathfrak{p}_{6}=(\mathfrak{r}_{2}^{2}\mathfrak{r}_{4})^{2}, & \mathfrak{p}_{7}=(r_{3}\mathfrak{r}_{4}\mathfrak{r}_{1})^{6}, & \mathfrak{p}_{8}=(\mathfrak{r}_{4}\mathfrak{r}_{1}\mathfrak{r}_{3})^{6}, \\ \mathfrak{p}_{9}=(\mathfrak{r}_{1}\mathfrak{r}_{2}\mathfrak{r}_{3}^{2})^{4}, & \mathfrak{p}_{10}=(\mathfrak{r}_{1}\mathfrak{r}_{2}\mathfrak{r}_{3}\mathfrak{r}_{4})^{4}, & \mathfrak{p}_{11}=(r_{1}\mathfrak{r}_{2}\mathfrak{r}_{1}\mathfrak{r}_{2}\mathfrak{r}_{4})^{5}, & \mathfrak{p}_{12}=(\mathfrak{r}_{1}\mathfrak{r}_{2}\mathfrak{r}_{3}^{2}\mathfrak{r}_{4})^{4}, \\ and & \mathfrak{z}=(\mathfrak{p}_{1}\mathfrak{p}_{5})^{2}. \end{array}$$

- (h)  $\mathfrak{O}/Z(\mathfrak{O})$  has a complement  $\mathfrak{K} \cong 3U_4(3) : 2$  in  $\mathfrak{H}/Z(\mathfrak{O})$ .
- (i)  $\mathfrak{H} = \langle \mathfrak{a}, \mathfrak{b}, \mathfrak{p}_i \mid 1 \leq i \leq 12 \rangle = \langle \mathfrak{b}, \mathfrak{c}, \mathfrak{d}, \mathfrak{f}, \mathfrak{z}, \mathfrak{p}_i \mid 1 \leq i \leq 12 \rangle$ , where  $\mathfrak{a} = \mathfrak{r}_1 \mathfrak{r}_3^3 \mathfrak{v} \mathfrak{r}_3$ ,  $\mathfrak{b} = [\mathfrak{r}_1 \mathfrak{r}_3 \mathfrak{r}_1 \mathfrak{r}_3^2 \mathfrak{v}]^6 [\mathfrak{r}_1 \mathfrak{r}_3 \mathfrak{r}_1 \mathfrak{r}_3 \mathfrak{v} \mathfrak{r}_3 \mathfrak{v}]^{12}$ ,  $\mathfrak{c} = (\mathfrak{a}\mathfrak{b})^2$ ,  $\mathfrak{d} = (\mathfrak{b}\mathfrak{c})^7$  and  $\mathfrak{f} = [(\mathfrak{b}\mathfrak{a}^3)^5 \cdot (\mathfrak{a}^4 \mathfrak{b}^2 \mathfrak{a})^9 \cdot (\mathfrak{a}^2 \mathfrak{b}^3 \mathfrak{a}^4 \mathfrak{b})^3 \cdot (\mathfrak{b}\mathfrak{a}^3)^5]^3$ .
- (j)  $\mathfrak{H}$  is isomorphic to the finitely presented group  $H = \langle b, c, d, f, z, p_i \mid 1 \leq i \leq 12 \rangle$  having the following set  $\mathcal{R}(H)$  of defining relations:

$$b^{6} = c^{9} = d^{3} = f^{2} = z^{2} = 1, \quad p_{i}^{2} = 1 \quad for \quad 1 \leq i \leq 12, \\ (z,b) = (z,c) = (z,d) = (z,f) = 1, \quad (z,p_{i}) = 1 \quad for \quad 1 \leq i \leq 12, \\ (b,d) = (c,d) = 1, \quad b^{f} = b^{5}, \quad c^{f} = d(b^{3}cb^{2}c^{6}bcbc), \quad d^{f} = d^{2}, \\ (c^{-1}b^{-1})^{7}d = 1, \quad (c^{-1}b)^{9} = z, \quad b^{-1}c^{-1}b^{-3}c^{-1}b^{3}c^{-1}b^{-2}d = 1, \\ (bc^{-2}b)^{4}d^{2} = (bc^{-3}bc^{2}b)^{2}d = bc^{3}b^{-2}cb^{3}c^{-1}bc^{-1}bcb^{-1}cbd^{2} = 1, \\ b^{-1}c^{-3}b^{-1}c^{-1}bc^{-1}b^{2}c^{-1}cbc^{3}b^{-1}d = b^{-1}c^{3}bc^{-1}bc^{-1}c^{2}c^{-1}cb^{-1}c^{-3}b^{-1}d = 1, \\ c^{-2}b^{-2}c^{-1}bc^{-1}b^{-3}c^{-1}bc^{-2}cbc^{-1}bd^{2} = z, \\ b^{-2}c^{-1}bc^{-1}b^{-2}c^{-1}bc^{-2}b^{-2}cbc^{-1}bc^{2}c^{2}d = cb^{-1}c^{4}bc^{-1}b^{-2}c^{-1}bc^{-1}c^{2}bc^{2}d^{2} = 1, \\ c^{-2}b^{-1}c^{2}b^{-1}c^{-2}b^{-2}c^{2}bc^{2}c^{2}c^{-2}b^{-1}d = b^{-1}c^{-3}bc^{-1}b^{-1}c^{-1}b^{-2}c^{-1}bc^{-1}c^{2}bc^{-1}c^{2}b^{-2}c^{-1}d^{2}c^{-1}c^{-1}b^{-2}c^{-1}bc^{-1}c^{2}b^{-1}c^{-2}b^{-2}c^{-1}d^{-1}b^{-1}c^{-1}b^{-1}c^{-1}b^{-1}c^{-1}b^{-1}c^{-1}b^{-1}c^{-1}b^{-1}c^{-1}b^{-1}c^{-1}b^{-1}c^{-1}b^{-1}c^{-1}b^{-1}c^{-1}b^{-1}b^{-1}c^{-1}b^{-1}c^{-1}b^{-1}c^{-1}b^{-1}c^{-1}b^{-1}c^{-1}b^{-1}c^{-1}b^{-1}c^{-1}b^{-1}c^{-1}b^{-1}c^{-1}b^{-1}b^{-1}c^{-1$$

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\begin{array}{ll} p_{7}^{c}p_{1}p_{2}p_{3}p_{6}p_{7}p_{8}p_{9}p_{10}=z, & p_{8}^{c}p_{2}p_{3}p_{4}p_{5}p_{6}p_{9}p_{12}=1, & p_{9}^{c}p_{1}p_{2}p_{3}p_{4}p_{7}p_{11}=z,\\ p_{10}^{c}p_{2}p_{3}p_{5}p_{7}p_{9}p_{10}p_{12}=z, & p_{11}^{c}p_{1}p_{4}p_{7}p_{8}p_{11}p_{12}=z, & p_{12}^{c}p_{1}p_{2}p_{8}p_{9}=1,\\ p_{1}^{d}p_{2}p_{6}p_{9}p_{10}=p_{2}^{d}p_{1}p_{4}p_{5}p_{8}p_{9}p_{12}=p_{3}^{d}p_{1}p_{2}p_{3}p_{4}p_{6}p_{9}=z,\\ p_{4}^{d}p_{1}p_{2}p_{3}p_{4}p_{9}p_{10}=p_{5}^{d}p_{1}p_{3}p_{5}p_{7}p_{10}p_{12}=z, & p_{6}^{d}p_{2}p_{3}p_{6}p_{9}=z,\\ p_{7}^{d}p_{3}p_{4}p_{5}p_{7}p_{8}p_{10}p_{11}=z, & p_{8}^{d}p_{2}p_{3}p_{4}p_{5}p_{6}p_{8}p_{11}p_{12}=z, & p_{9}^{d}p_{2}p_{3}p_{5}p_{8}p_{12}=z,\\ p_{10}^{d}p_{2}p_{4}p_{9}p_{10}=p_{11}^{d}p_{1}p_{8}p_{9}p_{10}p_{11}=1, & p_{12}^{d}p_{1}p_{2}p_{4}p_{7}p_{8}p_{9}p_{10}p_{11}=z,\\ p_{1}^{f}p_{3}p_{9}p_{11}p_{12}=z, & p_{2}^{f}p_{1}p_{5}p_{6}p_{9}p_{12}=z, & p_{3}^{f}p_{1}p_{2}p_{3}p_{7}p_{8}p_{9}=1,\\ p_{4}^{f}p_{1}p_{2}p_{4}p_{6}p_{7}p_{8}p_{9}p_{10}=p_{5}^{f}p_{1}p_{5}p_{6}p_{8}p_{10}p_{12}=1, & p_{6}^{f}p_{4}p_{5}p_{7}p_{9}=z,\\ p_{7}^{f}p_{3}p_{4}p_{5}p_{8}p_{10}p_{11}=p_{8}^{f}p_{3}p_{4}p_{8}p_{11}p_{12}=1, & p_{9}^{f}p_{2}p_{3}p_{6}p_{7}p_{8}p_{9}p_{10}p_{11}p_{12}=z,\\ p_{10}^{f}p_{4}p_{5}p_{6}p_{7}p_{9}p_{10}=1, & p_{11}^{f}p_{1}p_{4}p_{6}p_{9}p_{10}p_{11}=z, & p_{12}^{f}p_{1}p_{4}p_{9}p_{12}=1. \end{array}
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- (k)  $H = \langle b, c, d, f, z, p_i | 1 \le i \le 12 \rangle$  has a faithful permutation representation of degree 258048 with stabilizer  $L_1 = \langle b, c^3, f, p_5 p_6 p_7 p_9 p_{10} p_{11} \rangle$ .
- (l) In  $H = \langle b, c, d, f, z, p_i | 1 \le i \le 12 \rangle$  let  $r = [(fp_1)^2(cfb)^3(fp_1)^2(fc^4)]^6$  and  $p = [(fp_1)^2(cfb)^3(fp_1)^2(b^2fc)]^5$ . Then  $H = \langle b, p, r \rangle$ , and it has 167 conjugacy classes whose representatives are given in Table A.3.
- (m) Table B.4 is the character table of H.
- *Proof.* (a) By Lemma 6.2 there is an isomorphism  $\sigma: \mathfrak{G} \to G$  such that  $\sigma(\mathfrak{q}) = q_1$ ,  $\sigma(\mathfrak{y}) = y_1$ ,  $\sigma(\mathfrak{w}) = w_1$ , and  $\sigma(\mathfrak{t}) = t_1$ . In particular,  $G = \langle q_1, y_1, w_1, t_1 \rangle$ . Let  $x_1$  be as in Lemma 6.2(i). Let  $z_1 = \sigma(\mathfrak{z}) = (x_1y_1w_1)^8$ ). Using the faithful permutation representation PG of G given in Lemma 6.2(a) and MAGMA one sees that  $C_G(z_1)$  has order  $2^{21} \cdot 3^7 \cdot 5 \cdot 7$ . Hence  $\mathfrak{z}$  is a 2-central involution of  $\mathfrak{G}$  by Lemma 6.2(k).
- In G let  $r_1 = s_1 y_1^3$ ,  $r_2 = y_1^2 w_1 y_1 s_1$ ,  $r_3 = (s_1 y_1 w_1 s_1)^2$ ,  $r_4 = (s_1 w_1 y_1 s_1 w_1)^6$ , and  $v = (w_1 q_1 w_1 y_1 q_1 y_1)^7$ . Let  $H = \langle r_1, r_2, r_3, r_4, v \rangle = \sigma(\mathfrak{H})$ . Then Proposition 6.1(i), (j) and Lemma 6.2(l) imply that  $H \leq C_G(z_1)$ . Another application of PG and MAGMA yields that  $C_G(z_1) = H$  and that  $Z(H) = \langle z_1 \rangle$ .
- (b) From Proposition 6.1(j) we deduce that  $U=C_{G_1}(z_1)=\langle f_1,f_2,f_3,v\rangle$  has order  $2^{18}\cdot 3^5\cdot 5$ , where  $f_1=x_1y_1w_1$ ,  $f_2=(x_1y_1x_1w_1)^7$ ,  $f_3=(x_1w_1y_1w_1y_1^2)^7$ . By Proposition 6.1(k) U has a subgroup  $U_1$  of order  $2^9\cdot 3^5\cdot 5$  which does not contain z generated by the elements of the statement. Hence  $H_1=C_G(z_1)$  has a faithful permutation representation  $PH_1$  of degree 258048 by (a).
- (c) By Proposition 6.1(l) the subgroup  $\mathfrak{S}$  is a Sylow 2-subgroup of  $\mathfrak{E} = \langle \mathfrak{x}, \mathfrak{y}, \mathfrak{w}, \mathfrak{s} \rangle$ . Lemmas 6.2(h) states that  $\mathfrak{E} \cong E$  where E is the finitely presented group of Lemma 2.1. Thus  $|\mathfrak{S}| = 2^{21}$  by Table B.1. Hence  $\mathfrak{S}$  is a Sylow 2-subgroup of  $\mathfrak{G}$  by (a). The equality  $Z(\mathfrak{H}) = \langle \mathfrak{z} \rangle$  has been checked computationally in PG.
  - (d) This statement follows now immediately from Proposition 6.1(l).
  - (e) This assertion is true by (d) and Lemma 6.2(h), (i) and (k).
- (f) By Proposition 6.1(i) and Lemma 6.2(l) we know that  $\mathfrak{D} = C_{\mathfrak{C}}(\mathfrak{z}) = \langle \mathfrak{r}_i \mid 1 \leq i \leq 4 \rangle$ , where  $\mathfrak{z} = \kappa(z_1)$ . Thus it suffices to check that  $N_H(B_1) = \langle r_1, r_2, r_3, r_4 \rangle$ . This has been done using the faithful permutation representation PG and MAGMA.
- (g) We verified computationally that the Fitting subgroup O of H is extra-special of order  $2^{13}$  and has center  $Z(O) = \langle z \rangle$ . The twelve involutions  $p_i$  generating the subgroup O have been calculated with MAGMA by means of PG and the program GetShortGens(H,0). We also verified that  $z = (p_1 p_5)^2$ .

(h) Let  $\alpha: H \to H/Z(H) = H_1$  be the canonical epimorphism of  $H = C_G(z)$  with kernel  $Z(H) = \langle z \rangle$ . By (c) H has a faithful permutation representation PH with stabilizer  $U_1 = \langle vf_2v, (f_1f_2f_1vf_1)^4, (f_2f_1vf_1v)^4 \rangle$ . As z does not belong to  $U_1$  its cosets provide a faithful permutation representation PH of H having degree 258048 by (b). Using MAGMA we checked that the subgroup  $\alpha(U_1)$  of  $H_1$  is a stabilizer of a faithful permutation representation  $PH_1$  of  $H_1$  of degree 129024. Applying  $PH_1$  and the MAGMA command DegreeReduction(H\_1) MAGMA calculated a faithful permutation representation  $PH_{11}$  of  $H_1 = \langle \alpha(r_i), \alpha(v) \rangle$  of degree 504 with stabilizer

$$U_{11} = \langle \alpha(r_1)\alpha(r_3)\alpha(r_1), [\alpha(r_1)\alpha(v)\alpha(r_3)\alpha(v)]^3, [\alpha(r_1)\alpha(v)\alpha(r_1)\alpha(r_3)\alpha(v)\alpha(r_3)]^3 \rangle.$$

Clearly, V = O/Z(H) is an elementary abelian normal subgroup of  $H_1$  of order  $2^{12}$ . Using  $PH_{11}$  and the command HasComplement (H\_1,V) MAGMA established a complement  $K_1$  of V in  $H_1$ . By means of the command CompositionFactors (K\_1) we saw that  $|K_1:K_1'|=2$ ,  $|Z(K_1')|=3$ , and  $K'/Z(K')\cong U_4(3)$ .

- (i) Using  $PH_{11}$  a MAGMA calculation confirmed that  $H_1$  is generated by  $a_1 = \alpha(r_1)(\alpha(r_3)^3\alpha(v)\alpha(r_3))$  and  $b_1 = [\alpha(r_1)\alpha(r_3)\alpha(r_1)\alpha(r_3)^2\alpha(v)]^6[\alpha(r_1)\alpha(r_3)\alpha(r_1)\alpha(r_3)\alpha(v)\alpha(r_3)\alpha(v)]^{12}$ . Both generators have order 6. Furthermore,  $K'_1 = \langle b_1, c_1 \rangle$ , where  $c_1 = (a_1b_1)^2$ . Using the command GetShortGens(K\_1',Z(K\_1')) we observed that the center  $Z(K'_1)$  of  $K'_1$  is generated by the element  $d_1 = (b_1c_1)^7$  of order 3.
- Let  $\beta: K_1' \to K_1'/Z(K_1') = K_2$  be the canonical epimorphism of  $K_1$  with kernel  $Z(K_1') = \langle d_1 \rangle$ . Let  $U_{12} = U_{11} \cap K_1'$  and  $U_{13} = \langle U_{12}, d_1 \rangle$ . Then  $K_2$  has a faithful permutation representation  $PK_2$  of degree 126 with stabilizer  $\beta(U_{13})$ . Let  $a_1 = \beta(b_1)$  and  $a_2 = \beta(c_1)$ . Then  $K_2 = \langle a_1, a_2 \rangle$ . Using the command FPGroup(K\_2) MAGMA calculates the following set  $\mathcal{R}(K_2)$  of defining relations of  $K_2$ :

$$\begin{aligned} a_1^6&=1,\quad a_2^9&=1,\\ (a_2^{-1}a_1^{-1})^7&=1,\quad (a_1a_2^{-2}a_1)^4=1,\quad (a_2^{-1}a_1)^9=1,\\ a_1^{-1}a_2^{-1}a_1^{-3}a_2^{-1}a_1^3a_2^{-1}a_1^3a_2^{-1}a_1^{-2}=a_1a_2^3a_1^{-2}a_2a_1^3a_2^{-1}a_1a_2a_1^{-1}a_2a_1=1,\\ a_1^{-1}a_2^{-3}a_1^{-1}a_2^{-1}a_1a_2^{-1}a_1^2a_2a_1^{-1}a_2a_1a_2^3a_1^{-1}=1,\\ a_1^{-1}a_2^3a_1a_2^{-1}a_1a_2^{-1}a_1^2a_2a_1^{-1}a_2a_1^{-1}a_2^{-3}a_1^{-1}=(a_1a_2^{-3}a_1a_2^2a_1)^2=1,\\ a_2^{-2}a_1^{-2}a_2^{-1}a_1a_2^{-1}a_1^{-3}a_2^{-1}a_1a_2a_1^{-2}a_2a_1a_2^{-1}a_1=1,\\ a_1^{-2}a_2^{-1}a_1a_2^{-1}a_1^{-2}a_1^{-3}a_2^{-1}a_1a_2a_1^{-2}a_2a_1a_2^{-1}a_1=1,\\ a_2^{-2}a_1^{-1}a_2^4a_1a_2^{-1}a_1^{-2}a_1^{-2}a_2a_1^{-1}a_2a_1^{-1}a_2a_1^{2}a_2^2=1,\\ a_2a_1^{-1}a_2^4a_1a_2^{-1}a_1^{-2}a_1^{-2}a_1a_2a_1^{-1}a_2^2a_1a_2a_1=1,\\ a_2^{-2}a_1^{-1}a_2^2a_1^{-1}a_2^{-2}a_1^{-2}a_2^2a_1a_2a_1^{-2}a_2^{-1}a_1a_2&1=1,\\ a_1^{-1}a_2^{-3}a_1a_2^2a_1a_2^{-1}a_1^{-2}a_1^{-1}a_2a_1^{-1}a_2a_1^{-1}a_2^{-2}a_1^{-2}=1,\\ a_2^2a_1^{-1}a_2^{-1}a_1a_2^{-1}a_1^{-1}a_2^{-1}a_1^{-1}a_2a_1^{-1}a_2a_1^{-1}a_2^{-2}a_1^{-2}=1,\\ a_2^2a_1^{-1}a_2^{-1}a_1a_2^{-1}a_1^{-1}a_2^{-1}a_1^{-1}a_2^{-1}a_1a_1$$

Since  $K'_1 = \langle b_1, c \rangle$ ,  $Z(K'_1) = \langle d_1 \rangle$ ,  $d_1 = (b_1c_1)^7 \neq 1$ ,  $d_1^3 = 1$  we lift the relations of  $\mathcal{R}(K_2)$  to  $K'_1$  by evaluating them in the permutation representation of  $K'_1 = (b_1c_1)^7 \neq 1$ .

 $\langle b_1, c_1, d_1 \rangle$ . Thus we obtain the following set  $\mathcal{R}(K'_1)$  of defining relations of  $K'_1$ :

```
\begin{aligned} b_1^6 &= c_1^9 = d_1^3 = 1, \quad (b_1,d_1) = (c_1,d_1) = 1, \\ (c_1^{-1}b_1^{-1})^7 d_1 &= 1, \quad (c_1^{-1}b_1)^9 = 1, \quad b_1^{-1}c_1^{-1}b_1^{-3}c_1^{-1}b_1^3c_1^{-1}b_1^3c_1^{-1}b_1^{-2}d_1 = 1, \\ (b_1c_1^{-2}b_1)^4 d_1^2 &= (b_1c_1^{-3}b_1c_1^2b_1)^2 d_1 = b_1c_1^3b_1^{-2}c_1b_1^3c_1^{-1}b_1c_1^{-1}b_1c_1b_1^{-1}c_1b_1d_1^2 = 1, \\ b_1^{-1}c_1^{-3}b_1^{-1}c_1^{-1}b_1c_1^{-1}b_1^2c_1b_1^{-1}c_1b_1c_1^3b_1^{-1}d_1 = 1, \\ b_1^{-1}c_1^3b_1c_1^{-1}b_1c_1^{-1}b_1^2c_1b_1^{-1}c_1b_1^{-1}c_1^{-3}b_1^{-1}d_1 = 1, \\ c_1^{-2}b_1^{-2}c_1^{-1}b_1c_1^{-1}b_1^{-3}c_1^{-1}b_1c_1b_1^{-2}c_1b_1c_1^{-1}b_1d_1^2 = 1, \\ b_1^{-2}c_1^{-1}b_1c_1^{-1}b_1^{-2}c_1^{-1}b_1^{-2}c_1b_1^{-1}c_1b_1^{-2}c_1b_1c_1^{-1}b_1d_1^2 = 1, \\ b_1^{-2}c_1^{-1}b_1c_1^{-1}b_1^{-2}c_1^{-1}b_1^{-2}c_1b_1^{-1}c_1b_1^{-1}c_1b_1^{-2}c_1^2d_1^2 = 1, \\ c_1b_1^{-1}c_1^4b_1c_1^{-1}b_1^{-2}c_1^{-1}b_1c_1b_1^{-1}c_1^2b_1c_1b_1d_1^2 = 1, \\ c_1b_1^{-1}c_1^4b_1c_1^{-1}b_1^{-2}c_1^{-1}b_1c_1b_1^{-1}c_1^2b_1c_1b_1d_1^2 = 1, \\ c_1^{-2}b_1^{-1}c_1^2b_1^{-1}c_1^{-2}b_1^{-2}c_1^2b_1c_1b_1^2c_1^{-2}b_1^{-1}d_1 = 1, \\ b_1^{-1}c_1^{-3}b_1c_1^2b_1c_1^{-1}b_1^{-1}c_1b_1^{-1}c_1b_1^{-1}c_1^2b_1^{-1}c_1^{-2} = 1, \\ c_1^2b_1^{-1}c_1^{-1}b_1c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1 = 1, \\ b_1c_1^{-2}b_1c_1^{-2}b_1c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1 = 1, \\ b_1^{-2}c_1^2b_1^{-2}c_1^{-2}b_1c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1 = 1, \\ b_1^{-2}c_1^2b_1^{-2}c_1^{-2}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1^{-1}b_1^{-1}c_1
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Using the faithful permutation representation  $PH_{11}$  of  $H_1$  and the MAGMA command HasComplement (K\_1, K\_1')) we see that  $K'_1$  has a complement  $\langle f_1 \rangle$  of order 2 such that  $b_1^{f_1} \in \{b_1, b_1^{-1}\}$ . In order to find a generator  $f_1$  of a suitable complement we searched for an involution  $f_1$  of  $K_1$  so that at least one of the elements  $c_1^{f_1}$ ,  $d_1 \cdot c_1^{f_1}$ , or  $d_1^2 \cdot c_1^{f_1}$  is in the collection of all words in  $b_1$  and  $c_1$  of length at most 16. This search was successful. The involution  $f_1 = [(b_1 a_1^3)^5 \cdot (a_1^4 b_1^2 a_1)^9 \cdot (a_1^2 b_1^3 a_1^4 b_1)^3 \cdot (b_1 a_1^3)^5]^3$  of  $K_1 = \langle a_1, b_1 \rangle$  satisfies the following set  $\mathcal{R}(f_1)$  of relations:  $f_1^2 = 1$ ,  $b_1^{f_1} = b_1^5$ ,  $c_1^{f_1} = d_1(b_1^3 c_1 b_1^2 c_1^6 b_1 c_1 b_1 c_1)$ ,  $d_1^{f_1} = d_1^2$ . Hence  $K_1 = \langle a_1, b_1 \rangle = \langle b_1, c_1, d_1, f_1 \rangle$  has a set  $\mathcal{R}(K_1)$  of defining relations consisting of  $\mathcal{R}(K'_1)$  and  $\mathcal{R}(f_1)$ .

The elementary abelian normal subgroup  $V = \alpha(O)$  is generated by the involutions  $q_i = \alpha(p_i)$  which are the images of the twelve generating involutions  $r_i$  of the Fitting subgroup O of H. Using the faithful permutation representation  $PH_{11}$  we calculated the images  $q_i^x$  in V for all 4 generators  $x \in \{b_1, c_1, d_1, f_1\}$  of  $K_1$ . Thus we obtained the following set of essential relations  $\mathcal{R}_2(V \rtimes K_1)$  of the semi-direct product  $(V \rtimes K_1)$ :

$$\begin{split} q_4^{d_1}q_1q_2q_3q_4q_9q_{10} &= q_5^{d_1}q_1q_3q_5q_7q_{10}q_{12} = q_6^{d_1}q_2q_3q_6q_9 = 1, \\ q_7^{d_1}q_3q_4q_5q_7q_8q_{10}q_{11} &= q_8^{d_1}q_2q_3q_4q_5q_6q_8q_{11}q_{12} = q_9^{d_1}q_2q_3q_5q_8q_{12} = 1, \\ q_{10}^{d_1}q_2q_4q_9q_{10} &= q_{11}^{d_1}q_1q_8q_9q_{10}q_{11} = q_{12}^{d_1}q_1q_2q_4q_7q_8q_9q_{10}q_{11} = 1, \\ q_1^{f_1}q_3q_9q_{11}q_{12} &= q_2^{f_1}q_1q_5q_6q_9q_{12} = q_3^{f_1}q_1q_2q_3q_7q_8q_9 = 1, \\ q_4^{f_1}q_1q_2q_4q_6q_7q_8q_9q_{10} &= q_5^{f_1}q_1q_5q_6q_8q_{10}q_{12} = q_6^{f_1}q_4q_5q_7q_9 = 1, \\ q_7^{f_1}q_3q_4q_5q_8q_{10}q_{11} &= q_8^{f_1}q_3q_4q_8q_{11}q_{12} = q_9^{f_1}q_2q_3q_6q_7q_8q_9q_{10}q_{11}q_{12} = 1, \\ q_{10}^{f_1}q_4q_5q_6q_7q_9q_{10} &= q_{11}^{f_1}q_1q_4q_6q_9q_{10}q_{11} = q_{12}^{f_1}q_1q_4q_9q_{12} = 1. \end{split}$$

Hence the set  $\mathcal{R}(H_1)$  of defining relations of the semi-direct product  $H_1 = (V \rtimes K_1)$  consists  $\mathcal{R}(K_1)$ ,  $\mathcal{R}_2(V \rtimes K_1)$  and the following relations:

$$\begin{split} q_j^2 &= 1 \quad \text{for all} \quad 1 \leq j \leq 12, \\ q_k \cdot q_j &= q_j \cdot q_k \quad \text{for all} \quad 1 \leq j, k \leq 12. \end{split}$$

In order to get a presentation of H we lift the generators  $a_1$  and  $b_1$  of  $K_1$  to H. Clearly,  $a=r_1r_3^2vr_3$  and  $b=(r_1r_3r_1r_3^2v)^6(r_1r_3r_1r_3vr_3v)^{12}$  of H map onto  $a_1$  and  $b_1$  of  $H_1$ , respectively. Let  $c=(ab)^2$ ,  $d=(bc)^7$  and  $f=[(ba^3)^5\cdot (a^4b^2a)^9\cdot (a^2b^3a^4b)^3\cdot (ba^3)^5]^3$ . Then  $c_1$ ,  $d_1$ ,  $f_1$  in  $H_1$  are images of c, d, f in H under  $\alpha$ , respectively. Since  $z=(p_1p_5)^2$  generates the center Z(O) of the Frattini subgroup  $O=\langle p_i \mid 1\leq i\leq 12\rangle$  it follows that

$$H = \langle a, b, p_i \mid 1 \le i \le 12 \rangle = \langle b, c, d, f, p_i \mid 1 \le i \le 12 \rangle.$$

The set of defining relations  $\mathcal{R}(H)$  of  $H = \langle b, c, d, f, p_i \mid 1 \leq i \leq 12 \rangle$  has been obtained by evaluating the lifted equations of the presentation  $\mathcal{R}(H_1)$  of  $H_1$  in the permutation representation PH with stabilizer  $U_1$  defined in the proof of (c). The resulting equations of  $\mathcal{R}(H)$  are stated in assertion (k).

The map  $\mathfrak{H} \to H$  sending each generator  $\mathfrak{x}$  of  $\mathfrak{H}$  in (i) to the corresponding generator  $x \in H$  in (j) is an isomorphism by (a) and the order of H.

- (k) The given stabilizer of the group  $H = \langle b, c, d, f, p_i \mid 1 \leq i \leq 12 \rangle$  has been found as follows. In the original permutation representation of the finitely presented group G of degree 306936 we checked that the subgroup  $L = \langle b, c^3, f \rangle$  has index 1032192 in H and that  $z \notin L$ . Using then MAGMA and the command MyCosetAction(H,L: maxsize:=10000000) we verified that L has the same index in the finitely presented group  $H = \langle b, c, d, f, p_i \mid 1 \leq i \leq 12 \rangle$ . In the corresponding permutation representation pH of this group we searched then for an element  $p \in O = \langle p_i \rangle$  such that  $z \notin L_1 = \langle L, p \rangle$ . MAGMA found an involution  $p \in O$  with these properties. Since O is extra-special of order  $2^{13}$  the command LookupWord(O,p) worked well. The word of p is stated in the assertion. Using the command MyCosetAction(H,L\_1:maxsize:=10000000) MAGMA established in 70 seconds the index  $|H:L_1|=258048$ .
- (l) Both elements r and p of H have order 6. Using the faithful permutation representation PH of H with stabilizer  $L_1$  and MAGMA it has been verified that  $H = \langle r, p, b \rangle$ . Since  $H_1 = H/\langle z \rangle$  has a faithful permutation representation of degree 504 we used it and Kratzer's Algorithm 5.3.18 of [12] to calculate a system of representatives of the classes of  $H_1 = \langle \alpha(r), \alpha(p), \alpha(b) \rangle$ .  $H_1$  has 123 conjugacy classes. Their representatives have been lifted to H. Using PH we have checked the

conjugacy of the lifted representatives and the products with the central involution z of H.

(m) The character table of H was calculated automatically by MAGMA using PH.  $\hfill\Box$ 

#### 8. Group order

In this section we check the group order of  $\mathfrak{G}$  by means of Thompson's group order formula and Theorem 6.1.4 of [14].

**Proposition 8.1.** Let  $\mathfrak{G} = \langle \mathfrak{q}, \mathfrak{y}, \mathfrak{t}, \mathfrak{w} \rangle$  be the subgroup of  $GL_{8671}(13)$  constructed in Proposition 5.2. Let  $\mathfrak{x} = [(\mathfrak{yq^2yqqq^2})^{11}(\mathfrak{q^2y^2qqqq})^{11}(\mathfrak{qp^2qqqqqqq})^4]^{12}$  and  $\mathfrak{s} = (\mathfrak{y}^5\mathfrak{t})^7$ .

 $\begin{array}{lll} Let \ \mathfrak{r}_1 &= (\mathfrak{sh}^3)^3, \ \mathfrak{r}_2 &= (\mathfrak{p}^2\mathfrak{whs})^6, \ \mathfrak{r}_3 &= (\mathfrak{shhhs})^2, \ \mathfrak{r}_4 &= (\mathfrak{shhsh})^6, \ and \ \mathfrak{v} &= (\mathfrak{whhsh})^7. \ Let \ \mathfrak{a} &= \mathfrak{r}_1\mathfrak{r}_3^3\mathfrak{v}\mathfrak{r}_3, \ \mathfrak{b} &= [\mathfrak{r}_1\mathfrak{r}_3\mathfrak{r}_1\mathfrak{r}_3^2\mathfrak{v}]^6[\mathfrak{r}_1\mathfrak{r}_3\mathfrak{r}_1\mathfrak{r}_3\mathfrak{v}\mathfrak{r}_3\mathfrak{v}]^{12}, \\ \mathfrak{f} &= [(\mathfrak{ba}^3)^5(\mathfrak{a}^4\mathfrak{b}^2\mathfrak{a})^9(\mathfrak{a}^2\mathfrak{b}^3\mathfrak{a}^4\mathfrak{b})^3(\mathfrak{ba}^3)^5]^3, \ \mathfrak{r} &= [(\mathfrak{fr}_2^2)^2[(\mathfrak{ab})^2\mathfrak{fb}]^3(f\mathfrak{r}_2^2)^2(\mathfrak{f}(\mathfrak{ab})^8)]^6, \\ and \ \mathfrak{p} &= [(\mathfrak{fr}_2^2)^2[(\mathfrak{ab})^2\mathfrak{fb}]^3(\mathfrak{fr}_2^2)^2(\mathfrak{b}^2\mathfrak{f}(\mathfrak{ab})^2)]^5. \end{array}$ 

Then the following assertions hold:

- (a)  $\mathfrak{z} = (\mathfrak{xyw})^8$  is a 2-central involution of  $\mathfrak{G}$  with centralizer  $\mathfrak{H} = C_{\mathfrak{G}}(\mathfrak{z}) = \langle \mathfrak{b}, \mathfrak{p}, \mathfrak{r} \rangle$ . Furthermore,  $\mathfrak{H}$  has 9, conjugacy classes of involutions  $2_i$ ,  $1 \leq i \leq 9$  with representatives given in Table A.3, and  $|\mathfrak{H}| = 2^{21} \cdot 3^7 \cdot 5 \cdot 7$ .
- (b)  $\mathfrak{u} = (\mathfrak{ps})^{14}$  is an involution of  $\mathfrak{G}$  with centralizer  $\mathfrak{U} = C_{\mathfrak{G}}(\mathfrak{u}) = \mathfrak{A}_1 = \langle \mathfrak{q}, \mathfrak{p}, \mathfrak{s} \rangle$ . Furthermore,  $\mathfrak{U}$  has 7 conjugacy classes of involutions  $2_i$ ,  $1 \leq i \leq 7$ , with representatives given in Table A.2, and  $|\mathfrak{U}| = 2^{19} \cdot 3^9 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13$ .
- (c) The Fitting subgroup  $\mathfrak{B}$  of  $\mathfrak{E} = \langle \mathfrak{x}, \mathfrak{y}, \mathfrak{w}, \mathfrak{s} \rangle$  is an elementary abelian group of order  $2^{11}$  such that  $\mathfrak{E}/\mathfrak{B} \cong \mathcal{M}_{24}$ .
- (d)  $\mathfrak{D} = \langle \mathfrak{r}_j \mid 1 \leq j \leq 4 \rangle = N_{\mathfrak{H}}(\mathfrak{B}) = C_{\mathfrak{E}}(\mathfrak{z}_1) \text{ where } \mathfrak{z}_1 = \mathfrak{r}^2 \in \mathfrak{E}.$
- (e)  $\mathfrak{D}_1 = \langle \mathfrak{x}, \mathfrak{y}, \mathfrak{s} \rangle = N_{\mathfrak{A}_1}(\mathfrak{B}) = C_{\mathfrak{E}}(\mathfrak{z}_2)$  where  $\mathfrak{z}_2 = (\mathfrak{x}\mathfrak{y}^2)^7 \in \mathfrak{E}$ .
- (f) & has two conjugacy classes of involutions represented by 3 and u.
- (g) The conjugacy classes of involutions  $2_1$ ,  $2_3$ ,  $2_5$ ,  $2_6$ ,  $2_8$  and  $2_9$  of  $\mathfrak{H}$  fuse with  $\mathfrak{F}$  in  $\mathfrak{G}$ . Its classes  $2_2$ ,  $2_4$  and  $2_7$  fuse with  $\mathfrak{u}$  in  $\mathfrak{G}$ .
- (h) The conjugacy classes of involutions  $2_1$ ,  $2_2$ ,  $2_3$  and  $2_4$  of  $\mathfrak U$  fuse with  $\mathfrak u$  in  $\mathfrak G$ . Its classes  $2_5$ ,  $2_6$  and  $2_7$  fuse with  $\mathfrak z$  in  $\mathfrak G$ .
- (i)  $\mathfrak{G} = 2132400816 \cdot |\mathfrak{U}| + 4388805476055 \cdot |\mathfrak{H}| = 2^{21} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 23 \cdot 29.$

*Proof.* (a) holds by Proposition 7.1(a), (j), (l) and Table A.3.

- (b) By Proposition 6.1(c) we know that  $\mathfrak{A}_1 = \langle \mathfrak{q}, \mathfrak{y}, \mathfrak{s} \rangle$ . Its conjugacy classes are classified in Table A.2. It asserts that the involution  $\mathfrak{u} = (\mathfrak{y}\mathfrak{s})^{14}$  generates the center of  $\mathfrak{A}_1$ . The equation  $C_{\mathfrak{G}}(\mathfrak{u}) = \mathfrak{A}_1$  is a consequence of Lemma 6.2(e) and (f). All other assertions hold by Table A.2.
  - (c) and (d) hold by Proposition 7.1(d) and (g), respectively.
  - (e) This is true by Proposition 6.1(d), (f) and Table A.1.
- (f) Table A.1 shows that E has 5 conjugacy classes of involutions and that  $z_1 = x^2$  is the representative of the unique 2-central conjugacy class of  $E = \langle x, y, e \rangle$ . By (d) the subgroup  $\mathfrak{D} = \langle \mathfrak{x}, \mathfrak{y}, \mathfrak{s} \rangle = \mathfrak{H} \cap \mathfrak{E}$ . It has 18 conjugacy classes  $2_k$ ,  $1 \leq k \leq 18$  of involutions. Using MAGMA and the faithful permutation representations  $P\mathfrak{H}$  and  $P\mathfrak{E}$  of Propositions 7.1(c) and 6.1(h), respectively, we calculated the fusion of the classes  $2_k$  of  $\mathfrak{D}$  in  $\mathfrak{H}$  and also in  $\mathfrak{E}$ . Thus we obtained a fusion graph  $\mathcal{G}(H)$  of the fusion of the  $\mathfrak{H}$ -classes and  $\mathfrak{E}$ -classes of order 2 in the matrix group  $\mathfrak{G}$ . It follows

that  $\mathfrak{G}$  has 2 conjugacy classes of involutions represented by  $\mathfrak{z}$  and  $\mathfrak{u}$  belonging to the classes  $2_1$  and  $2_2$  of  $\mathfrak{H}$ , respectively.

- (g) This statement follows also from the fusion graph  $\mathcal{G}$ .
- (h) In view of (e) we now study the fusion of the 11 classes of involutions of  $\mathfrak{R} = N_{\mathfrak{A}_1}(\mathfrak{B}) = \mathfrak{A}_1 \cap \mathfrak{E}$  in the two over groups  $\mathfrak{A}_1$  and  $\mathfrak{E}$ . Using MAGMA and the faithful permutation representation  $PA_1$  of Lemma 3.1(c) it follows that the classes  $2_1, 2_2, 2_3$  and  $2_4$  of  $\mathfrak{U}$  fuse with  $\mathfrak{u}$ , and that the remaining three classes of  $\mathfrak{U}$  fuse in  $\mathfrak{G}$  with  $\mathfrak{z}$ .
- (i) In order to simplify notation we replace the Gothic letters by Roman ones. Let  $r(z,u,z) = \left|\left\{(x,y) \in (z^G \cap H) \times (u^G \cap H) \middle| z \in \langle xy \rangle\right\}\right|$  and  $r(z,u,u) = \left|\left\{(x,y) \in (z^G \cap U) \times (u^G \cap U) \middle| u \in \langle xy \rangle\right\}\right|$ .

By Table B.4 H has 45 real z-special conjugacy classes. Here their representatives are denoted by  $\{t_j \mid 1 \leq j \leq 45\}$ . Let  $z_i$  and  $u_k$  be representatives of the H-classes of involutions fusing to z and u in G, respectively. For each triple  $(z_i, u_k, t_j)$  let

$$d(z_i, u_k, t_j) = \frac{|H|^2}{|C_H(z_i)| \cdot |C_H(u_k)| \cdot |C_H(t_j)|} \cdot \sum_{\psi \in Irr_{\mathbb{C}}(H)} \psi(z_i) \psi(u_k) \psi(t_j) \psi(1)^{-1}.$$

Then Theorem 1.6.4 of [14] and (g) imply that

$$r(z, u, z) = \sum_{i=1}^{6} \sum_{k=1}^{3} \sum_{j=1}^{45} d(z_i, u_k, t_j).$$

Using (g) and the values of the character Table B.4 of H these formulas yield that r(z, u, z) = 2132400816.

By Table B.5  $U = C_G(u) = A_1$  has 22 real u-special conjugacy classes. Denote their representatives by  $\{s_n \mid 1 \leq n \leq 22\}$ . Let  $u_i$  and  $z_k$  be representatives of the U-classes of involutions fusing to u and z in G, respectively. For each triple  $(u_i, z_k, s_n)$  let

$$d(u_i, z_k, s_n) = \frac{|U|^2}{|C_U(u_i)| \cdot |C_U(z_k)| \cdot |C_U(s_n)|} \cdot \sum_{\psi \in Irr_{\mathbb{C}}(U)} \psi(u_i)\psi(z_k)\psi(s_n)\psi(1)^{-1}.$$

Then Theorem 1.6.4 of [14] and (h) imply that

$$r(z, u, u) = \sum_{i=1}^{4} \sum_{k=1}^{3} \sum_{n=1}^{22} d(u_i, z_k, s_n).$$

Using (h) and the values of the character Table B.5 of U these formulas yields that r(z,u,u)=4388805476055. Now Theorem 4.2.1 of [12] due to J. G. Thompson implies

$$|G| = r(z, u, z) \cdot |C_G(u)| + r(z, u, u) \cdot |C_G(z)| = 2^{21} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 23 \cdot 29.$$

#### APPENDIX A. REPRESENTATIVES OF CONJUGACY CLASSES

# **A.1.** Conjugacy classes of $E(\mathrm{Fi}'_{24}) = \langle x, y, e \rangle$

Class	Representative	Class	Centralizer	2P	3P	5P	7P	11P	23P
1	1	1	$2^{21} \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 23$	1	1	1	1	1	1
$2_1$	$(e)^{2}$	276	$2^{19} \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$	1	21	21	21	$^{2_{1}}$	$2_1$
$2_2$	$(x)^2$	1771	$2^{21} \cdot 3^3 \cdot 5$	1	$2_2$	22	22	$2_2$	$2_2$
23	$(y)^3$	182160	$2^{17} \cdot 3 \cdot 7$	1	23	23	23	23	23
$2_{4}$	$(ye^2)^3$	1275120	$2^{17} \cdot 3$	1	24	24	$2_{4}$	$2_{4}$	$2_{4}$
$2_{5}$	$(xey)^3$	2040192	$2^{14} \cdot 3 \cdot 5$	1	$^{2_{5}}$	25	25	25	25
31	$(y)^2$	14508032	$2^8 \cdot 3^3 \cdot 5$	$3_{1}$	1	31	31	31	31
$3_2$	$(xy)^4$	124354560	$2^6 \cdot 3^2 \cdot 7$	$3_2$	1	$3_2$	$3_2$	$3_2$	$3_2$
41	$(xy)^3$	1020096	$2^{15} \cdot 3 \cdot 5$	22	41	41	41	41	41
$4_{2}$	$(xyey)^3$	1020096	$2^{15} \cdot 3 \cdot 5$	$2_2$	$4_2$	$4_2$	$4_2$	42	$4_2$
43	$(xe)^3$	5100480	$2^{15} \cdot 3$	$2_2$	43	43	43	43	$4_{3}$
$4_4$	$(xye)^6$	5100480	$2^{15} \cdot 3$	$\overline{2}_2$	$4_{4}$	$4_4$	$4_4$	$4_{4}$	$4_{4}$
45	e	11658240	$2^{11} \cdot 3 \cdot 7$	$2_{1}$	$4_{5}$	$4_{5}$	$4_{5}$	$4_{5}$	$4_{5}$
46	x	61205760	213	$\frac{-1}{2_2}$	46	46	$4_{6}$	46	46
47	$(x^2yxy^2)^3$	81607680	$2^{11} \cdot 3$	$\overline{2_3}$	47	47	47	47	47
48	$(x^2yxy^2e^2)^3$	81607680	$2^{11} \cdot 3$	23	48	48	48	48	48
49	$(xyeyey)^2$	244823040	211	$2_4$	49	49	49	49	49
$4_{10}$	$(x^3 exey)^2$	244823040	211	24	$4_{10}$	$4_{10}$	$4_{10}$	$4_{10}$	$4_{10}$
$4_{11}$	$\frac{(x^3y^2xe)}{x^3y^2xe}$	489646080	$\frac{2}{2^{10}}$	$\frac{24}{2_3}$	$4_{11}$	$4_{11}$	$4_{11}$	$4_{11}$	$4_{11}$
$4_{12}$	xyxyxey	489646080	$\frac{2}{2^{10}}$	$\frac{2_3}{2_4}$	$4_{12}$	$4_{12}$	$4_{12}$	$4_{12}$	$4_{12}$
5	$(xy^2)^3$	1044578304	$2^5 \cdot 3 \cdot 5$	5	5	1	5	5	5
61	$(xye^2y)^5$	14508032	$2^8 \cdot 3^3 \cdot 5$	31	$2_2$	61	61	61	61
$\frac{6_{1}}{6_{2}}$	$(xge^{-g})^2$	217620480	$2^8 \cdot 3^2$	31	$\frac{2}{2}$	$6_{2}$	$\frac{6_1}{6_2}$	$6_{2}$	$6_2$
63	$\frac{(x^2y^2)}{x^2y^2}$	217620480	$2^8 \cdot 3^2$	31	21	63	63	63	63
64	$(xy)^2$	870481920	$\frac{2}{2^{6} \cdot 3^{2}}$	$3_2$	$\frac{2_{1}}{2_{2}}$	64	64	64	64
65	(209)	2611445760	$2^{6} \cdot 3$	$\frac{3_{2}}{3_{1}}$	23	65	65	65	$6_{5}$
66	$ye^2$	2611445760	$\frac{2^{6} \cdot 3}{2^{6} \cdot 3}$	$\frac{3_1}{3_1}$	$\frac{2_3}{2_4}$	66	66	$6_{6}$	$6_{6}$
67	xey	10445783040	$2^4 \cdot 3$	$\frac{3_1}{3_2}$	25	67	67	67	67
$7_1$	$(xyxe)^2$	2984509440	$2^3 \cdot 3 \cdot 7$	$7_1$	$7_{2}$	$7_2$	1	$7_1$	$7_1$
$7_2$	$(xyexe)^2$	2984509440	$2^3 \cdot 3 \cdot 7$	$7_{2}$	$7_1$	$7_1$	1	$7_{2}$	$7_2$
81	$(xye)^3$	652861440	$\frac{2^{8} \cdot 3}{2^{8} \cdot 3}$	$\frac{12}{4_4}$	81	81	81	81	81
82	$(xey^2)^3$	652861440	$2^8 \cdot 3$	41	82	82	82	82	82
83	$(xyxy^2)^3$	652861440	$\frac{2^{8} \cdot 3}{2^{8} \cdot 3}$	$\frac{4_{1}}{4_{2}}$	83	83	83	83	83
84	$(xexey)^2$	979292160	29	$\frac{1}{4}$	84	84	84	84	84
85	$xy^2xe$	1958584320	$\frac{2}{2^8}$	43	85	85	85	85	85
86	$xyxy^2xy^2$	3917168640	27	$\frac{4}{6}$	86	86	86	86	86
87	xyeyey	7834337280	$\frac{2}{2^{6}}$	$\frac{4_{6}}{4_{9}}$	87	87	87	87	87
88	$x^3 exey$	7834337280	$\frac{2}{2^{6}}$	$4_{10}$	88	88	88	88	88
$\frac{08}{10_1}$	$(xye^2y)^3$	1044578304	$2^5 \cdot 3 \cdot 5$	5	$10_{1}$	$\frac{08}{22}$	$10_{1}$	101	$10_{1}$
$\frac{10_1}{10_2}$	$(xye \ y)$ $y^2e$	6267469824	$2 \cdot 3 \cdot 5$ $2^4 \cdot 5$	5	$10_{1}$	$\frac{22}{2_1}$	$10_{1}$	$10_{1}$	$10_{1}$
$10_{3}$		6267469824	$2^4 \cdot 5$	5	$10_{4}$	$\frac{2_1}{2_5}$	$10_{4}$	$10_{3}$	$10_{4}$
$\frac{10_{3}}{10_{4}}$	$\frac{xyxeyey}{xyxexe^3}$	6267469824	$2 \cdot 5$ $2^4 \cdot 5$	5	$10_{4}$ $10_{3}$	$\frac{25}{25}$	$10_{4}$ $10_{3}$	$10_{4}$	$10_{4}$ $10_{3}$
11	$\frac{xyxexe}{(ye)^2}$	22790799360	$2 \cdot 3$ $2 \cdot 11$	11	11	11	11	104	11
$\frac{11}{12_1}$	xe	2611445760	$2^6 \cdot 3$	62	43	121	121	121	121
$\frac{12_1}{12_2}$	$(xye)^2$	2611445760	$2^6 \cdot 3$	$\frac{6_2}{6_2}$	$\frac{43}{4_4}$	$\frac{12_1}{12_2}$	$\frac{12_1}{12_2}$	$\frac{12_1}{12_2}$	$\frac{12_1}{12_2}$
$\frac{12_2}{12_3}$	(xye) $xy$	5222891520	$2 \cdot 3$ $2^5 \cdot 3$	$\frac{6_2}{6_4}$	44	$\frac{12_2}{12_3}$	$\frac{12_2}{12_3}$	$\frac{12_2}{12_3}$	$\frac{12_2}{12_3}$
$\frac{12_3}{12_4}$	xy $xyey$	5222891520	$\frac{2^5 \cdot 3}{2^5 \cdot 3}$	64	$\frac{4_1}{4_2}$	$\frac{12_3}{12_4}$	$\frac{12_3}{12_4}$	$\frac{12_3}{12_4}$	$\frac{12_3}{12_4}$
$\frac{124}{125}$	$\frac{xyey}{y^3e}$	10445783040	$2 \cdot 3$ $2^4 \cdot 3$	$6_{3}$	$\frac{4_{2}}{4_{5}}$	$12_{4}$ $12_{5}$	$\frac{124}{125}$	$12_{4}$ $12_{5}$	$\frac{12_4}{12_5}$
$\frac{12_{5}}{12_{6}}$		10445783040	$2 \cdot 3$ $2^4 \cdot 3$		$\frac{45}{47}$	$12_{6}$	$12_{6}$	$12_{6}$	$12_{6}$
$\frac{126}{127}$	$\frac{x^2yxy^2}{x^2yxy^2e^2}$	10445783040	$2 \cdot 3$ $2^4 \cdot 3$	6 <sub>5</sub>	$\frac{47}{48}$	$\frac{126}{127}$	$\frac{126}{127}$	$\frac{126}{127}$	$\frac{126}{127}$
$\frac{127}{141}$		8953528320	$2 \cdot 3$ $2^3 \cdot 7$	$7_{2}$	$\frac{48}{142}$	$\frac{127}{142}$	$\frac{127}{21}$	$\frac{127}{141}$	$\frac{127}{141}$
$\frac{14_1}{14_2}$	$\frac{(xyey^2)^2}{(xye^2xe)^2}$	8953528320 8953528320	$\frac{2^3 \cdot 7}{2^3 \cdot 7}$	$\frac{7_2}{7_1}$	$\frac{14_2}{14_1}$	$\frac{14_2}{14_1}$	$\frac{2_1}{2_1}$	$\frac{14_1}{14_2}$	$\frac{14_1}{14_2}$
$\frac{14_{2}}{14_{3}}$		17907056640	$\frac{2^{2}\cdot 7}{2^{2}\cdot 7}$	_	$\frac{14_1}{14_4}$	$\frac{14_1}{14_4}$	_	$\frac{14_2}{14_3}$	$\frac{14_2}{14_3}$
$\frac{14_{3}}{14_{4}}$	xyxe	17907056640	$\frac{2\cdot 7}{2^2\cdot 7}$	71	$\frac{14_4}{14_3}$	_	23	_	_
$\frac{14_4}{15_1}$	xyexe	16713252864	$2 \cdot 7$ $2 \cdot 3 \cdot 5$	72	14 <sub>3</sub>	$\frac{14_3}{3_1}$	23	$\frac{14_4}{15_2}$	$\frac{14_4}{15_1}$
	$xy^2$		$2 \cdot 3 \cdot 5$ $2 \cdot 3 \cdot 5$	151	5	•	152		-
152	$xy^4$	16713252864	$\frac{2 \cdot 3 \cdot 5}{2^5}$	152		31	151	151	152
16	xexey	15668674560		84	16	16	16	16	16
$20_1$	xyxexe	6267469824	$\begin{array}{c} 2^4 \cdot 5 \\ 2^4 \cdot 5 \end{array}$	$10_1$	$20_{1}$	42	$20_1$	$20_{1}$	$20_1$
$20_{2}$	$xy^2xy^4$	6267469824	2 · 5	$10_{1}$	$20_{2}$	$4_1$	$20_{2}$	$20_{2}$	$20_{2}$

Conjugacy classes of  $E(Fi'_{24}) = \langle x, y, e \rangle$  (continued)

Class	Representative	Class	Centralizer	2P	3P	5P	7P	11P	23P
$21_{1}$	xeye	23876075520	$3 \cdot 7$	$21_{1}$	$7_1$	$21_{2}$	$3_2$	$21_{1}$	$21_{1}$
$21_{2}$	$xy^2eye$	23876075520	$3 \cdot 7$	$21_{2}$	$7_2$	$21_{1}$	$3_2$	$21_{2}$	$21_{2}$
22	ye	22790799360	$2 \cdot 11$	11	22	22	22	$2_1$	22
$23_{1}$	$xy^2e$	21799895040	23	$23_{1}$	$23_{1}$	$23_{2}$	$23_{2}$	$23_{2}$	1
$23_{2}$	$xey^3$	21799895040	23	$23_{2}$	$23_{2}$	$23_{1}$	$23_{1}$	$23_{1}$	1
$24_{1}$	xye	10445783040	$2^4 \cdot 3$	$12_{2}$	81	$24_{2}$	$24_{2}$	$24_{1}$	$24_{1}$
$24_{2}$	$xy^3$	10445783040	$2^4 \cdot 3$	$12_{2}$	81	$24_{1}$	$24_{1}$	$24_{2}$	$24_{2}$
$24_{3}$	$xey^2$	20891566080	$2^3 \cdot 3$	$12_{3}$	82	$24_{3}$	$24_{3}$	$24_{3}$	$24_{3}$
$24_{4}$	$xyxy^2$	20891566080	$2^3 \cdot 3$	$12_{4}$	83	$24_{4}$	$24_{4}$	$24_{4}$	$24_{4}$
$28_{1}$	$xyey^2$	17907056640	$2^2 \cdot 7$	$14_{1}$	$28_{2}$	$28_{2}$	$4_5$	$28_{1}$	$28_{1}$
$28_{2}$	$xye^2xe$	17907056640	$2^2 \cdot 7$	$14_{2}$	$28_{1}$	$28_{1}$	$4_5$	$28_{2}$	$28_{2}$
$30_{1}$	$xye^2y$	16713252864	$2 \cdot 3 \cdot 5$	$15_{1}$	$10_{1}$	$6_{1}$	$30_{2}$	$30_{2}$	$30_{1}$
$30_{2}$	$x^2y^2xy^2$	16713252864	$2 \cdot 3 \cdot 5$	$15_{2}$	$10_{1}$	$6_1$	$30_{1}$	$30_{1}$	$30_{2}$

**A.2.** Conjugacy classes of  $A_1 = Aut(2 \operatorname{Fi}_{22}) = \langle y, q, s \rangle$ 

Class	Representative	Centralizer	$^{2P}$	3P	5P	7P	11P	13P
1	1	$2^{19} \cdot 3^9 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13$	1	1	1	1	1	1
21	$(ys)^{14}$	$2^{19} \cdot 3^9 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13$	1	21	21	21	21	21
$\frac{2_{1}}{2_{2}}$	$\frac{(gs)}{(y)^7}$	$2^{17} \cdot 3^{6} \cdot 5 \cdot 7 \cdot 11$	1	$\frac{2_1}{2_2}$	$\frac{2_1}{2_2}$	$\frac{2_1}{2_2}$	$\frac{2_1}{2_2}$	$\frac{2_1}{2_2}$
$\frac{22}{2_3}$	(9)	$2^{14} \cdot 3^{6} \cdot 5^{2} \cdot 7$	1	$\frac{22}{2_3}$	$\frac{22}{2_3}$	$2_{3}$	23	$\frac{22}{2_3}$
$\frac{23}{24}$	$(yq)^{10}$	$2^{19} \cdot 3^4 \cdot 5$	1	$\frac{23}{24}$	$\frac{23}{24}$	$\frac{23}{24}$	$\frac{23}{24}$	$\frac{23}{24}$
$\frac{24}{2_5}$	$\frac{(yq)}{(y^2q)^8}$	$2^{19} \cdot 3^4 \cdot 5$	1	$\frac{24}{25}$	25	$\frac{24}{25}$	25	$\frac{24}{25}$
	$\frac{(y^{3}qs)^{9}}{(y^{3}qs)^{9}}$	$\frac{2^{14} \cdot 3^{4} \cdot 5}{2^{14} \cdot 3^{4} \cdot 5}$	1	$\frac{25}{26}$	$\frac{25}{26}$	$\frac{25}{26}$	$\frac{25}{26}$	$\frac{25}{26}$
$\frac{2_6}{2_7}$		$2^{17} \cdot 3^3$	1	$\frac{26}{27}$	$\frac{26}{27}$	$\frac{26}{27}$	$\frac{26}{27}$	$\frac{26}{27}$
$\frac{27}{3_1}$	$\frac{(ysq)}{(qs)^4}$	$2^{10} \cdot 3^7 \cdot 5 \cdot 7$	31	1	$\frac{27}{3_1}$	$\frac{27}{3_1}$	31	31
	(48)	$\begin{array}{c} 2 \cdot 3 \cdot 3 \cdot 7 \\ 2^9 \cdot 3^9 \end{array}$	_	1	$\frac{3_1}{3_2}$	$3_2$	$3_{2}$	
$\frac{3_2}{3_3}$	$\frac{q}{(ysq)^8}$	$\frac{2 \cdot 3}{2^8 \cdot 3^7}$	$\frac{3_2}{3_3}$	1	$\frac{3_2}{3_3}$	$3_{3}$	$\frac{3_{2}}{3_{3}}$	$\frac{3_2}{3_3}$
$\frac{3_{3}}{3_{4}}$	(ysq)	$\frac{2 \cdot 3}{2^5 \cdot 3^7}$	$3_4$	1	$\frac{3_3}{3_4}$	$3_4$	$3_{4}$	$3_{4}$
	(9 43)	$2^{11} \cdot 3^4 \cdot 5 \cdot 7$			$\frac{3_4}{4_1}$			
$\frac{4_1}{4}$	$(ys)^{\gamma}$	$\begin{array}{c} 2 \cdot 3 \cdot 3 \cdot 7 \\ \hline 2^{14} \cdot 3^4 \end{array}$	$\frac{2_1}{2_5}$	$\frac{4_1}{4_2}$	$\frac{41}{42}$	$\frac{4_1}{4_2}$	$\frac{4_1}{4_2}$	$\frac{4_1}{4_2}$
42	$\frac{(yqs)^9}{(y^3sq^2)^5}$	$2^{14} \cdot 3 \cdot 5$						
$\frac{4_3}{4_4}$	$(y \ sq)$ $(yqyq^2)^6$	$\begin{array}{c} 2 \cdot 3 \cdot 3 \\ 2^{13} \cdot 3^3 \end{array}$	$2_{5}$	43	43	$\frac{4_3}{4_4}$	43	43
		$2^{11} \cdot 3^2 \cdot 5$	$2_{5}$	$\frac{4_4}{4}$	44		44	$\frac{4_4}{4}$
45	(94)	$2^{13} \cdot 3 \cdot 5$ $2^{13} \cdot 3$	24	45	45	45	45	45
46	$(y^2q)^4$ $(as)^3$	$2^{11} \cdot 3^2$	25	46	46	46	46	46
47		$\frac{2^{11} \cdot 3^{2}}{2^{11} \cdot 3^{2}}$	24	47	47	47	47	47
48	$(ysq)^6$	$2^{11} \cdot 3^{2}$	27	48	48	48	48	48
49	$y^2 sys$	$2^{12} \cdot 3$ $2^{12} \cdot 3$	27	$4_{9}$	$4_9$	$4_{9}$	$4_{9}$	$4_{9}$
$\frac{4_{10}}{4_{10}}$	$(y^4sq)^3$	2 - 3	25	$4_{10}$	410	410	$4_{10}$	410
$4_{11}$	$(y^2sy^2sq)^3$ $(y^2asa)^3$	$2^{11} \cdot 3$	24	$4_{11}$	$4_{11}$	$4_{11}$	$4_{11}$	$4_{11}$
$4_{12}$	(9 404)	$2^{10} \cdot 3$	$^{27}$	$4_{12}$	$4_{12}$	$4_{12}$	$4_{12}$	$4_{12}$
5	$(yq)^4$	$2^5 \cdot 3 \cdot 5^2$	5	5	1	5	5	5
61	$(y^3q^2)^5$	$2^{10} \cdot 3^7 \cdot 5 \cdot 7$	$3_1$	21	61	61	61	61
62	(9 490)	$2^9 \cdot 3^9$	$3_2$	21	62	62	62	62
63	$(y^4s)^2$	$2^8 \cdot 3^7$	$3_3$	$2_1$	63	63	63	63
64	$(yqsys)^5$	$2^8 \cdot 3^5 \cdot 5$	31	$\frac{2}{2}$	64	64	64	64
65	$q^2 sqsqs$	$2^8 \cdot 3^5 \cdot 5$	$3_1$	23	65	65	65	65
66	$(y^3 sq^2 s)^3$ $(y^3 q^2 s)^7$	$2^8 \cdot 3^6$	$3_2$	22	66	66	66	66
67	(9 9 0)	$   \begin{array}{r}     2^8 \cdot 3^4 \cdot 7 \\     2^8 \cdot 3^4 \cdot 7   \end{array} $	$3_1$	$^{2_3}$	68	67	68	67
68	(3 13-1 /		31	23	67	68	67	68
69	$(y^2sqsq^2)^3$	$2^5 \cdot 3^7$	34	21	69	69	69	69
610		$2^9 \cdot 3^4$	$3_2$	$2_5$	$6_{10}$	$6_{10}$	$6_{10}$	$6_{10}$
611	(9 409 0)	$2^9 \cdot 3^4$	$3_2$	24	611	611	611	611
612		$2^{10} \cdot 3^3$	31	$\frac{2_4}{2}$	$\frac{6_{12}}{c}$	$6_{12}$	612	$\frac{6_{12}}{c}$
613	$(yqyq^2)^4$	$2^{10} \cdot 3^3$	31	25	$6_{13}$	$6_{13}$	$6_{13}$	613
614	$(y^2sq^2)^3$	$2^5 \cdot 3^6$	32	23	614	614	614	614
615	$\frac{y^3s}{7}$	$2^5 \cdot 3^5$	33	23	$6_{15}$	$6_{15}$	615	$6_{15}$
616	$\frac{y^7q}{(ysq)^4}$	$2^5 \cdot 3^5$	33	$\frac{2}{2}$	$6_{16}$	616	616	616
617	$(ysq)^{\frac{1}{2}}$	$2^8 \cdot 3^3$	$3_3$	27	$6_{17}$	$6_{17}$	617	617
618	$y^-qsys$	$2^8 \cdot 3^3$	33	27	618	618	618	618
619	3 2 2	$2^8 \cdot 3^3$	32	27	$6_{19}$	619	$6_{19}$	619
620	$y^3 sqy^2 sq^2 s$	$2^8 \cdot 3^3$	$3_1$	26	$6_{20}$	620	620	620
621	$\begin{array}{r} y^3 sqy^2 sq^2 s \\ y^2 q^2 sqsq^2 yq \\ (y^4 sq)^2 \end{array}$	$2^8 \cdot 3^3$	31	27	$6_{21}$	$6_{21}$	$6_{21}$	$6_{21}$
622	$(y^4 sq)^2$	$2^7 \cdot 3^3$	33	25	$6_{22}$	$6_{22}$	$6_{22}$	$6_{22}$
623		$2^7 \cdot 3^3$	33	24	623	$6_{23}$	623	623
$6_{24}$	$(y^3qs)^3$	$2^5 \cdot 3^4$	$3_4$	26	$6_{25}$	$6_{24}$	$6_{25}$	$6_{24}$
$6_{25}$	$(y^3 sqyq)^3$	$2^5 \cdot 3^4$	34	26	$6_{24}$	$6_{25}$	$6_{24}$	$6_{25}$
626	$y^3q^2syq$	$2^5 \cdot 3^4$	$3_4$	23	$6_{27}$	$6_{26}$	$6_{27}$	626
627	$y^4q^2sqsys$	$2^5 \cdot 3^4$	$3_4$	23	$6_{26}$	$6_{27}$	$6_{26}$	$6_{27}$
$6_{28}$	$(y^2qsq)^2$	$2^5 \cdot 3^3$	$3_4$	$^{27}$	$6_{28}$	$6_{28}$	$6_{28}$	$6_{28}$

Conjugacy classes of  $A_1 = Aut(2\operatorname{Fi}_{22}) = \langle y,q,s \rangle$  (continued)

Class	Representative	Centralizer	2P	3P	5P	7P	11P	13P
629	$y^3qsqs$	$2^{5} \cdot 3^{3}$	34	27	629	629	629	629
630	$y^4qy^3q$	$2^{5} \cdot 3^{3}$	$3_3$	27	630	630	630	630
631	$y^3sysqys$	$2^{5} \cdot 3^{3}$	$3_3$	26	631	631	631	631
7	$(y)^2$	$2^3 \cdot 3 \cdot 7$	7	7	7	1	7	7
81	$y^2s$	$2^8 \cdot 3$	$4_{6}$	81	81	81	81	81
82	$(uaua^2)^3$	$2^8 \cdot 3$	$4_4$	82	82	82	82	82
83	$(y^4qs)^3$	$2^8 \cdot 3$	$4_4$	83	83	83	83	83
84	$ \begin{array}{c} (y^4qs)^3 \\ (ysq^2sq)^3 \end{array} $	$2^8 \cdot 3$	$4_4$	84	84	84	84	84
85	$y^3qy^2s$	$2^8 \cdot 3$	$4_4$	85	85	85	85	85
86	$(y^2q)^2$	28	$^{46}$	86	86	86	86	86
87	$(ysq)^3$	$2^{6} \cdot 3$	$4_{8}$	87	87	87	87	87
88	$y^2 sqys$	26	$4_9$	88	88	88	88	88
$9_{1}$	$(yqs)^4$	$2^4 \cdot 3^4$	$9_{1}$	$3_2$	$9_{1}$	$9_{1}$	$9_{1}$	$9_{1}$
$9_{2}$	$(y^2 s q^2)^2$	$2^3 \cdot 3^4$	$9_{2}$	$_{3_{2}}$	$9_{2}$	$9_{2}$	$9_{2}$	$9_{2}$
93	$(y^{3}qs)^{2}$	$2^2 \cdot 3^3$	93	34	93	93	93	93
101	$(y^3q^2)^3$	$2^5 \cdot 3 \cdot 5^2$	5	101	21	101	101	101
102	$(y^4qysq)^3$	$2^4 \cdot 3 \cdot 5^2$	5	102	23	102	102	102
103	$(yq)^2$	$2^{5} \cdot 5$	5	103	24	103	103	103
104	$\frac{(y^3sq^2)^2}{yq^2}$	$2^5 \cdot 5$ $2^3 \cdot 3 \cdot 5$	5	104	25	104	104	104
105	$\frac{yq^2}{y^2q^2s}$	$2^4 \cdot 3 \cdot 5$	5	105	22	105	105	105
106	$\frac{y q s}{(y^3 q)^2}$	$2 \cdot 3$ $2^2 \cdot 11$	5 11	106	26	106	106	$10_{6}$
$\frac{11}{12_1}$	$\frac{(y^*q)}{(yq^2sq)^5}$	$2^6 \cdot 3^3 \cdot 5$	61	11 4 <sub>1</sub>	$\frac{11}{12_1}$	$\frac{11}{12_1}$	$\frac{1}{12_1}$	$\frac{11}{12_1}$
	$yq^2ysysq^2ys$	$\begin{array}{c} 2 \cdot 3 \cdot 3 \\ 2^8 \cdot 3^3 \end{array}$	_	_				
$\frac{12_2}{12_3}$	$yq \ ysysq \ ys \ (yqs)^3$	$\frac{2^5 \cdot 3^4}{2^5 \cdot 3^4}$	$6_{10}$	$\frac{4_2}{4_2}$	$\frac{12_2}{12_3}$	$\frac{12_2}{12_3}$	$\frac{12_2}{12_3}$	$\frac{12_2}{12_3}$
$\frac{12_{3}}{12_{4}}$	$\frac{(yqs)}{(y^4qys)^3}$	$\frac{2 \cdot 3}{2^5 \cdot 3^4}$	$6_{2}$	$\frac{4_{2}}{4_{1}}$	$12_{4}$	$\frac{12_{3}}{12_{4}}$	$\frac{12_{3}}{12_{4}}$	$12_{4}$
$12_{5}$	$\frac{(yqys)}{(yqyq^2)^2}$	$\frac{2^8 \cdot 3^2}{2^8 \cdot 3^2}$	$6_{13}$	$\frac{4}{4}$	$12_{5}$	$12_{5}$	$12_{5}$	$12_{5}$
$\frac{125}{126}$	$\frac{(gqgq^{-})}{(y^4qs)^2}$	$\frac{2^{8} \cdot 3^{2}}{2^{8} \cdot 3^{2}}$	$6_{13}$	$4_4$	$12_{6}$	$12_{6}$	$12_{6}$	$12_{6}$
127	$y^2qsyqys$	$2^{6} \cdot 3^{3}$	$6_{10}$	44	$12_{7}$	$12_{7}$	$12_{7}$	$12_{7}$
128	$yq^2sqysq^2$	$2^7 \cdot 3^2$	613	$4_4$	$12_{8}$	$12_{8}$	128	$12_{8}$
129	$y^2sysqsqyq$	$\frac{2^7 \cdot 3^2}{2^7 \cdot 3^2}$	$6_{13}$	$4_2$	129	129	$12_{9}$	$12_{9}$
$12_{10}$	$y^4qy^3qys$	$2^{5} \cdot 3^{3}$	$6_{22}$	$4_2$	$12_{10}$	$12_{10}$	$12_{10}$	$12_{10}$
$12_{11}$	$y^2qy^2qy^2sq$	$2^8 \cdot 3$	$6_{13}$	$4_{3}$	$12_{11}$	$12_{11}$	$12_{11}$	$12_{11}$
$12_{12}$	as	$2^6 \cdot 3^2$	$6_{12}$	$4_7$	$12_{12}$	$12_{12}$	$12_{12}$	$12_{12}$
$12_{13}$	$(ysq)^2$	$2^6 \cdot 3^2$	$6_{17}$	$4_{8}$	$12_{13}$	$12_{13}$	$12_{13}$	$12_{13}$
$12_{14}$	$y^2sqsqyq$	$2^6 \cdot 3^2$	$6_{12}$	$4_5$	$12_{14}$	$12_{14}$	$12_{14}$	$12_{14}$
$12_{15}$	$y^3 sqsyq^2$	$2^6 \cdot 3^2$	$6_{17}$	$4_{9}$	$12_{15}$	$12_{15}$	$12_{15}$	$12_{15}$
$12_{16}$	$y^4s$	$2^4 \cdot 3^3$	$6_3$	$4_1$	$12_{16}$	$12_{16}$	$12_{16}$	$12_{16}$
$12_{17}$	yqysys	$2^{5} \cdot 3^{2}$	$6_{19}$	$4_9$	$12_{17}$	$12_{17}$	$12_{17}$	$12_{17}$
$12_{18}$	$y^4sysq$	$2^{5} \cdot 3^{2}$	$6_{19}$	$4_{8}$	$12_{18}$	$12_{18}$	$12_{18}$	$12_{18}$
$12_{19}$	$y^2 q s q s q s$	$2^{5} \cdot 3^{2}$	$6_{11}$	47	$12_{19}$	$12_{19}$	$12_{19}$	$12_{19}$
$12_{20}$	$(yqys)^2$	$2^{6} \cdot 3$	$6_{10}$	46	$12_{20}$	$12_{20}$	$12_{20}$	$12_{20}$
1221	$y^2 s y^2 s q$ $y^3 q^2 y s$	$2^{6} \cdot 3$	$6_{12}$	$4_{11}$	$12_{21}$	$12_{21}$	$12_{21}$	$12_{21}$
1222	$y^3q^2ys$	$2^4 \cdot 3^2$	623	47	$12_{22}$	$12_{22}$	$12_{22}$	$12_{22}$
1223	$y^2 sqysq$	$2^4 \cdot 3^2$	623	45	$12_{23}$	$12_{23}$	$12_{23}$	1223
1224	yqysqys	$2^4 \cdot 3^2$ $2^4 \cdot 3^2$	$6_{28}$	49	1224	1224	1224	1224
1225	$yqysyq^2s$		628	$4_{8}$	$12_{25}$	$12_{25}$	$12_{25}$	$12_{25}$
1226	$y^4sq$	$2^5 \cdot 3$	$6_{22}$	410	1226	1226	1226	1226
1227	$y^2qsq$	$2^4 \cdot 3$	$6_{28}$	412	1228	1227	1228	1227
1228	$y^3qsq$ $y^4q$	$2^4 \cdot 3$ $2 \cdot 13$	628	412	$12_{27}$	1228	1227	1228
13	$\frac{y \cdot q}{(ys)^2}$	$2 \cdot 13$ $2^3 \cdot 3 \cdot 7$	13	13 14 <sub>1</sub>	13	$\frac{13}{2_1}$	13 14 <sub>1</sub>	1 14 <sub>1</sub>
141		$\frac{2^3 \cdot 3 \cdot 7}{2^2 \cdot 3 \cdot 7}$	7		141			
$14_{2}$	yqsq	2 . 3 . 7	7	$14_{2}$	$14_{2}$	$^{2_{3}}$	$14_{2}$	$14_{2}$

Conjugacy classes of  $A_1 = Aut(2\operatorname{Fi}_{22}) = \langle y,q,s \rangle$  (continued)

Class	Representative	Centralizer	2P	3P	5P	7P	11P	13P
143		$2^2 \cdot 7$	7	$14_{3}$	$14_{3}$	$2_2$	$14_{3}$	$14_{3}$
15	$(y^3q^2)^2$	$2^3 \cdot 3 \cdot 5$	15	5	31	15	15	15
161	$y^2q$	$2^{5}$	86	$16_{1}$	$16_{1}$	$16_{1}$	$16_{1}$	$16_{1}$
162	$y^2qs$	$2^{5}$	86	$16_{2}$	$16_{2}$	$16_{2}$	$16_{2}$	162
181	$u^4sugugus$	$2^4 \cdot 3^4$	91	$6_2$	181	181	181	181
182	$(u^4 aus)^2$	$2^{3} \cdot 3^{4}$	$9_{2}$	62	182	182	182	182
183	$y^4qsq$	$2^{3} \cdot 3^{3}$	91	$6_{14}$	186	183	186	183
184	$y^3 sq^2 s$	$2^{3} \cdot 3^{3}$	91	66	185	184	185	184
185	$u^4 as us$	$2^{3} \cdot 3^{3}$	$9_{1}$	66	184	$18_{5}$	$18_{4}$	$18_{5}$
186	$y^2q^2syq^2$	$2^{3} \cdot 3^{3}$	$9_{1}$	$6_{14}$	$18_{3}$	186	$18_{3}$	186
187	$(yqs)^2$	$2^4 \cdot 3^2$	$9_{1}$	$6_{10}$	187	187	187	187
188	$u^2 a s u^2 s$	$2^4 \cdot 3^2$	$9_{1}$	$6_{11}$	188	188	188	188
189	$y^2sq^2$	$2^2 \cdot 3^3$	$9_{2}$	$6_{14}$	$18_{9}$	189	189	$18_{9}$
1810	$y^2 sq sq^2$	$2^{2} \cdot 3^{3}$	93	$6_9$	$18_{10}$	$18_{10}$	$18_{10}$	$18_{10}$
1811	$usausa^2$	$2^2 \cdot 3^3$	$9_{2}$	66	$18_{11}$	$18_{11}$	$18_{11}$	$18_{11}$
$18_{12}$	$y^3qs$	$2^2 \cdot 3^2$	93	$6_{24}$	$18_{13}$	$18_{12}$	$18_{13}$	$18_{12}$
$18_{13}$	$y^3sqyq$	$2^2 \cdot 3^2$	$9_{3}$	$6_{25}$	$18_{12}$	$18_{13}$	$18_{12}$	$18_{13}$
$20_{1}$	$yq^2ys$	$2^3 \cdot 3 \cdot 5$	$10_{1}$	$20_{1}$	$4_1$	$20_{1}$	$20_{1}$	$20_{1}$
$20_{2}$	yq	$2^3 \cdot 5$	$10_{3}$	$20_{2}$	$4_{5}$	$20_{2}$	$20_{2}$	$20_{2}$
$20_{3}$	$yq$ $y^3sq^2$	$2^3 \cdot 5$	$10_{4}$	$20_{3}$	$4_{3}$	$20_{3}$	$20_{3}$	$20_{3}$
21	$(y^3q^2s)^2$	$2^2 \cdot 3 \cdot 7$	21	7	21	$_{3_{1}}$	21	21
$22_{1}$	$y^3q$	$2^2 \cdot 11$	11	$22_{1}$	$22_{1}$	$22_{3}$	$2_2$	$22_{3}$
$22_{2}$	ysqs	$2^2 \cdot 11$	11	$22_{2}$	$22_{2}$	$22_{2}$	$2_1$	$22_{2}$
$22_{3}$	$y^2 sqs$	$2^2 \cdot 11$	11	$22_{3}$	$22_{3}$	$22_{1}$	$2_2$	$22_{1}$
$24_{1}$	$yqyq^2$	$2^5 \cdot 3$	$12_{5}$	82	$24_{1}$	$24_{1}$	$24_{1}$	$24_{1}$
$24_{2}$	$y^4qs$	$2^5 \cdot 3$	$12_{6}$	83	$24_{2}$	$24_{2}$	$24_{2}$	$24_{2}$
$24_{3}$	$ysq^2sq$	$2^5 \cdot 3$	$12_{6}$	84	$24_{3}$	$24_{3}$	$24_{3}$	$24_{3}$
$24_{4}$	$y^2qysyq$	$2^5 \cdot 3$	$12_{5}$	85	$24_{4}$	$24_{4}$	$24_{4}$	$24_{4}$
$24_{5}$	ysq	$2^4 \cdot 3$	$12_{13}$	87	$24_{7}$	$24_{7}$	$24_{5}$	$24_{5}$
$24_{6}$	yqys	$2^4 \cdot 3$	$12_{20}$	81	$24_{8}$	$24_{8}$	$24_{6}$	$24_{6}$
$24_{7}$	$ysq^2$	$2^4 \cdot 3$	$12_{13}$	87	$24_{5}$	$24_{5}$	$24_{7}$	$24_{7}$
$24_{8}$	$y^2qys$	$2^4 \cdot 3$	$12_{20}$	81	$24_{6}$	$24_{6}$	$24_{8}$	$24_{8}$
26	$y^5q$	$2 \cdot 13$	13	26	26	26	26	$2_1$
28	$\frac{ys}{y^3q^2}$	$2^2 \cdot 7$	$14_{1}$	28	28	$4_1$	28	28
$30_{1}$		$2^3 \cdot 3 \cdot 5$	15	$10_{1}$	$6_{1}$	$30_{1}$	$30_{1}$	$30_{1}$
$30_{2}$	yqsys	$2^2 \cdot 3 \cdot 5$	15	$10_{5}$	64	$30_{2}$	$30_{2}$	$30_{2}$
$30_{3}$	$y^4qysq$	$2^2 \cdot 3 \cdot 5$	15	$10_{2}$	65	$30_{3}$	$30_{3}$	$30_{3}$
$36_{1}$	yqs	$2^3 \cdot 3^2$	$18_{7}$	$12_{3}$	$36_{1}$	$36_{1}$	$36_{1}$	$36_{1}$
$36_{2}$	ysysqs	$2^3 \cdot 3^2$	187	$12_{3}$	$36_{2}$	$36_{2}$	$36_{2}$	$36_{2}$
$36_{3}$	$y^4qys$	$2^2 \cdot 3^2$	$18_{2}$	$12_{4}$	$36_{3}$	$36_{3}$	$36_{3}$	$36_{3}$
421	$y^3q^2s$	$2^2 \cdot 3 \cdot 7$	21	$14_{2}$	$42_{3}$	67	$42_{3}$	$42_{1}$
$42_{2}$	$y^3 sqs$	$2^2 \cdot 3 \cdot 7$	21	$14_{1}$	$42_{2}$	$6_{1}$	$42_{2}$	$42_{2}$
$42_{3}$	$y^2qysq^2$	$2^2 \cdot 3 \cdot 7$	21	$14_{2}$	$42_{1}$	68	$42_{1}$	$42_{3}$
60	$yq^2sq$	$2^2 \cdot 3 \cdot 5$	$30_{1}$	$20_{1}$	$12_{1}$	60	60	60

**A.3.** Conjugacy classes of  $H(\mathrm{Fi}'_{24}) = \langle r, p, b \rangle$ 

No.	Class	Representative	Centralizer	2P	3P	5P	7P
1	1	1	$2^{21} \cdot 3^7 \cdot 5^1 \cdot 7^1$	1	1	1	1
2	$2_{1}$	$z = (p^4 r^2)^6$	$2^{21} \cdot 3^7 \cdot 5^1 \cdot 7^1$	1	21	21	21
3	$2_2$	$(pr)^6$	$2^{19} \cdot 3^4 \cdot 5^1$	1	$2_2$	$2_2$	$2_2$
4	23	$r^6$	$2^{20} \cdot 3^2 \cdot 5^1$	1	23	$2_3$	$2_3$
5	$2_{4}$	$(p^2br)^9$	$2^{14} \cdot 3^4 \cdot 5^1$	1	$^{2_4}$	$^{2_4}$	$2_4$
6	$2_{5}$	$\gamma (n^2 hr)^9$	$2^{14} \cdot 3^4 \cdot 5^1$	1	25	$2_{5}$	25
7	26	$p^6$	$2^{17} \cdot 3^3$	1	26	26	26
8	$2_7$	$zp^{o}$	$2^{17} \cdot 3^3$	1	27	27	27
9	28	$(rb^3)^4$	$2^{16} \cdot 3^{1}$	1	$^{2_{8}}$	$^{2_{8}}$	28
10	29	$(p^2r)^3$	$2^{13} \cdot 3^2$	1	29	29	29
11	31	$p^4$	$2^8 \cdot 3^7 \cdot 5^1 \cdot 7^1$	$3_{1}$	1	31	31
12	$3_2$	$z(prpb^2)^4$	$2^{12} \cdot 3^{6}$	$3_2$	1	$3_2$	$3_2$
13	$3_{3}$	$z(pr^2b)^4$	$2^{10} \cdot 3^7$	$3_{3}$	1	$3_{3}$	33
14	$3_{4}$	$z(pb^2)^3$	$2^5 \cdot 3^7$	$3_{4}$	1	$3_{4}$	$3_{4}$
15	$_{3_{5}}$	$b^2$	$2^8 \cdot 3^5$	$3_{5}$	1	$_{3_{5}}$	$_{3_{5}}$
16	$_{3_{6}}$	$(prbr)^2$	$2^5 \cdot 3^6$	36	1	$_{3_{6}}$	$_{3_{6}}$
17	$_{37}$	$r^4$	$2^6 \cdot 3^4$	37	1	37	37
18	$4_{1}$	$(p^{3}br)^{6}$	$2^{15} \cdot 3^5 \cdot 5^1$	$2_1$	$4_1$	$4_{1}$	$4_1$
19	$4_{2}$	$(rb)^4$	$2^{15} \cdot 3^2$	$2_1$	$4_2$	$4_2$	$4_2$
20	$4_{3}$	$(prb)^5$	$2^{13} \cdot 3^1 \cdot 5^1$	$2_3$	$4_{3}$	$4_{3}$	$4_3$
21	$4_{4}$	$(pr)^3$	$2^{11} \cdot 3^2 \cdot 5^1$	$2_2$	$4_4$	$4_4$	$4_4$
22	$4_{5}$	$r^3$	$2^{13} \cdot 3^2$	$2_3$	$4_5$	$4_5$	$4_5$
23	$4_{6}$	$(p^2brb)^2$	$2^{14} \cdot 3^{1}$	$2_3$	$4_{6}$	$4_{6}$	$4_{6}$
24	$4_{7}$	$p^3$	$2^{11} \cdot 3^2$	$^{26}$	$4_7$	$4_7$	47
25	$4_{8}$	$(p^4rprb)^3$	$2^{11} \cdot 3^2$	$2_2$	$4_{8}$	$4_{8}$	$4_{8}$
26	$4_{9}$	$zp^3$	$2^{11} \cdot 3^2$	$^{2_{6}}$	$4_9$	$4_{9}$	$4_9$
27	$4_{10}$	$(p^2 r p r^2)^2$	$2^{14}$	$2_3$	$4_{10}$	$4_{10}$	$4_{10}$
28	$4_{11}$	$(p^2r^2bprb)^3$	$2^{11} \cdot 3^{1}$	$2_2$	$4_{11}$	$4_{11}$	$4_{11}$
29	$4_{12}$	$p^4r^3$	$2^{12}$	$2_3$	$4_{12}$	$4_{12}$	$4_{12}$
30	$4_{13}$	$z(p^2b^3r)^3$	$2^{10} \cdot 3^1$	$2_8$	$4_{13}$	$4_{13}$	$4_{13}$
31	$4_{14}$	$z(p^2rb)^3$	$2^{10} \cdot 3^{1}$	$^{26}$	$4_{14}$	$4_{14}$	$4_{14}$
32	$4_{15}$	$(p^2b^3r)^3$	$2^{10} \cdot 3^{1}$	$2_8$	$4_{15}$	$4_{15}$	$4_{15}$
33	$4_{16}$	$(p^2b^3)^3$	$2^9 \cdot 3^1$	$2_7$	$4_{16}$	$4_{16}$	$4_{16}$
34	$4_{17}$	$(p^2bprb)^3$	$2^9 \cdot 3^1$	$2_7$	$4_{17}$	$4_{17}$	$4_{17}$
35	$4_{18}$	$(rb^{3})^{2}$	$2^{10}$	$2_8$	$4_{18}$	$4_{18}$	$4_{18}$
36	$4_{19}$	$p^2r^3$	210	$2_8$	$4_{19}$	$4_{19}$	$4_{19}$
37	$4_{20}$	$prpr^3$	29	$2_8$	$4_{20}$	$4_{20}$	$4_{20}$
38	5	$\frac{(p^2b)^6}{(p^2b)^5}$	$2^6 \cdot 3^1 \cdot 5^1$	5	5	1	5
39	61	$(p^2b)^5$	$2^8 \cdot 3^7 \cdot 5^1 \cdot 7^1$	31	21	61	61
40	62	$(prpb^2)^4$	$2^{12} \cdot 3^{6}$	32	21	62	62
41	63	$(p^2brpr)^3$	$2^{10} \cdot 3^7$	33	$2_1$	63	63
42	64	(po)	2 <sup>5</sup> · 3 <sup>7</sup>	$3_4$	$2_1$	64	$6_4$
43	65	$(p^4rb^2r)^2$	$2^8 \cdot 3^5$ $2^9 \cdot 3^4$	$\frac{3}{5}$	$2_1$	65	65
44	66	$\frac{(prbpbr)^3}{(p^2r^2bnrb)^2}$		$3_{3}$	22	66	66
45	67			$\frac{3}{2}$	22	67	67
46	68	$\frac{(pr^2b^2pb^2)^2}{(pr^4b^2pb^2)^2}$	$\frac{2^5 \cdot 3^6}{2^{11} \cdot 3^2}$	36	$\frac{2_1}{2}$	68	68
47	69	$(p^4bprpb)^2$		$3_2$	23	69	69
48	610	$\frac{p^2 r b p b r^2 b p r}{(p^2 b^3)^2}$	$2^8 \cdot 3^3$	$\frac{3}{2}$	26	610	610
49	611	(p o°)-	$2^8 \cdot 3^3$ $2^8 \cdot 3^3$	31	27	611	$6_{11}$
50	612	$z(p^3bpbrb)$		32	25	612	612
51 52	613	$p^3bpbrb$		32	24	613	613
	614	p- (-212-12\2	$\frac{2^8 \cdot 3^3}{2^8 \cdot 3^3}$	31	26	614	614
53 54	615	$\frac{(p^2b^2pb^2)^2}{z(p^2b^2pb^2)^2}$	$\frac{2^{8} \cdot 3^{5}}{2^{8} \cdot 3^{3}}$	$3_3$ $3_3$	26	615	615
	616	$\frac{z(p^-b^-pb^-)^-}{b}$	$\frac{2^8 \cdot 3^3}{2^8 \cdot 3^3}$		27	616	616
55	$6_{17}$	b	2 . 3	$3_5$	$2_7$	$6_{17}$	$6_{17}$

Conjugacy classes of  $H(Fi'_{24}) = \langle r, p, b \rangle$  (continued)

[No | Class | Representative | | Centralizer | 2P | 3P | 5P | 7P ]

No	Class	Representative	Centralizer	2P	3P	5P	7P
56	$6_{18}$	zb	$2^8 \cdot 3^3$	$_{3_{5}}$	$^{2_{6}}$	$6_{18}$	$6_{18}$
57	619	$z(p^2rbpbr^2bpr)$	$2^8 \cdot 3^3$ $2^6 \cdot 3^4$	$3_2$	$^{27}$	$6_{19}$	$6_{19}$
58	$6_{20}$	$(p^{3}br)^{4}$	$2^{6} \cdot 3^{4}$	37	$2_1$	$6_{20}$	$6_{20}$
59	$6_{21}$	$(pr)^2$	$2^7 \cdot 3^3$	$_{3_{5}}$	$2_2$	$6_{21}$	$6_{21}$
60	622	$(p^2br)^3$	$2^5 \cdot 3^4$	34	24	623	622
61	623	$(p^2br)^{15}$	$2^5 \cdot 3^4$	34	24	622	623
62	624	$z(n^2br)^{15}$	$2^5 \cdot 3^4$	34	$2_5$	$6_{25}$	$6_{24}$
63	$6_{25}$	$z(p^2br)^3$	$2^5 \cdot 3^4$	34	$2_{5}$	$6_{24}$	$6_{25}$
64	626	$p^3r^2prpb$	$2^7 \cdot 3^2$	$3_2$	$^{29}$	626	626
65	627	$p^5r$	$2^5 \cdot 3^3$	35	$\frac{-3}{2_4}$	627	627
66	628	$z(p^2rb)^2$	$2^{5} \cdot 3^{3}$	34	$2_7$	628	628
67	629	$\frac{z(p+b)}{p^4b}$	$2^5 \cdot 3^3$	35	27	$6_{29}$	$6_{29}$
68	630	$z(p^4b)$	$2^5 \cdot 3^3$	$3_{5}$	26	630	630
69	631	$(p^2r^2bpb)^2$	$2^{5} \cdot 3^{3}$	34	$^{-6}$	631	631
70	632	$\frac{(p^{7} \cdot p^{5})}{z(p^{5}r)}$	$2^5 \cdot 3^3$	35	$2_{5}$	$6_{32}$	$6_{32}$
71	633	prbr	$2^5 \cdot 3^3$	$\frac{3_{6}}{3_{6}}$	$\frac{25}{27}$	633	633
72	$6_{34}$	z(prbr)	$\frac{2^{5} \cdot 3^{3}}{2^{5} \cdot 3^{3}}$	$3_{6}$	26	$6_{34}$	$6_{34}$
73		$\frac{z(pror)}{(p^2b^3r)^2}$	$\frac{2^{7} \cdot 3^{1}}{2^{7} \cdot 3^{1}}$	$3_{5}$	$\frac{26}{28}$	$6_{35}$	6
74	$6_{35}$ $6_{36}$	$\frac{(p\ b\ r)}{r^2}$	$\frac{2 \cdot 3}{2^5 \cdot 3^2}$	$\frac{3_{5}}{3_{7}}$	$\frac{28}{23}$		635
75	$6_{37}$	$\frac{r}{p^2r}$	$\frac{2 \cdot 3}{2^3 \cdot 3^2}$	$\frac{37}{37}$	$\frac{23}{29}$	$6_{36}$ $6_{37}$	636
76		$(pb)^3$	$2^1 \cdot 3^1 \cdot 7^1$	7	7	7	637
	7					7	1
77	81	$\frac{(p^3br)^3}{(pr^2b)^3}$	$\frac{2^8 \cdot 3^2}{2^9 \cdot 3^1}$	41	81	81	81
78	82	(pr b)	$\frac{2^8 \cdot 3}{2^8 \cdot 3^1}$	42	82	82	82
79	83	$(prbpb^2)^3$		42	83	83	83
80	84	$p^2bpbr$	$\frac{2^8}{2^8}$	410	84	84	84
81	85	$\frac{p^2 r p r^2}{\frac{2}{3} \frac{2}{3} \frac{1}{1} \frac{1}{1}}$		410	85	85	85
82	86	$p^2r^2brbr$	2 <sup>8</sup>	46	86	86	86
83	87	$p^3br^4$	2 <sup>8</sup>	$4_{10}$	87	87	87
84	88	$p^2brb$ $(nr^2)^3$	28	46	88	88	88
85	89		$2^6 \cdot 3^1$	47	89	89	89
86	810	$rb^3$	2 <sup>6</sup>	$4_{18}$	810	810	810
87	811	$p^2bpb$	26	$4_{18}$	811	811	811
88	812	$pr^3$	26	49	812	812	812
89	$9_{1}$	$(prbpbr)^2$	$2^5 \cdot 3^4$	$9_{1}$	$3_{3}$	$9_{1}$	$9_{1}$
90	$9_{2}$	$(p^2b^2rbr)^4$	$2^{3} \cdot 3^{4}$	$9_{2}$	$3_3$	$9_{2}$	$9_{2}$
91	$9_{3}$	$(p^2brpr)^2$	$2^1 \cdot 3^4$	$9_{3}$	$3_3$	$9_{3}$	$9_{3}$
92	$9_{4}$	$(pb^2)^{10}$	$2^2 \cdot 3^3$	$9_{5}$	$3_{4}$	$9_{5}$	$9_{4}$
93	$9_{5}$	$(pb^2)^2$	$2^2 \cdot 3^3$	$9_{4}$	$3_{4}$	$9_{4}$	$9_{5}$
94	$10_{1}$	$(p^2b)^3$	$2^6 \cdot 3^1 \cdot 5^1$	5	$10_{1}$	$2_1$	$10_{1}$
95	$10_{2}$	$(prb)^6$	$2^{5} \cdot 5^{1}$	5	$10_{3}$	$^{2_{3}}$	$10_{3}$
96	$10_{3}$	$(prb)^2$	$2^5 \cdot 5^1$	5	$10_{2}$	$2_3$	$10_{2}$
97	$10_{4}$	$(p^3rb)^2$	$2^{5} \cdot 5^{1}$	5	$10_{4}$	$2_2$	$10_{4}$
98	$10_{5}$	pbrb	$2^4 \cdot 5^1$	5	$10_{5}$	$^{2_{4}}$	$10_{5}$
99	$10_{6}$	z(pbrb)	$2^4 \cdot 5^1$	5	$10_{6}$	$^{2_{5}}$	$10_{6}$
100	$12_{1}$	$(prpb^2)^2$	$2^9 \cdot 3^5$	$6_2$	$4_1$	$12_{1}$	$12_{1}$
101	$12_{2}$	$(pbpb^3)^3$	$2^7 \cdot 3^5$	63	$4_1$	$12_{2}$	$12_{2}$
102	$12_{3}$	$(p^2rbrprb)^2$	$2^9 \cdot 3^3$	$6_2$	$4_1$	$12_{3}$	$12_{3}$
103	$12_{4}$	$p^4rb^2r$	$2^6 \cdot 3^4$	$6_{5}$	$4_1$	$12_{4}$	124
104	$12_{5}$	$pr^2b^2pb^2$	$2^4 \cdot 3^5$ $2^8 \cdot 3^2$	$6_8$	$4_1$	$12_{5}$	$12_{5}$
105	$12_{6}$	$z(p^4bprpb)$		69	$4_5$	$12_{6}$	$12_{6}$
106	$12_{7}$	$p^4bprpb$	$2^8 \cdot 3^2$	69	$4_{5}$	127	$12_{7}$
107	128	$(pr^2b)^2$	$2^{7} \cdot 3^{2}$	63	$4_2$	$12_{8}$	128
108	129	$(p^3br)^2$	$2^5 \cdot 3^3$	$6_{20}$	$4_1$	$12_{9}$	$12_{9}$
109	$12_{10}$	$(pb^2rb)^2$	$2^{5} \cdot 3^{3}$	$6_{20}$	$4_1$	$12_{10}$	1210
110	1211	$p^2rpr^2b^2$	$2^{6} \cdot 3^{2}$	67	$4_4$	$12_{11}$	12 <sub>11</sub>
111	1212	p	$2^{6} \cdot 3^{2}$	614	47	$12_{12}$	$12_{12}$
	12	P	2 3	V14	-1	12	12

Conjugacy classes of  $H(Fi'_{24}) = \langle r, p, b \rangle$  (continued)

No	Class	Representative	Centralizer	2P	3P	5P	7P
112	$12_{13}$	$p^4rprb$	$2^6 \cdot 3^2$	67	$4_{8}$	$12_{13}$	12 <sub>13</sub>
		p rpro	$2^6 \cdot 3^2$	_			
113	$12_{14}$	$(prbpb^2)^2$		65	42	$12_{14}$	1214
114	$12_{15}$	zp	$2^6 \cdot 3^2$	$6_{14}$	$4_{9}$	$12_{15}$	$12_{15}$
115	$12_{16}$	$p^2r^2bpr$	$2^7 \cdot 3^1$	$6_{9}$	$4_3$	$12_{16}$	$12_{16}$
116	$12_{17}$	$p^2rp^2br$	$2^7 \cdot 3^1$	69	$4_{6}$	$12_{17}$	$12_{17}$
117	$12_{18}$	$z(p^2b^2pb^2)$	$2^5 \cdot 3^2$	$6_{15}$	$4_7$	$12_{18}$	$12_{18}$
118	$12_{19}$	$p^{2}rp^{2}b^{2}r$	$2^5 \cdot 3^2$	66	48	$12_{19}$	$12_{19}$
119	$12_{20}$	$p^2b^2pb^2$	$2^5 \cdot 3^2$	615	$4_9$	$12_{20}$	$12_{20}$
120	$12_{21}$	$p^2r^2bprb$	$2^{6} \cdot 3^{1}$	67	$4_{11}$	$12_{21}$	$12_{21}$
121	1222	$\frac{p \cdot sprs}{pr}$	$2^4 \cdot 3^2$	621	$4_4$	$12_{22}$	$12_{22}$
122	$12_{23}$	$p^2b^2pr^2$	$2^4 \cdot 3^2$	$6_{21}$	$\frac{14}{48}$	$12_{23}$	$12_{23}$
123	$12_{24}$		$2^4 \cdot 3^2$ $2^4 \cdot 3^2$	$6_{36}$	$4_{5}$		$12_{24}$
	1224	$\frac{r}{z(p^2r^2bpb)}$	$\frac{2 \cdot 3}{2^4 \cdot 3^2}$			$12_{24}$	
124	1225	$z(p \ r \ opo)$		631	47	$12_{25}$	$12_{25}$
125	$12_{26}$	$p^2r^2bpb$	$2^4 \cdot 3^2$	631	$4_{9}$	$12_{26}$	$12_{26}$
126	$12_{27}$	zr	$2^4 \cdot 3^2$	$6_{36}$	$4_5$	$12_{27}$	$12_{27}$
127	$12_{28}$	$p^5rbr$	$2^4 \cdot 3^2$	68	$4_2$	$12_{28}$	$12_{28}$
128	$12_{29}$	$p^2b^3r$	$2^5 \cdot 3^1$	$6_{35}$	$4_{15}$	$12_{29}$	$12_{29}$
129	$12_{30}$	$z(p^2b^3r)$	$2^{5} \cdot 3^{1}$	$6_{35}$	$4_{13}$	$12_{30}$	$12_{30}$
130	1231	$p^2bprb$	$2^4 \cdot 3^1$	617	417	1231	1231
131	1232	$z(p^2rb)^5$	$2^4 \cdot 3^1$	631	$4_{14}$	$12_{34}$	$12_{32}$
132	$12_{33}$	$\frac{z(p+b)}{p^2b^3}$	$2^4 \cdot 3^1$	611	$4_{16}$	$12_{34}$ $12_{33}$	$12_{33}$
133	$12_{34}$	$z(p^2rb)$	$2^4 \cdot 3^1$	631	$4_{14}$	$12_{32}$	$12_{34}$
134		$\frac{z(p + b)}{z(pb)^3}$					
	14	z(po)	$\frac{2^{1} \cdot 3^{1} \cdot 7^{1}}{2^{1} \cdot 3^{1} \cdot 5^{1}}$	7	14	14	$\frac{2_1}{15}$
135	15	$(p^2b)^2$		15	5	31	15
136	16	rb	25	82	16	16	16
137	$18_{1}$	$(pbpb^5)^2$	$2^{5} \cdot 3^{4}$	$9_{1}$	63	$18_{1}$	$18_{1}$
138	$18_{2}$	$(p^2b^2rbr)^2$	$2^{3} \cdot 3^{4}$	$9_{2}$	$6_3$	$18_{2}$	$18_{2}$
139	183	$p^2brpr$	$2^1 \cdot 3^4$	93	63	$18_{3}$	$18_{3}$
140	$18_{4}$	prbpbr	$2^4 \cdot 3^2$	$9_{1}$	$6_{6}$	$18_{4}$	$18_{4}$
141	185	$pb^2$	$2^2 \cdot 3^3$	$9_{5}$	64	186	185
142	186	$(pb^2)^5$	$2^2 \cdot 3^3$	94	64	185	186
143	187	$p^2br$	$   \begin{array}{r}     2^2 \cdot 3^3 \\     2^2 \cdot 3^3 \\     2^2 \cdot 3^2   \end{array} $	95	$6_{22}$	189	187
144	188	$z(p^2br)^5$	$2^2 \cdot 3^2$	94	$6_{24}$	1810	188
145	189	$\frac{z(pbr)^5}{(p^2br)^5}$	$2^2 \cdot 3^2$	94	$6_{23}$	187	189
146		$z(p^2br)$	$\frac{2 \cdot 3}{2^2 \cdot 3^2}$	95	$6_{25}$		
147	1810	$\frac{z(p\ br)}{prbrb}$	$     \begin{array}{r}       2^2 \cdot 3^2 \\       2^4 \cdot 5^1   \end{array} $			188	1810
-	$20_{1}$	proro		101	$20_{1}$	$4_1$	201
148	$20_{2}$	$(prb)^3$	$2^3 \cdot 5^1$	$10_{2}$	$20_{4}$	43	204
149	$20_{3}$	$p^3rb$	$2^{3} \cdot 5^{1}$	$10_{4}$	$20_{3}$	$4_4$	$20_{3}$
150	$20_{4}$	prb	$2^{3} \cdot 5^{1}$	$10_{3}$	$20_{2}$	$4_3$	$20_{2}$
151	$21_{1}$	pb	$2^1 \cdot 3^1 \cdot 7^1$	$21_{2}$	7	$21_{1}$	$_{3_1}$
152	$21_{2}$	$(pb)^2$	$2^1 \cdot 3^1 \cdot 7^1$	$21_{1}$	7	$21_{2}$	$3_1$
153	$24_{1}$	$p^2rbrprb$	$2^5 \cdot 3^2$	$12_{3}$	81	$24_{1}$	$24_{1}$
154	$24_{2}$	$prpb^2$	$2^5 \cdot 3^2$	$12_{1}$	81	$24_{2}$	$24_{2}$
155	243	$pb^2rb$	$2^{3} \cdot 3^{2}$	$12_{10}$	81	$24_{3}$	$24_{3}$
156	$24_{4}$	$p^3br$	$2^{3} \cdot 3^{2}$	$12_{9}$	81	$24_{4}$	$24_4$
157	$24_{5}$	$(pr^2)^5$	$2^4 \cdot 3^1$	$12_{12}$	89	$24_{9}$	$24_{9}$
158	$24_{6}$	$prbpb^2$	$\frac{2 \cdot 3}{2^4 \cdot 3^1}$	$12_{12}$ $12_{14}$	_	247	247
159	$\frac{246}{247}$	$\frac{propo}{(prbpb^2)^5}$	$\frac{2 \cdot 3}{2^4 \cdot 3^1}$	1214	83	$\frac{247}{24_6}$	
-		(propo )	$2^4 \cdot 3^1$	1214	83		246
160	248	$pr^2b$		128	82	248	248
161	$24_{9}$	$pr^2$	$2^4 \cdot 3^1$	$12_{12}$	89	$24_{5}$	$24_{5}$
162	30	$p^2b$	$2^1 \cdot 3^1 \cdot 5^1$	15	$10_{1}$	$6_{1}$	30
163	$36_{1}$	$pbpb^5$	$   \begin{array}{r}     2^4 \cdot 3^3 \\     2^4 \cdot 3^3   \end{array} $	$18_{1}$	$12_{2}$	$36_{1}$	$36_{1}$
164	$36_{2}$	$p^3r^2b$	$2^4 \cdot 3^3$	$18_{1}$	$12_{2}$	$36_{2}$	$36_{2}$
165	363	$p^2b^2rbr$	$2^2 \cdot 3^3$	$18_{2}$	$12_{2}$	363	$36_{3}$
166	$42_{1}$	$z(pb)^2$	$2^{1} \cdot 3^{1} \cdot 7^{1}$	211	14	$42_{1}$	61
167	422	z(pb)	$2^1 \cdot 3^1 \cdot 7^1$	212	14	$42_{2}$	61
101	742	~ (po)	2 .0 .1	412	1.1	722	ΟŢ

#### APPENDIX B. CHARACTER TABLES

## **B.1.** Character table of $E(\mathrm{Fi}'_{24}) = \langle x, y, e \rangle$

6	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	19 2 1	21 3 1	17 1 i	17 1	14 1 1	8 3 1	6 2	15 1 1	15 1 1	15 1	15 1	11 1	13	11 1	11 1	11	11 10	10
11 23	$\begin{array}{c c} & 1 \\ \hline 1 \\ \hline 1a \end{array}$	1 2a	2b	2c	2d	2e	3a	3b	4a	4b	4c	4d.	4e.	4 f	4a	4h	4 <i>i</i>	41 41	41
2F 3F 5F 7F 11F 23F	$\begin{array}{ccc} 1a \\ 1a \\ 1a \\ 1a \\ 1a \\ 1a \end{array}$	1a 2a 2a 2a 2a 2a 2a	$ \begin{array}{c} 1a \\ 2b \\ 2b$	1a $2c$ $2c$ $2c$ $2c$ $2c$	1a $2d$ $2d$ $2d$ $2d$ $2d$ $2d$ $2d$	1a $2e$ $2e$ $2e$ $2e$ $2e$ $2e$	3a 1a 3a 3a 3a 3a 3a	3b 1a 3b 3b 3b 3b 3b	2b $4a$ $4a$ $4a$ $4a$ $4a$	2b 4b 4b 4b 4b 4b 4b	$\begin{array}{c} 2b \\ 4c \\ 4c \\ 4c \\ 4c \\ 4c \\ 4c \end{array}$	2b $4d$ $4d$ $4d$ $4d$ $4d$ $4d$	2a 4e 4e 4e 4e 4e 4e	2b 4f 4f 4f 4f 4f 4f	2c $4g$ $4g$ $4g$ $4g$ $4g$ $4g$	$\begin{array}{c} 2c \\ 4h \\ 4h \\ 4h \\ 4h \\ 4h \\ 4h \end{array}$	$\begin{array}{c} 2d \\ 4i \\ 4i \\ 4i \\ 4i \\ 4i \\ 4i \end{array}$	2d 2d 4j 4k 4j 4k 4j 4k 4j 4k 4j 4k	$\begin{array}{ccc} & 4l \\ & 4l \end{array}$
X.1. X.1. X.1. X.1. X.1. X.1. X.1. X.1.	1 2 23 45 45 45 45 45 45 45 45 45 45 45 45 45	$\begin{array}{c} 2a \\ \hline 1 \\ \hline 1 \\ \hline 23 \\ 45 \\ 45 \\ 45 \\ 231 \\ 253 \\ 45 \\ 231 \\ 253 \\ 483 \\ 570 \\ 990 \\ 990 \\ 990 \\ 990 \\ 1035 \\ 1035 \\ 1035 \\ 1035 \\ 1035 \\ 1035 \\ 1035 \\ 1035 \\ 1035 \\ 1105 \\ 1155 \\ 1540 \\ 1155 \\ 1155 \\ 1155 \\ 1155 \\ 1155 \\ 12475 \\ 24$	1 23 45 45 45 231 231 231 231 231 231 231 231 231 231	$\begin{array}{c} 2c\\ -2c\\ -1\\ -7\\ -3\\ -3\\ -3\\ -3\\ -7\\ -7\\ -2\\ -1\\ -14\\ -18\\ -18\\ -12\\ -1\\ -21\\ -21\\ -21\\ -21\\ -21\\ -21\\$	$\begin{array}{c} 2d\\ 1\\ 1\\ 7\\ -3\\ -3\\ 3\\ 7\\ 7\\ 7\\ 7\\ 8\\ 28\\ 13\\ 35\\ 5\\ 7\\ -14\\ 4-18\\ 27\\ -21\\ -21\\ 49\\ 82\\ 21\\ 48\\ 22\\ 1\\ -21\\ 82\\ 21\\ 48\\ 49\\ -21\\ 82\\ 21\\ 48\\ 49\\ -21\\ 1-56\\ 6\\ -28\\ 82\\ 21\\ 1-56\\ 62\\ 44\\ 49\\ -21\\ 1-56\\ 62\\ 44\\ 49\\ -21\\ 1-56\\ 62\\ 44\\ 49\\ -21\\ 1-7\\ 1-6\\ 8\\ 22\\ 1-7\\ 1-21\\ 1-6\\ 8\\ 22\\ 1-7\\ 1-21\\ 1-6\\ 8\\ 23\\ 1-7\\ 1-7\\ 1-21\\ 1-6\\ 8\\ 24\\ 4-1\\ 1-9\\ 8\\ 23\\ 1-17\\ 1-17\\ 1-16\\ 64\\ 4-12$	$\begin{array}{c} 2ee \\ -2e \\ 1 \\ -1 \\ -1 \\ -1 \\ 5 \\ 5 \\ -9 \\ -9 \\ 12 \\ 2 \\ -11 \\ 3 \\ 15 \\ 10 \\ 0 \\ -10 \\ 35 \\ -5 \\15 \\ -16 \\ -16 \\ -16 \\ -16 \\ -16 \\ -16 \\ -25 \\ -45 \\ -45 \\ -45 \\ -45 \\ -45 \\ -45 \\ -45 \\ -45 \\ -45 \\ -45 \\ -45 \\ -45 \\ -45 \\ -45 \\ -45 \\ -128 \\ 80 \\ 80 \\ -96 \\ 80 \\ -96 \\ -96 \\ -96 \\ -96 \\ -105 \\ -96 \\ -105 \\ -$	-40 -45 -45 -30 -45 -45 -39 -9 -9 -9 -9 -9 -9 -9 -9 -9 -9 -15 -15 -10 -15 -15 -10 -15 -15 -10 -15 -15 -10 -15 -15 -10 -10 -10 -10 -10 -10 -10 -10 -10 -10	36	$\begin{array}{c} 4a\\ 4\\ 1\\ 1\\ -1\\ 1\\ -1\\ 5\\ 5\\ 5\\ 9\\ -9\\ 9\\ 12\\ 2\\ -11\\ 3\\ 15\\ 10\\ 0\\ 10\\ -10\\ 35\\ 5\\ -5\\ -5\\ -15\\ 10\\ 0\\ 10\\ -10\\ 35\\ -5\\ -5\\ -15\\ -15\\ 24\\ 4\\ 30\\ -15\\ 60\\ 0\\ -15\\ 60\\ 0\\ -15\\ 60\\ 0\\ -15\\ -45\\ -45\\ -45\\ -45\\ -45\\ -45\\ -45\\ -4$	$\begin{array}{c} 1\\ -1\\ -1\\ -1\\ 5\\ 5\\ -9\\ -9\\ -9\\ -9\\ 12\\ 2\\ -11\\ 1\\ 3\\ 3\\ 15\\ 5\\ 10\\ 0\\ -10\\ -10\\ -10\\ -10\\ -10\\ -15\\ -16\\ -15\\ -16\\ -15\\ -24\\ 4\\ -19\\ 12\\ 4\\ -19\\ 6\\ 0\\ -45\\ -45\\ -45\\ -45\\ -45\\ -45\\ -45\\ -45$	$\begin{array}{c} 1\\ 7\\ -3\\ -3\\ -3\\ 7\\ 7\\ 28\\ 13\\ 3\\ 5\\ -14\\ -14\\ -18\\ -18\\ -21\\ -21\\ -21\\ -21\\ -21\\ -21\\ -21\\ -3\\ -3\\ -56\\ -21\\ -3\\ -3\\ -56\\ -21\\ -48\\ -21\\ -48\\ -48\\ -48\\ -48\\ -56\\ -56\\ -56\\ -56\\ -5\\ -3\\ -3\\ -3\\ -3\\ -3\\ -3\\ -16\\ -16\\ -16\\ -16\\ -16\\ -16\\ -16\\ -16$	7 -3 -3 -7 -7 -28 13 35 -9 -14 -14 -18 -21 -21 -21 -48 8 8 -21 48 49 -56 -56 -21 -15 -52 8 -21 -19 -19 -21 -21 -21 -21 -21 -21 -21 -21 -21 -21	$ \begin{array}{c} 1\\ 7\\ -3\\ -3\\ 7\\ 7\\ 28\\ 13\\ -1\\ -14\\ -14\\ -18 \end{array} $	$\begin{array}{c} 4f \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -$	$\begin{array}{c} 4g \\ -1 \\ -1 \\ -3 \\ -3 \\ -1 \\ -1 \\ -3 \\ -3$	$\begin{array}{c} 4h \\ \hline 1 \\ -1 \\ -3 \\ -3 \\ -1 \\ 14 \\ -3 \\ 37 \\ -2 \\ 22 \\ 66 \\ 63 \\ 33 \\ 37 \\ -8 \\ 83 \\ 83 \\ -3 \\ -17 \\ -8 \\ 84 \\ -43 \\ -24 \\ 24 \\ 24 \\ 24 \\ 66 \\ -17 \\ -16 \\ -16 \\ 88 \\ -17 \\ -17 \\ 61 \\ -16 \\ -16 \\ -16 \\ -10 \\ -$	$\begin{array}{c} 4i \\ \hline 1 \\ \hline 3 \\ 1 \\ \hline 1 \\ -1 \\ 4 \\ 1 \\ 3 \\ 1 \\ 1 \\ -2 \\ 2 \\ 2 \\ -1 \\ 3 \\ 3 \\ 1 \\ 4 \\ 4 \\ -5 \\ 1 \\ 1 \\ -5 \\ 5 \\ 4 \\ 1 \\ 1 \\ -5 \\ -5 \\ 4 \\ 1 \\ 1 \\ -5 \\ -5 \\ 4 \\ 1 \\ 1 \\ -5 \\ -5 \\ 5 \\ 4 \\ 1 \\ 1 \\ 1 \\ -5 \\ -5 \\ 5 \\ 4 \\ 1 \\ 1 \\ 1 \\ -5 \\ -5 \\ 5 \\ 4 \\ 4 \\ -5 \\ -5 \\ 1 \\ -1 \\ 1 \\ 1 \\ -1 \\ -1 \\ $	$\begin{array}{c} 47 \\ 48 \\ 1 \\ 31 \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\$	$ \begin{array}{c} -1 \\ -1 \\ -3 \\ -3 \\ -1 \\ -1 \\ 4 \\ -3 \\ 3 \\ -1 \\ 2 \\ 2 \\ 6 \\ 6 \\ 3 \\ 3 \\ 3 \\ -7 \\ -7 \\ -1 \\ -8 \\ -1 \\ -5 \\ -5 \\ -3 \\ 3 \\ -5 \\ -5 \\ -3 \\ 3 \\ -8 \\ -2 \\ -1 \\ -8 \\ -8 \\ -8 \\ -8 \\ -8 \\ -8 \\ -8$

## ${\it Character\ table\ of\ E(Fi'_{24})\ (continued)}$

$\begin{array}{cccccccccccccccccccccccccccccccccccc$	8 6 6 6 4 2 2 1 1 1 	3 3 8 1 1 1 1	8 8 9 1 1 . 	8 7 6 6	5 5 4 4 1 1 1 1	4 1 i i i	6 6 5 5 1 1 1 1 
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	6c 6d 6e 6f 6g 3a 3b 3a 3a 36 2a 2b 2c 2d 2e 6c 6d 6e 6f 6g 6c 6d 6e 6f 6g 6c 6d 6e 6f 6g 6c 6d 6e 6f 6g	7b 7a 8a 7b 7a 8a 1 1a 1a 8a 1 7a 7b 8a	8b 8c 8d 8b 8c 8d 8b 8c 8d	8e 8f 8g 8h 8e 8f 8g 8h 8e 8f 8g 8h	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$egin{array}{ll} 10c \ 11a \\ 2e \ 11a \ 1 \\ 10c \ 11a \ 1 \\ 10d \ 1a \ 1 \end{array}$	2a 12b 12c 12d 6b 6b 6d 6d 4c 4d 4a 4b 2a 12b 12c 12d 2a 12b 12c 12d 2a 12b 12c 12d 2a 12b 12c 12d 2a 12b 12c 12d
X.1 1 1 1 X.2 3 5 5 X.3	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{c} 1 & 1 & 1 \\ -1 - 1 & 3 & 1 \\ 1 & 1 & 1 \\ 3 & 3 & -1 \\ & & & 4 \\ 1 & 1 & 1 \\ 3 & 3 & 3 \\ -2 & -2 & -2 \\ -2 & -2 & -2 \\ -2 & -2 &$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$			$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{c} -2 -2 \\ 1 \\ 1 \\ -4 \\ -4 \\ -1 -1 -1 \\ -1 -1 \\ -1 -1 \\ -1 -1 \\ -1 $	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	2 2 -1	-3 -3 -3 -3 -1 -5 -1 -5 -1 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	3 3 2	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		$\begin{array}{cccccccccccccccccccccccccccccccccccc$
X 56	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	3 3 -1 -3 3 3 7 -3 3 3 3 7 -4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$		$\begin{array}{cccccccccccccccccccccccccccccccccccc$

$Character\ table$	$of \ E(Fi'_{24})$	(continued)
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$ \begin{array}{c} 12 \cdot 12 \cdot 12 \cdot 12 \cdot 12 \cdot 12 \cdot 14 \cdot 14 \cdot$	2 3 5 7 11 23	4 1	4 1	4 1	3 · i ·	3 i	2 i	2 i	1 1 1	1 1 1	5	4 i :	4 i	1	1	1			4 1	4 1	3 1	3 1	i			:
$\begin{array}{c} X.21 & 1-1-1 & 2 & 2 & & 1 & -1 & -1 & -1 & -1 & -1 & -1$	2P 3P 5P 7P 11P 23P	60	60	60	76	7a $14a$ $14a$ $2a$ $14b$ $14b$	7a $14d$ $14d$ $2c$ $14c$ $14c$	7b $14c$ $14c$ $2c$ $14d$ $14d$	15a $5a$ $3a$ $15b$ $15b$ $15a$	15b 5a 3a 15a 15a 15b	8d 16a 16a 16a 16a 16a	100	100	91.0	917	11a 22a 22a 22a 22a 22a 22a	23a 23a 23b 23b 23b 23b	226	12b 8a 24b 24b 24a 24a	$     \begin{array}{r}       12b \\       8a \\       24a \\       24a \\       24b \\       24b \\     \end{array} $	12c $8b$ $24c$ $24c$ $24c$ $24c$	12d $8c$ $24d$ $24d$ $24d$ $24d$	14a 28b 28b 4e 28a 28a	1.46	150	156
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	X.1 X.2 X.3 X.4 X.5 X.6 X.7 X.8 X.10 X.11 X.12 X.13 X.14 X.15 X.16 X.17 X.18 X.20 X.22 X.23 X.23	1 1 1 1 1 1 1 2 2 2 1 1 1 1 1 1 	1 -11 · · · · · · · · · · · · · · · · ·	-1 -1 -1 -1 -1 -1 -1 -1 -1 -2 2	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccc}  & -A \\  & -\bar{A} \\  & \cdot \\  & \cdot \\  & -1 \\  & \cdot \\  & 1 \\  & \cdot \\  & A \\  & \bar{A} \end{array} $	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$			1 -1 -1 -1 -1 -1 -1 -1 -1	-1 -1 -1 1 1 2 2 -1 -2	1 -11 -11 -11 -11 -11 -11 -11 -11 -11 -	-1 -1 A A A -1 -A -A -A 1 1	1 A A A A A A A A A A A A A A A A A A A			-1 -1 -1 1 -1 -1 	-1 -1 -1 -1 -1 -1 -1	-1 -1 -1 -1 1 -1 -1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	-1 1 1 	-A $-A$ $-A$ $-1$ $-1$ $-1$ $-1$ $-1$ $-1$ $-1$ $-1$	-A		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	X.26 X.27 X.28 X.29 X.30 X.31 X.32 X.33 X.34 X.35 X.36 X.37 X.38 X.39	$\begin{bmatrix} -1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$ $\begin{bmatrix} -2 \\ 1 \\ 1 \\ \vdots \\ -1 \\ -2 \\ \vdots \\ \vdots \\ \vdots \\ -1 \end{bmatrix}$	-1 -1 -1 -1 1	-1 -1 -1 -1 1	1 	$\begin{array}{c} 1\\ \cdot\\ \cdot\\ \cdot\\ \cdot\\ \cdot\\ -\dot{\bar{A}}\end{array}$	$\begin{array}{c} -1 \\ \vdots \\ \vdots \\ -\bar{A} \end{array}$	-1 -1 -1 -1		1 -1	-1 -1 -1 : : : : : : : : : : : : : : : :	-1 -1 1 	-1 -1 1 	-1	_1			1	1 1 1 -1 -1 -1	1 1 1 -1 -1 -1 -1				Ā	-B -1	$-\bar{B}$
A.001	X. 42 X. 44 X. 45 X. 46 X. 47 X. 50 X. 51 X. 52 X. 53 X. 54 X. 55 X. 56 X. 57 X. 58 X. 60 X. 61 X. 62 X. 63 X. 64 X. 65 X. 66 X. 66		$-1\\-1\\1\\2$	$-\frac{1}{2}$	$-\bar{A}$ $-\bar{A}$ $\vdots$ $-1$ $\vdots$	-A	A	$-\frac{\bar{A}}{\bar{A}}$			-11 11	-1 i	-3						-1 -1 -1		-1 -1 -1 -1	-11 -11 -11 -11 -11 -11 -11	$-\bar{A}$	-A		

where  $A = \zeta(7)^4 + \zeta(7)^2 + \zeta(7)$ ,  $B = -2\zeta(15)_3\zeta(15)_5^3 - 2\zeta(15)_3\zeta(15)_5^2 - \zeta(15)_3 - \zeta(15)_5^3 - \zeta(15)_5^2 - 1$ ,  $C = -\zeta(23)^{18} - \zeta(23)^{16} - \zeta(23)^{13} - \zeta(23)^{12} - \zeta(23)^9 - \zeta(23)^8 - \zeta(23)^6 - \zeta(23)^4 - \zeta(23)^3 - \zeta(23)^2 - \zeta(23) - 1$ ,  $D = -2\zeta(7)^4 - 2\zeta(7)^2 - 2\zeta(7) - 2$ ,  $E = -3\zeta(7)^4 - 3\zeta(7)^2 - 3\zeta(7) - 3$ ,  $F = -4\zeta(5)^3 - 4\zeta(5)^2 - 3$ ,  $G = 4\zeta(5)^3 + 4\zeta(5)^2 + 1$ ,  $H = -4\zeta(12)_4\zeta(12)_3 - 2\zeta(12)_4$ .

**B.2.** Character table of  $mH = \langle r, u, v \rangle$ 

2 3 5 7	13 13 2 1	10 9 1 1	$^{11}_{\ 6}_{\ 1}_{\ 1}$	13 5 :	10 5 1	8 13 :	$\begin{array}{c} 7 \\ 10 \\ 1 \\ \cdot \end{array}$	$\begin{smallmatrix} 7\\10\\1\\ \cdot\end{smallmatrix}$	6 7 i	4 11 :	4 10 :	4 9 :	2 9	3 7	9 4 1	10 4	8 3 i
$ \begin{array}{r} 13 \\ 2P \\ 3P \\ 5P \end{array} $	$ \begin{array}{c} 1a \\ 1a \\ 1a \\ 1a \\ 1a \end{array} $	$ \begin{array}{r} 1 \\ 2a \\ 1a \\ 2a \\ 2a \end{array} $	2b 1a 2b 2b	$\begin{array}{c} 2c \\ 1a \\ 2c \\ 2c \\ 2c \end{array}$	2d 1a 2d 2d 2d	3a 3a 1a 3a	3b 3b 1a 3b	3c $3c$ $1a$ $3c$	$ \begin{array}{r} 1\\ 3d\\ 3d\\ 1a\\ 3d \end{array} $	3e $3e$ $1a$ $3e$	$\begin{array}{c} 3f \\ 3f \\ 1a \\ 3f \\ \end{array}$	$\begin{array}{c} 3g \\ 3g \\ 1a \\ 3g \end{array}$	3h 3h 1a 3h	3i $3i$ $1a$ $3i$	$\begin{array}{c} 4a \\ 2b \\ 4a \\ 4a \end{array}$	$\begin{array}{c} 4b \\ 2c \\ 4b \\ 4b \\ \end{array}$	$\begin{array}{c} 4c \\ 2b \\ 4c \\ 4c \end{array}$
7P 13P X.1 X.2 X.3	$ \begin{array}{r} 1a \\ 1a \\ 1 \\ 2 \\ 200 \end{array} $	$ \begin{array}{r} 2a \\ 2a \\ 1 \\ -1 \\ -90 \end{array} $	$ \begin{array}{r} 2b \\ 2b \end{array} $	$ \begin{array}{r} 2c \\ 2c \\ 1 \\ 1 \\ 2 \\ 10 \end{array} $	2d 2d 1 -1	$     \begin{array}{r}       3a \\       3a \\       \hline       1 \\       2 \\       \hline       7 \\     \end{array} $	$     \begin{array}{r}       3b \\       3b \\       \hline       1 \\       1 \\       2 \\       \hline       20 \\     \end{array} $	3c 3c 1 1 2	$     \begin{array}{r}       3d \\       3d \\       \hline       1 \\       -1 \\       \hline       27 \\    \end{array} $	3e 3e 1 1 2	$\frac{3f}{3f}$ 1  1 2	3g 3g 1 1 2	$\frac{3h}{3h}$ $\frac{1}{2}$	$     \begin{array}{r}       3i \\       3i \\       \hline       1 \\       -1     \end{array} $	$ \begin{array}{r} 4a \\ 4a \\ -1 \\ -1 \end{array} $	$\frac{4b}{4b}$ $\frac{1}{2}$	$ \begin{array}{c} 4c \\ 4c \\ 1 \\ -1 \\ \vdots \end{array} $
$X.4 \\ X.5 \\ X.6 \\ X.7 \\ X.8 \\ Y.0$	300 300 600 780 780 780	78 -104 104	20 20 40 76 76 76	12 12 24 12 12 12	-10 10	57 57 114 51 51	$   \begin{array}{r}     30 \\     30 \\     60 \\     -3 \\     24 \\     24   \end{array} $	30 30 60 24 -3 -3	27 27 -27	3 6 51 -30 -30	$\begin{array}{c} 3 \\ 3 \\ 6 \\ -3 \\ 24 \\ 24 \end{array}$	$ \begin{array}{r} -6 \\ -6 \\ -12 \\ 24 \\ 24 \\ 24 \end{array} $	12 12 24 -3 -3 -3	:	$-10 \\ 10 \\ \\ \\ \\ \\ 6 \\ -16 \\ \\ 16$	8 16 12 12 12	-6 6 -8 8
$X.9 \\ X.10 \\ X.11 \\ X.12 \\ X.13 \\ X.14$	780 780 2275 2275 2457 2457	$     \begin{array}{r}       104 \\       -78 \\       -455 \\       455 \\       -273 \\       273    \end{array} $	76 35 35 49 49	$     \begin{array}{r}       12 \\       -29 \\       -29 \\       57 \\       57     \end{array} $	$     \begin{array}{r}       -14 \\       25 \\       -25 \\       -33 \\       33     \end{array} $	51 88 88 270 270	-3 115 115 108 108	$ \begin{array}{r} -3 \\ 24 \\ -20 \\ -20 \\ 108 \\ 108 \end{array} $	91 91	51 7 7 27 27	$ \begin{array}{r}     24 \\     -3 \\     34 \\     34 \\     27 \\     27 \end{array} $	24 7 7 27 27	-3 7 7 27 27	10 10	$     \begin{array}{r}       -6 \\       -15 \\       15 \\       -29 \\       29   \end{array} $	12 3 3 21 21	$ \begin{array}{c} -6 \\ -7 \\ 7 \\ -7 \\ -7 \end{array} $
X.15 X.16 X.17 X.18 X.19	2808 2808 4550 5616 6825	468 -468 -455	120 120 70 240 105	$     \begin{array}{r}       -8 \\       -8 \\       -58 \\       -16 \\       -87     \end{array} $	-12 12	-108 $-108$ $176$ $-216$ $264$	54 54 230 108 75	54 54 -40 108 210	78 78 -91 -78	$ \begin{array}{r} -27 \\ -27 \\ 14 \\ -54 \\ 102 \end{array} $	$     \begin{array}{r}       -27 \\       -27 \\       \hline       68 \\       -54 \\       \hline       75     \end{array} $	27 27 14 54 21	14 21	$ \begin{array}{r}  -3 \\  -3 \\  -10 \\  3 \end{array} $	12 -12 -15	8 8 6 16 9	$ \begin{array}{c} 20 \\ -20 \\ \cdot \\ -7 \end{array} $
$X.20 \\ X.21 \\ X.22 \\ X.23 \\ X.24$	6825 9450 9450 16380 16380	$\begin{array}{r} 455 \\ 1260 \\ -1260 \\ 546 \\ 1092 \end{array}$	105 $210$ $210$ $-84$ $-84$	$     \begin{array}{r}       -87 \\       74 \\       74 \\       60 \\       60     \end{array} $	$     \begin{array}{r}     -25 \\     60 \\     -60 \\     2 \\     36     \end{array} $	$     \begin{array}{r}       264 \\       -27 \\       -27 \\       -387 \\       -387     \end{array} $	75 135 135 99 342	210 135 135 342 99	168 168	102 54 54 18 18	75 54 54 18 18	21 27 27 45 45	21 -9 -9	6 6	15 -6 44	36 -	$     \begin{array}{r}       7 \\       28 \\       -28 \\       -14 \\       -28     \end{array} $
X.25 X.26 X.27 X.28 X.29 X.30	16380 16380 17550 17550 18200 18200	$     \begin{array}{r}       -546 \\       -1092 \\       1170 \\       -1170 \\       1820 \\       -1820     \end{array} $	$     \begin{array}{r}     -84 \\     -84 \\     -90 \\     -90 \\     280 \\     280   \end{array} $	60 46 46 24 24	$     \begin{array}{r}       -2 \\       -36 \\       -30 \\       30 \\       60 \\       -60 \\     \end{array} $	-387 -387 783 783 461 -25	99 342 135 135 110 110	342 99 135 135 110 -160	78 78 182 182	$     \begin{array}{r}       18 \\       18 \\       -27 \\       -27 \\       -79 \\       137     \end{array} $	$     \begin{array}{r}       18 \\       18 \\       -27 \\       -27 \\       -79 \\       -52     \end{array} $	45 45 -7 29	$     \begin{array}{r}       -9 \\       -9 \\       27 \\       27 \\       11 \\       2   \end{array} $	$ \begin{array}{r}     -3 \\     -3 \\     -7 \\     20 \end{array} $	$     \begin{array}{r}       6 \\       -44 \\       30 \\       -30 \\       20 \\       -20 \\    \end{array} $	36 36 26 26 8 8	$     \begin{array}{r}       14 \\       28 \\       -6 \\       6 \\       28 \\       -28     \end{array} $
X.30 X.31 X.32 X.33 X.34 X.35	18200 18200 18200 18900 24192 24192	-1820 -1820 1820 -2016 -2016	280 280 280 420	24 24 148 128 128	-60 -60 -60 -96 -96	$     \begin{array}{r}       -25 \\       461 \\       -25 \\       -54 \\       864 \\       864    \end{array} $	110 110 110 270 216 216	-160 $110$ $-160$ $270$ $216$ $216$	$     \begin{array}{r}       182 \\       182 \\       183 \\       -168 \\       168 \\       168 \\     \end{array} $	-79 137 108 54 54	-52 $-79$ $-52$ $108$ $54$ $54$	-7 29 54	11 2 27 27	-7 20 -6 6	-20 -20 20	-36	-28 -28 28
X.36 X.37 X.38 X.39 X.40	27300 27300 35100 36400 36400	2730 -2730	140 - 140 - -180 560 560	-124 -124 -92 48 48	-70 70	$\begin{array}{c} 813 \\ 813 \\ 1566 \\ -50 \\ 922 \end{array}$	300 300 270 220 220	$\begin{array}{r} 300 \\ 300 \\ 270 \\ -320 \end{array}$	273 273 -78 -182 -182	$     \begin{array}{r}       30 \\       30 \\       -54 \\       274 \\       -158     \end{array} $	$\begin{array}{r} 30 \\ 30 \\ -54 \\ -104 \\ -158 \end{array}$	21 21 58 -14	39 39 54 4 22	$\begin{array}{c} 3\\ 3\\ 3\\ -20\\ 7\end{array}$	10 -10	52 16 16	14 -14
X.41 $X.42$ $X.43$ $X.44$ $X.45$	48384 54600 54600 54600	$-5460 \\ -1820 \\ 1820$	280 840 280 - 840	$ \begin{array}{r} 256 \\ 8 \\ 72 \\ -248 \\ 72 \end{array} $	$     \begin{array}{r}       -20 \\       -60 \\       \hline       60 \\     \end{array} $	$     \begin{array}{r}       1728 \\       -561 \\       -75 \\       1626 \\       -75     \end{array} $	432 $ 600 $ $ -210 $ $ 600 $ $ -210$	$-210 \\ 60 \\ 600 \\ 60$	$-168 \\ 546 \\ -273$	$     \begin{array}{r}       108 \\       60 \\       -156 \\       \hline       60 \\       -156     \end{array} $	$     \begin{array}{r}       108 \\       -21 \\       33 \\       60 \\       33     \end{array} $	-39 87 42 87	54 -3 6 78 6	-6 6 -3	$ \begin{array}{r} -60 \\ -20 \\ 20 \end{array} $	24 · 24	-28 -28 -28
X.46 X.47 X.48 X.49 X.50 X.51	54600 70200 70200 70200 70200 87360	5460 1560 3900 -3900 -1560 4368	280 680 680 680 680 448 -	8 120 120 120 120 -192	$     \begin{array}{r}       20 \\       120 \\       60 \\       -60 \\       -120 \\       -48     \end{array} $	$ \begin{array}{r} -561 \\ 216 \\ 216 \\ 216 \\ 216 \\ -120 \end{array} $	$     \begin{array}{r}       600 \\       -270 \\       540 \\       540 \\       -270 \\       636     \end{array} $	-210 $540$ $-270$ $-270$ $540$ $96$	546	$ \begin{array}{r} 60 \\ 216 \\ -27 \\ -27 \\ 216 \\ 42 \end{array} $	-21 54 135 135 54 150	$     \begin{array}{r}       -39 \\       -27 \\       -27 \\       -27 \\       -27 \\       42     \end{array} $	$     \begin{array}{r}       -3 \\       -27 \\       -27 \\       -27 \\       -27 \\       -39     \end{array} $	6	100 -100 -80	16 48 48 48 48	$     \begin{array}{r}       28 \\       8 \\       20 \\       -20 \\       -8     \end{array} $
$X.52 \\ X.53 \\ X.54 \\ X.55$	87360 87360 87360 109200 122850	-2912 2912 -4368	448 - 448 - 448 - 560	-192 -192 -192 -192 -16 -126	$ \begin{array}{r}     -48 \\     32 \\     -32 \\     48 \\     -60 \end{array} $	-120 $-120$ $-120$ $-120$ $-1122$ $2565$	96 96 636 1200 945	636 636 96 -420	-546	204 204 42 120 -108	96 96 150 -42 54	$\begin{array}{r} 42 \\ 42 \\ 42 \\ 42 \\ -78 \\ -27 \end{array}$	-39 -39 -39 -39 -6 54	-6	-80 80	32 30	-28
X.57 X.58 X.59 X.60 X.61	122850 122850 122850 139776 139776	2730 $-2730$ $-5460$ $-5824$ $5824$	-70 - -70 - -70 - 896 896	-126 -126 -126	$^{90}_{-90}_{60}_{-64}$	2565 $2565$ $2565$ $-192$ $-192$	135 135 945 456 456	945 945 135 456 456		135 $135$ $-108$ $-192$ $-192$	-27 $-27$ $54$ $-192$ $-192$	-27 $-27$ $-27$ $132$ $132$	54 54 54 -30 -30		$     \begin{array}{r}       30 \\       -30 \\       -80 \\       -64 \\       64     \end{array} $	30 30 30	-14 14 28
$X.64 \\ X.65 \\ X.66$	147420 147420 147420 147420 163800	-1638 1638 -3276 3276 -5460	924 924 924 924 840	$-36 \\ -36 \\ 24$	$     \begin{array}{r}       186 \\       -186 \\       -12 \\       12 \\       -20 \\     \end{array} $	891 891 891 2691	-567 $-567$ $-324$ $-324$ $180$	-324 $-324$ $-567$ $-567$ $180$		405 $ 405 $ $ -324 $ $ -63$	-81 $-81$ $162$ $162$ $-63$	81 81 81 -117	-9		$ \begin{array}{r} -6 \\ 6 \\ 36 \\ -36 \\ -60 \\ 60 \end{array} $	12 · 12 · 12 · 12 · 48 · 48 ·	$     \begin{array}{r}       -14 \\       14 \\       -28 \\       28 \\       -28 \\     \end{array} $
X.68	163800 163800 163800 184275 184275	5460 $5460$ $-5460$ $-12285$ $12285$ $7371$	840 840 840 315 315 441	24 24 24 51 51 297	20	-1683 $2691$ $-1683$ $567$ $567$ $2187$		990 180 990 -405 -405 729	819	$     \begin{array}{r}     -63 \\     -63 \\     -63 \\     -162 \\     -162     \end{array} $	-63	-117 -117 -117	-9 -9 -9	9 9	-15	$ \begin{array}{r} 48 \\ 48 \\ -25 \\ -25 \\ -63 \end{array} $	$     \begin{array}{r}       28 \\       28 \\       -28 \\       -21 \\       21 \\       -21    \end{array} $
X.73 X.74 X.75 X.76 X.77	184275 - 184275 - 199017 - 199017 - 218700 - 245700 - 245700 -	-7371 $-2430$	$     \begin{array}{r}       441 \\       -540 \\       -540 \\       -420 \\       -420     \end{array} $	$   \begin{array}{r}     297 \\     108 \\     108 \\     -28 \\     -28 \\     \end{array} $	-171 $-270$ $270$ $210$ $-210$	2187 2187 2187 2943 2943	729 729	729 729	27 27 27 273 273	27 27	27 27	27 27	27 27	27 27 27 30 30	-51 90 -90 90 -90	-63 -63 -8	$\begin{array}{c} 21 \\ 6 \\ -6 \\ 14 \end{array}$
1.04	245700 245700 245700 291200 - 291200 291200 - 332800 -	-10040	1280	-128	160 -160 -160 160	-2344 $-2344$ $1544$ $1544$ $-1568$	680 680 680 680 160	$     \begin{array}{r}       680 \\       680 \\       -400 \\       -400 \\       160     \end{array} $	728 728 728 728 832	$     \begin{array}{r}     -22 \\     -22 \\     194 \\     194 \\     -56     \end{array} $	$     \begin{array}{r}     -22 \\     -22 \\     -76 \\     -76 \\     -56     \end{array} $	$     \begin{array}{r}     -40 \\     -40 \\     -4 \\     -4 \\     34   \end{array} $	$     \begin{array}{r}     -4 \\     -4 \\     -13 \\     -13 \\     16   \end{array} $	$ \begin{array}{r} 26 \\ 26 \\ -28 \\ -28 \\ -32 \end{array} $	:		
$X.83 \\ X.84 \\ X.85$	332800 368550 437400 491400	16640	1280 $630$ $-1080$ $-840$	$^{102}_{216}_{-56}$	•	-1568	160 1620 540		$     \begin{array}{r}       832 \\       -819 \\       -27 \\       -273     \end{array} $	$-56 \\ -324 \\ 54$	-56 $162$ $54$	34 54		$     \begin{array}{r}       -32 \\       -9 \\       -27 \\       -30     \end{array} $	•	-50 -16	64

2 3 5 7	10 2	7 2 1	9 2	8 2	4 2 2	6 9	5 7 1	7 7	7 5 1	7 6	8 5	4 7	4 7	6 5	6 5	6 5	7 4	7 4	3 6	6 4	4 5
$\frac{13}{2P}$	$\frac{4d}{2c}$	$\frac{4e}{2b}$	$\frac{4f}{2b}$	$\frac{4g}{2c}$	5a 5a	6 <sub>1</sub> 3a	$\frac{6_2}{3b}$	6 <sub>3</sub>	$\frac{6_4}{3c}$	6 <sub>5</sub> 3a	6 <sub>6</sub> 3a	6 <sub>7</sub> 3e	6 <sub>8</sub>	6 <sub>9</sub>	$\frac{6_{10}}{3b}$	$\frac{6_{11}}{3c}$	$\frac{6_{12}}{3b}$	$\frac{6_{13}}{3c}$	$\frac{6_{14}}{3g}$	$\frac{6_{15}}{3c}$	$\frac{6_{16}}{3e}$
3P 5P 7P 13P	$\begin{array}{c} 4d \\ 4d \\ 4d \\ 4d \\ 4d \end{array}$	4e 4e 4e 4e	4f $4f$ $4f$ $4f$	4g $4g$ $4g$ $4g$	5a 1a 5a 5a	$\begin{array}{c} 2a \\ 6_1 \\ 6_1 \\ 6_1 \end{array}$	2a 6 <sub>2</sub> 6 <sub>2</sub> 6 <sub>2</sub>	2a 6 <sub>3</sub> 6 <sub>3</sub>	2d 6 <sub>4</sub> 6 <sub>4</sub> 6 <sub>4</sub>	$\begin{array}{c} 2b \\ 6_5 \\ 6_5 \\ 6_5 \end{array}$	2c 66 66 66	$\begin{array}{c} 2a \\ 67 \\ 67 \\ 67 \end{array}$	$     \begin{array}{c}       3f \\       2a \\       68 \\       68 \\       68   \end{array} $	2d 69 69 69	$ \begin{array}{c} 2b \\ 6_{10} \\ 6_{10} \\ 6_{10} \end{array} $	$ \begin{array}{c} 2b \\ 6_{11} \\ 6_{11} \\ 6_{11} \end{array} $	$\begin{array}{c} 2c \\ 6_{12} \\ 6_{12} \\ 6_{12} \end{array}$	$\begin{array}{c} 2c \\ 6_{13} \\ 6_{13} \\ 6_{13} \end{array}$	2a 6 <sub>14</sub> 6 <sub>14</sub>	$\begin{array}{c} 2d \\ 6_{15} \\ 6_{15} \\ 6_{15} \end{array}$	$\begin{array}{c} 2c \\ 6_{16} \\ 6_{16} \\ 6_{16} \end{array}$
X.1 $X.2$ $X.3$	1 1 2	1 1 2	1 -1	1 -1	1 1 2	1 -1	1 -1	1 -1	1 -1	1 1 2	1 1 2	-1 -1	1 -1	1 -1	1 1 2	1	1 1 2	1 1 2	1 -1	1 -1	1 1 2
X.4 X.5 X.6		-	$-\frac{1}{2}$	$-\frac{2}{2}$	-	-9 9		-18 18	-10 10	$-7 \\ -7 \\ -14$	9 9 18	-9 9	-9 9	$-1 \\ 1 \\$	2 2 4	2 2 2 4	6 6 12	6 6 12		$-\frac{2}{2}$	2 3 3 6
X.7 X.8 X.9	12 12 12	4 4 4	6	-2 :	5 5 5	$-3 \\ -23 \\ 23$	$^{15}_{-14}$	6 -5 5	14 -5 5	$-5 \\ -5 \\ -5$	3 3 3	$-3 \\ 4 \\ -4$	$-3 \\ 4 \\ -4$	5 1 -1	13 4 4	13 13	-3	-3 -3	6 4 -4	$-\frac{2}{5}$	$\begin{array}{c} 3 \\ -6 \\ -6 \end{array}$
$X.10 \\ X.11 \\ X.12$	12 3 3	$     \begin{array}{r}       4 \\       -5 \\       -5     \end{array} $	$-6 \\ 1 \\ -1$	$\begin{array}{c} 2\\1\\-1\end{array}$	5	$-50 \\ 50$	$-15 \\ -5 \\ 5$	$^{-6}_{-14}$	$-14 \\ 10 \\ -10$	$-\frac{5}{8}$	$-\frac{3}{8}$	$^{3}_{-23}$	$^{3}_{-4}$	$-5 \\ -2 \\ 2$	$     \begin{array}{r}       13 \\       -1 \\       -1     \end{array} $	4 8 8	$     \begin{array}{r}       -3 \\       -5 \\       -5     \end{array} $	4 4	$-6 \\ -5 \\ 5$	$-2 \\ -2 \\ 2$	3 7 7
X.13 X.14 X.15	-3 -3 8	9	$-5 \\ -4 \\ -4$	$-5 \\ -4 \\ -4$	7 7 8	$-30 \\ 30 \\ -18$	$-30 \\ 30 \\ 36$	$-30 \\ 30 \\ -18$	-30 30 6	22 22 12	30 30 4	-3 9	$-3 \\ 3 \\ 9$	-6 6	$\begin{array}{c} 4 \\ 4 \\ -6 \end{array}$	$\begin{array}{c} 4 \\ 4 \\ -6 \end{array}$	$12 \\ 12 \\ -2$	$12 \\ 12 \\ -2$	$-3 \\ -9 \\ -9$	$-6 \\ -6 \\ -6$	3 1
$X.16 \\ X.17 \\ X.18 \\ X.19$	16	-10 $-15$	4 1	4	8 16	18 -50	-36	18 -14	-6 10	12 16 24 24	$^{4}_{-16} \\ ^{8}_{-24}$	-9 4 .	-9 -23	-6 -2	$-6 \\ -2 \\ -12 \\ 15$	$^{-6}_{16}$ $^{-12}_{6}$	$^{-2}_{-10}$ $^{-4}_{3}$	$     \begin{array}{r}       -2 \\       8 \\       -4 \\       -6     \end{array} $	9 -5	6 -2	$14 \\ 2 \\ -6$
$X.20 \\ X.21 \\ X.22$		$-15 \\ 10 \\ 10$	$-\frac{1}{8} \\ -8$	-1		50 45 -45	5 45 -45	14 -9 9	$-10 \\ 15 \\ -15$	24 21 21	$-24 \\ -5 \\ 5$	$-4 \\ 18 \\ -18 \\ -$	23 18 -18	$-\frac{2}{3}$	15 3 3	6 3 3	$\begin{array}{c} 3 \\ -1 \\ -1 \end{array}$	$-6 \\ -1 \\ -1$	5 9 -9	$\frac{2}{3}$ $-3$	$-\frac{6}{2}$
$X.23 \\ X.24 \\ X.25$	$-12 \\ -12 \\ -12$	4 4 4	$^{10}_{-4}$	$^{-6}_{-4}$	5 5 5	$-21 \\ 39 \\ 21$	$^{51}_{30}_{-51}$	$-12 \\ -33 \\ 12$	$^{20}_{-9}$	$-3 \\ -3 \\ -3$	$-3 \\ -3 \\ -3$	$^{6}_{12}$ $^{-6}$	$\begin{array}{c} 6 \\ 12 \\ -6 \end{array}$	$^{11}_{-9}$	15 6 15	6 15 6	$^{-21}_{\ \ 6}_{\ \ -21}$	$-2_{6}^{6}$	$-\frac{3}{3}$	$-4 \\ 3 \\ 4$	6 6 6
$X.26 \\ X.27 \\ X.28$	-12	$^{4}_{-10}$	$^{4}_{-6}$	$^{-10}_{10}$	5	-39 $-45$ $45$	$-30 \\ -45 \\ 45$	33 63 -63	$^{9}_{-15}$	$-3 \\ -9 \\ -9$	-3 - 31 - 31	-12 - 9 -9	$^{-12}_{-9}$	$-21 \\ 21 \\ 21$	-9 -9	15 -9 -9	6 7 7	$-21 \\ 7 \\ 7 \\ 7$	-3 <u>.</u>	-3 -9 9	6 1 1
$X.29 \\ X.30 \\ X.31 \\ X.32$	8 8 8	:	$\begin{array}{r} 4 \\ -4 \\ -4 \end{array}$	$-4 \\ -4 \\ -4$	:	-43 43 43 -43	$     \begin{array}{r}       20 \\       -20 \\       -20 \\       20     \end{array} $	$     \begin{array}{r}       56 \\       -2 \\       -56 \\       2     \end{array} $	30 -30	$     \begin{array}{r}       37 \\       -17 \\       37 \\       -17     \end{array} $	-3 -9 -3 -9	11 -11 -11 · 11 ·	$     \begin{array}{r}       11 \\       16 \\       -11 \\       -16     \end{array} $	-3 3 -3	10 10 10 10	10 -8 10 -8	6 6 6	6 6	$-7 \\ -11 \\ 7 \\ 11$	$ \begin{array}{r} 12 \\ 6 \\ -12 \\ -6 \end{array} $	-3 -3 -3
X.33 X.34 X.35	12	20	•	•	-8 -8	72 -72	-36 36	72 -72	-30 $-24$ $-24$	42	$\begin{array}{c} -9 \\ 10 \\ 32 \\ 32 \end{array}$	18 -18	18 -18	-3 $-24$ $-24$	6	6	$-\frac{0}{8}$	$-\frac{1}{8}$		-0	-3 4 2 2 2
$X.36 \\ X.37 \\ X.38$	$-8 \\ -8 \\ 4$	-20	$^{-14}_{14}$	$^{6}_{-6}$	:	$-57 \\ -57 \\ .$	$-30 \\ -30 \\ \cdot$	$^{84}_{-84}$	$-{}^{20}_{-20}$	$-18^{-5}$	$^{-19}_{-19}$	-30 -30	30 -30	$^{-7}_{7}$	$^{-4}_{-4}$	$^{-4}_{-4}$	$-4 \\ -4 \\ 14$	$-4 \\ -4 \\ 14$	$-\frac{3}{3}$	$^{-4}_{4}$	$\frac{2}{2}$
X.39 X.40 X.41	16 16	:	:	:	-16	-33				-34 74	$     \begin{array}{r}       -18 \\       -6 \\       \hline       64     \end{array} $			:	20 20	-16 20	12 12 16	12 16			$-6 \\ -6 \\ 4$
X.42 X.43 X.44 X.45	$^{24}_{-16}$	:	-4	-4 -4	:	-33 43 -43	-20 $20$	66 -2	10 30 -30		$ \begin{array}{r} -1 \\ -27 \\ -38 \\ -27 \end{array} $	-60 16 - -16	$^{21}_{-11}$	3 -3	$     \begin{array}{r}     -8 \\     -6 \\     -8 \\     -6     \end{array} $	$10 \\ 12 \\ -8 \\ 12$	$^{8}_{-8}$	$-10 \\ 12 \\ -8 \\ 12$	-11 $11$	$-\frac{2}{6}$ $-\frac{1}{6}$	8 4
X.46 X.47 X.48			$-4 \\ 16 \\ 4$	4 -4		$\begin{array}{c} 33 \\ -60 \\ 174 \end{array}$	$-30 \\ 30 \\ 30 \\ 30$	$     \begin{array}{r}       -66 \\       12 \\       -6     \end{array} $	$-10 \\ 60 \\ -30$	$^{-17}_{32}$	$-1 \\ -24 \\ -24$	60 - -6 -15 -	$     \begin{array}{r}       -21 \\       -6 \\       -15     \end{array} $	$-7 \\ 12 \\ 6$	$-8 \\ 14 \\ -4$	10 -4 14	$^{8}_{-6}$	$-10 \\ 12 \\ -6$	-3 3 3	12 6	8 12 -15
$X.49 \\ X.50 \\ X.51$			$-4 \\ -16 \\ -16$	4	10	$-174 \\ 60 \\ 156$	$-30 \\ -30 \\ 66$	$^{6}_{-12}$ $^{-24}$	$^{30}_{-60}$	$\frac{32}{32}$	$     \begin{array}{r}       -24 \\       -24 \\       \hline       24     \end{array} $	$\begin{array}{c} 15 \\ 6 \\ -6 \end{array}$	15 6 -6	$-6 \\ -12 \\ -12$	$^{-4}_{14}_{-20}$	$^{14}_{-4}$ $^{16}$	$^{12}_{-6}$ $^{12}$	$^{-6}_{12}$	$-3 \\ -6$	$^{-6}_{-12}$	$-15 \\ 12 \\ 6$
$X.52 \\ X.53 \\ X.54$	:	:	16	:	10 10 10	$^{4}_{-4}$	$-104 \\ 104 \\ -66$	$\begin{array}{c} 4 \\ -4 \\ 24 \end{array}$	$-76 \\ -76 \\ -24$	16 16 16	24 24 24	$-\frac{4}{6}$	$-\frac{4}{6}$	$-4 \\ 4 \\ 12$	$^{16}_{16}$ $^{-20}$	-20 $-20$ $16$	12	12 12	$^{-14}_{14}_{6}$	$-{20 \atop -20}$	$-12 \\ -12 \\ 6$
X.55 X.56 X.57 X.58	6 6 6	10 10 10	$-10 \\ 10$	-10	:	195 -105 105	$^{15}_{-15}$ $^{15}_{15}$	105 75 -75		-34 $-43$ $-43$ $-43$	$     \begin{array}{r}       -2 \\       -27 \\       -27 \\       -27 \\     \end{array} $	6 3 -3	6 3 -3	-999	$-16 \\ -7 \\ 11 \\ 11$	$   \begin{array}{r}     20 \\     11 \\     -7 \\     -7   \end{array} $	$^{16}_{-15}$ $^{15}_{15}$	-20 $15$ $-15$ $-15$	$-\frac{1}{3}$	$-\frac{1}{3}$	16 -9
X.59 X.60 X.61	6	10 16 16	-8		1 1	$-195 \\ -8$	$-15 \\ -154 \\ 154$	$-105 \\ -8 \\ -8$	$-64 \\ -64 \\ 64$	-43 $-43$ $-32$ $-32$	-27 -27	-6 8 -8	-6 -8 -8	$-\frac{3}{8}$	-7 $-4$ $-4$	11 -4 -4	-15 :	15	-3 8 -8	$-3 \\ -4 \\ 4$	-3 :
$X.62 \\ X.63 \\ X.64$	$\frac{12}{12}$ $\frac{12}{12}$	$-4 \\ -4 \\ -4$	$-{10\atop 4}$	$\begin{array}{c} 2 \\ -2 \\ -12 \end{array}$	$-5 \\ -5 \\ -5$	$     \begin{array}{r}       63 \\       -63 \\       -117     \end{array} $	$-45 \\ 45 \\ -36$	$^{-18}_{18}_{-9}$	$^{6}_{-6}$	$     \begin{array}{r}       -21 \\       -21 \\       -21     \end{array} $	$\frac{27}{27}$	$^{9}_{-9}$	-9 18	$^{15}_{-15}$ $^{-21}$	$-3 \\ -3 \\ 24$	$^{24}_{24}_{-3}$	$-15 \\ -12$	$^{-12}_{-12}_{-15}$	-9 9 9	$^{-6}_{\ \ \ 3}$	9
$X.65 \\ X.66 \\ X.67$	12	-4 :	$-4 \\ -4 \\ -4$	$^{12}_{-4}$	-5 :	$     \begin{array}{r}       117 \\       -33 \\       33 \\     \end{array} $	$\begin{array}{r} 36 \\ 30 \\ -30 \\ \end{array}$	$-96 \\ -66$	$-80 \\ -10$	$-21 \\ -51 \\ -$	27 · 51 · -3 ·	-18 - 21 - 21	-18 $21$ $60$	$^{21}_{7}_{-7}$	24 12 12	$-3 \\ 12 \\ -6 \\ 13$		$-15 \\ -12 \\ 6$	$-9 \\ -3 \\ -3$	$^{-3}_{16}$	$-\frac{1}{3}$
X.68 X.69 X.70 X.71	-1 -1	$-5 \\ -5$	$-4 \\ -9 \\ 9$	$-\frac{4}{7} \\ -7$	:	-33 $-135$ $135$	$-30 \\ 30 \\ \cdot$	$   \begin{array}{r}     96 \\     66 \\     -27 \\     27   \end{array} $			51 - -3 -9 -	-54	$     \begin{array}{r}     -21 \\     -60 \\     27 \\     -27     \end{array} $	-7 7 9 -9	12 12 18 18	$     \begin{array}{r}       12 \\       -6 \\       -9 \\       -9     \end{array} $	-12 $-6$ $-6$	-12 6 3 3	-3 3	-16 $-2$ $-9$	-3 -3 6 6
X 72	-15	1 1	$\begin{array}{c} 3 \\ -3 \\ 18 \end{array}$	$-\frac{7}{3}$ $-\frac{3}{6}$	17 17	$   \begin{array}{r}     81 \\     -81 \\     -243   \end{array} $	-81 -81	$^{81}_{-81}$	81	$-45 \\ -45$	$\begin{array}{c} -9 \\ 27 \\ 27 \\ 27 \end{array}$		:	$-9 \\ -9 \\ -27$	9	-9 9 9	9	9	:	-9 -9	
X.73 X.74 X.75 X.76 X.77	8		$-18 \\ -2 \\ -2$	$-\frac{6}{2}$		$     \begin{array}{r}       243 \\       -171 \\       171     \end{array} $	$-90_{90}$	-90 90	30 -30	27 27 39 39	$     \begin{array}{r}       27 \\       -1 \\       -1     \end{array} $	-9 9	$-\frac{1}{9}$	$\begin{array}{c} 27 \\ 21 \\ -21 \end{array}$			-10 -10	$-10 \\ -10$	-9 9	-6 6	-1 -1
X.79 X.80	•	:	:	:	:	$-20 \\ -20 \\ -20$	$ \begin{array}{c} 20 \\ -20 \\ -20 \end{array} $	$^{128}_{-128}$	$-80 \\ 80 \\ -40$	:	40 · 40 -8	-34 - 34 - 34 - 34 - 34 - 34 - 34 - 34	-34 - 34 - 20	$-20 \\ 20 \\ 20$	•	:	$-8 \\ -8 \\ -8$	$^{-8}_{-8}$ $^{16}$	$^{-16}_{16}_{-2}$	-8 8 8	$ \begin{array}{r} -1 \\ -1 \\ -2 \\ -2 \\ -2 \\ -2 \end{array} $
X.81 X.82 X.83	-2	_10	:	:	:	$\begin{array}{c} 20 \\ 208 \\ -208 \end{array}$	$^{20}_{-80}_{80}$	$     \begin{array}{r}     -88 \\     64 \\     -64   \end{array} $	40	-16 -16 -18 -18 54 78	-8 · ·	-34 -8 8	$\frac{20}{-8}$	:	$^{-16}_{-16}$	-16 $-16$	-8 -12	16 6		-8 : :	-2 12
$X.84 \\ X.85 \\ X.86$	16	-10		:	:	:		:		-18 54 78	$   \begin{array}{r}     -18 \\     54 \\     -2   \end{array} $	:	:				-12 $-20$				-2

2	4 5	$\frac{4}{5}$	$\frac{4}{5}$	2 6	$\frac{5}{4}$	6 3	$\frac{4}{4}$	$_{4}^{4}$	$^4_4$	2 5	$\frac{3}{4}$	$\frac{2}{4}$	3	$\frac{3}{1}$	6	6 1	6 1	6	3 6	3 6	$\frac{1}{7}$	3 5	$\frac{1}{6}$	$_{6}^{1}$	6
7 13	617	610	610	600	601	600	600	604	605	600	607	600	600	1 7a	8a	8h	80	8d	90	9 <i>h</i>	96	9.4	90 0	a # 0	
2P 3P 5P	$\begin{array}{c} 3g \\ 2c \\ 6_{17} \end{array}$	$\begin{array}{c} 3e \\ 2d \\ 6_{18} \end{array}$	$ \begin{array}{c} 3f \\ 2d \\ 6_{19} \end{array} $	$\frac{3h}{2a}$	$\frac{3b}{2d}$	$\frac{3d}{2c}$	3g $2b$ $6aa$	$\frac{3f}{2c}$	3g 2c 6ar	$\begin{array}{c} 3h \\ 2c \\ 6_{26} \end{array}$	$\frac{3g}{2d}$	$\frac{3h}{2d}$	3i $2c$ $620$	7a 7a 7a		$\begin{array}{c} 4d \\ 8b \\ 8b \end{array}$		4d $8d$ $8d$	9a 3a 9a	9b 3a 9b	9c 3a 9c	9d 3e 9d	3a	9f 9 3e 3 9f 9	g e g
5P 7P 13P	$\frac{617}{617}$	$\frac{6_{18}}{6_{18}}$	6 <sub>19</sub> 6 <sub>19</sub>	$6_{20} \\ 6_{20} \\ 6_{20}$	$\begin{array}{c} 2d \\ 6_{21} \\ 6_{21} \\ 6_{21} \end{array}$	$6_{22}$ $6_{22}$	$6_{23}^{23}$ $6_{23}^{23}$	$624 \\ 624 \\ 624$	$625 \\ 625 \\ 625$	$\frac{626}{626}$	$\frac{6_{27}}{6_{27}}$	$^{628}_{628}_{628}$	$6_{29} \\ 6_{29} \\ 1$	$\frac{1a}{7a}$	8a 8a	8b 8b	8c	8d 8d	9a 9a	9b 9b	9c 9c 1	9d 9d	9e 9	9f 9 9f 9	9 9 9
X.2 X.3 X 4	1 2	-1 $-1$	-1 $-1$	-1	$-1 \\ -4$	$-\frac{1}{3}$	1 1 2 2	1 2 3	1 2	1 2	$-\frac{1}{2}$	$-1 \\ -4$	$-1 \\ -1$	$\frac{1}{2}$	-1	$\frac{1}{2}$	2 2	-Î	1 2 3	1 2 3	1 2 3	$-\frac{1}{3}$	1 2 - 3		1 1 3
X.5 X.6 X 7	12	1	1	_ ;	4 _ i	$\begin{array}{c} 3 \\ 3 \\ -3 \end{array}$	2 2 4 4	3 6 -3			$-\frac{2}{2}$	4 _ i	:	-1		$-\frac{2}{-4}$	$\frac{2}{4}$		3 6 6	3 6 -3	3 6 -3 -3	$-3^{3}$	2 - 3 3 6 -		3
X.8 X.9 X.10	$ \begin{array}{r}     -6 \\     -6 \\     12 \end{array} $	5 4 -4 -5	5 4 -4 -5	-5 -5 3	$-\frac{1}{2}$	:	4	_3 _3	6 6	3 3 3 3	$-\frac{2}{2}$	-1 -1	:	$-2$ $\frac{3}{3}$ $\frac{3}{3}$	_?	:		_?	$-3 \\ -3$	6	-3 -3	:	-3 -3 -3	:	:
X.11 X.12 X 13	1	$-1 \\ -1 \\ -3$	-8 8 -3	-3 -5 -5 -5 -3 -3	$     \begin{array}{r}       -2 \\       2 \\       7 \\       -7 \\       -6 \\       6     \end{array} $	$-5 \\ -5$	$ \begin{array}{r} 4 \\ -1 \\ -1 \\ -5 \end{array} $	$-\frac{2}{-2}$	1 1 3	1 1	$-1 \\ -3$	$\begin{array}{c} 1 \\ 1 \\ -1 \\ -3 \end{array}$	$-\frac{1}{2}$		$-\frac{1}{-3}$	$-1 \\ -1 \\ 3 \\ 3$	$-\frac{1}{-\frac{1}{3}}$	$-\frac{1}{1}$	$\begin{array}{c} 6 \\ -2 \\ -2 \\ 0 \end{array}$	$-\frac{6}{7}$	$     \begin{array}{r}       -3 \\       -3 \\       7 \\       7   \end{array} $	10 10	$-\frac{3}{2}$	1 1	1 1
X.14 X.15 X.16	$     \begin{array}{r}       1 \\       3 \\       3 \\       -5 \\       -5 \\       2     \end{array} $	$     \begin{array}{r}       -4 \\       -5 \\       1 \\       -1 \\       -3 \\       3 \\       -3 \\       3     \end{array} $	$     \begin{array}{r}       -4 \\       -5 \\       -8 \\       8 \\       -3 \\       -3 \\       3     \end{array} $	3	6	$-\frac{1}{2}$	$-\frac{5}{3}$	3 1 1	3 -5 -5	3 4 4	$ \begin{array}{c} 2 \\ -2 \\ 2 \\ -2 \\ 1 \\ -1 \\ -3 \\ 3 \\ 3 \\ -3 \end{array} $	-3 3	1 1	1 1	3	3	3 -	-1	9	9		$-3 \\ -3$	-	-3 – -3 –	3
X.17 X.18 X.19	$-10^{2}_{3}$			-5	7	$     \begin{array}{r}       -2 \\       -2 \\       5 \\       2     \end{array} $	$-\frac{5}{6}$	-4	$-10^{2}_{3}$	2 8 3 3	1	i	$-\frac{2}{1}$	2	1	-2 $-3$	$-\frac{1}{2}$	1	-4 12	14	14 -6	-10 3	-4 - 3	-1 - 3	3
$X.20 \\ X.21 \\ X.22$	$     \begin{array}{r}       -10 \\       3 \\       5 \\       5 \\       -3 \\       -3     \end{array} $	8 6 -6	$-1 \\ 6 \\ -6$	5	-7 $-9$ $-9$	8 8	$ \begin{array}{r} -5 \\ -5 \\ 3 \\ -2 \\ 6 \\ -3 \\ -3 \\ 3 \\ -3 \end{array} $	2 3 2 2 6 6	3 5 5	$     \begin{array}{r}       3 \\       -4 \\       -4     \end{array} $	$-1 \\ -3 \\ -3$	-1 :	2 2		$-1 \\ -2 \\ 2$	-3	-3 ·	$-1 \\ 2 \\ -2$	12	3	-6 :	6 6	3 	-3 – -3 –	3
$X.23 \\ X.24 \\ X.25$	-3 -3 -3	$ \begin{array}{r} -8 \\ 8 \\ 6 \\ -6 \\ 2 \end{array} $ $ \begin{array}{r} -2 \\ -3 \\ -9 \\ -15 \\ 9 \\ 15 \end{array} $	$-\frac{6}{2}$ $-\frac{1}{2}$	-3 3 -3 -9 9 -7	$^{-1}_{\ 6}_{\ 1}$		$-3 \\ -3$	6	$ \begin{array}{c} 1 \\ 3 \\ -5 \\ 2 \\ -10 \\ 3 \\ 5 \\ 5 \\ -3 \\ -3 \\ -3 \\ 4 \\ 4 \\ -3 \\ -3 \\ -3 \\ -3 \\ -3 \\ -3 \\ -3 \\ -3$	$-3 \\ -3 \\ -3$	$     \begin{array}{r}       -1 \\       3 \\       -3 \\       5 \\       3 \\       -5 \\       -3 \\    \end{array} $	$-1 \\ -3 \\ 1$	:		$-2$ $\dot{2}$	:		$-2$ $\dot{2}$	$-\overset{\cdot}{9}$	-9 $-9$	$^{-9}_{18}$ $^{-9}$	:	9 9	:	:
$X.26 \\ X.27 \\ X.28$	$-3 \\ 4 \\ 4$	$-\frac{1}{3}$	$-3 \\ 3 \\ -9$	$-3 \\ -9 \\ 9$	$-6 \\ -3 \\ -3$	$-\frac{1}{2}$	-3 ·	6 1 1	$-3 \\ 4 \\ 4$	-3 -3 -3 -5 -5 -3		$     \begin{array}{r}       -1 \\       -3 \\       1 \\       3 \\       -3 \\       3     \end{array} $	1	i 1	$-\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{4}}}$	:	:	:	-9		18	$-\frac{1}{3}$		6	6
X.29 X.30 X.31	$ \begin{array}{r} 4 \\ 4 \\ -3 \\ 9 \\ -3 \end{array} $	$^{-9}_{-15}$	-9 12 9	$     \begin{array}{r}       -7 \\       -2 \\       7 \\       2   \end{array} $	:	$     \begin{array}{r}       -2 \\       -2 \\       6 \\       6     \end{array} $	1 1 1	-3 $-3$	-3 -3 -3	-3 $-3$	$-3 \\ -3 \\ 3$	-3	-3 $-3$	:		:	:	:	$-7 \\ -7 \\ -7$	$-7 \\ -7 \\ -7$	2 2 2 2	$     \begin{array}{r}     -4 \\     -7 \\     -4     \end{array} $	2 2 2 2	8 – 8 –	1 2 1
X.32 X.33 X.34	9 10 -4		6	9	12 -12	$-\frac{6}{8}$	1 6	4 2	-3 10 $-4$	-8 5 5 -1 -1		3	$-\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}$	:	:	:	:	:	-7	2		$-7 \\ -6 \\ 6$	:	2 3 6	2 3 6
X.36 X.37 X.38	$     \begin{array}{r}       -4 \\       -1 \\       -1 \\       8     \end{array} $	$\begin{array}{r} 6 \\ -6 \\ 2 \\ -2 \end{array}$	$-\frac{6}{2} \\ -2$	-9 -3 -3	$-12 \\ -10 \\ 10$	$     \begin{array}{r}       -8 \\       8 \\       -7 \\       -7 \\       -7 \\       -6     \end{array} $	5 5	2 2 2 2 2	$     \begin{array}{r}       -4 \\       -1 \\       -1 \\     \end{array} $	$-1 \\ -1 \\ -10$	$\begin{array}{c} \cdot \\ 5 \\ -5 \end{array}$	$-3 \\ -1 \\ 1$	$-\frac{1}{1}$		:	2 2	$-\frac{1}{2}$	:	3 3	3 3	3 3	6 6 6	3 3	6 9 9 -6 –	
X.39 X.40 X.41	18 -6 -8			:		-6	2 2	$-\frac{1}{6}$	8 -6 -6 -8				3		:	:	:		$-14 \\ -14$	-14	$\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}$	3 7 4 -6	4 - 4 -	-2 - -8 -6 -	2
X.42 X.43 X.44	$-1 \\ -9$	$-8 \\ 12$	$-15^{1}$	$-\frac{3}{2}$	-2 :	$-\frac{8}{2}$	1 3 10	$-\frac{1}{-3}$	$ \begin{array}{r} -6 \\ -8 \\ -1 \\ 3 \\ -2 \\ 3 \\ 3 \\ 3 \\ 3 \\ -6 \\ -6 \end{array} $	-1 -2	$-\frac{1}{3}$	1	2 i		:	4	: -4	•	6 -3 6 -3	$-3 \\ -12 \\ 6$	6 6 6	$-6 \\ -6 \\ -6$	-3 - 6 -	-9 -9	
X.45 X.46 X.47	$-9 \\ -1 \\ 3$	$-12^{\circ}_{8}_{6}$	15 -1 6	$-\frac{2}{3}$	-6	2	3 1	$-3 \\ -1 \\ -6$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-1 \\ -3$	$-1 \\ -3$	2	-3					$-\frac{3}{6}$	$-12 \\ -3 \\ -9$	6	3	6 -3 -	- 9 - 9	
$X.48 \\ X.49 \\ X.50$	$ \begin{array}{c} -1 \\ 3 \\ 3 \\ 3 \\ -12 \\ 6 \\ 6 \end{array} $	8 6 -3 -6 6	$     \begin{array}{r}       6 \\       -3 \\       3 \\       -6 \\       6     \end{array} $	-3 3 -3 -3 -5 -5 -3	$-\frac{6}{6}$	:	5 5 5 -2 -2 -2	$\frac{3}{3}$ $-6$	3 3	-1 3 3 3 -3 -3 -3 -3	$-3 \\ -3 \\ 3 \\ 3$	-3 -3 3 3	:	$-3 \\ -3 \\ -3$	:	:	:	:	$-9 \\ -9 \\ 18$	18 18 -9		:	:	:	
$X.51 \\ X.52 \\ X.53$	$^{-12}_{\  \   6}$	$_{-4}^{6}$ $_{-6}^{4}$	$     \begin{array}{r}       6 \\       -4 \\       4 \\       -6     \end{array} $	-555	$ \begin{array}{r}     6 \\     -6 \\     -6 \\     8 \\     -8 \\     6 \end{array} $	:	$-2 \\ -2 \\ -2$	-6 :	$-\frac{.}{6}$	$-3 \\ -3 \\ -3$	$-\frac{1}{2}$	$-1 \\ 1$	:	:			:	:	$-3 \\ 6 \\ 6 \\ -3$	-3 -3 6	$-12 \\ 15 \\ 15$	:	$     \begin{array}{r}       6 \\       -3 \\       -3     \end{array} $	:	:
$X.54 \\ X.55 \\ X.56$	$     \begin{array}{r}     -12 \\     -2 \\     9 \\     -9     \end{array} $	-6 -6		6		$-\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\cdot}}}$	$-\frac{2}{2}$	$-6 \\ -2 \\ -6$	$ \begin{array}{c} -2 \\ -3 \\ 3 \\ 3 \\ -3 \end{array} $	$-3 \\ -2 \\ .$	-3	-3 :	$-\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\cdot}}}$	:	- <u>2</u>	:	:	2	$^{12}_{-9}$	$\frac{-6}{18}$	$^{-12}_{12}$	$-\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}$	$^{6}_{-6}$	9	:
X.57 X.58 X.59	-9 -9 9	-3 6	-3 6	$-6 \\ -6 \\ -6$	-9 -3 3 9	:	-7 $-7$ $-7$	$-3 \\ -6$	$-3 \\ -3$		-3 3		:	:	$-\frac{4}{2}$	:	:	-2	18 18 -9	-9 -9 18		:		:	:
X.61 X.62 X.63	9 9 -9	$ \begin{array}{r} -6 \\ 3 \\ -3 \\ 6 \\ 8 \\ -3 \\ 3 \\ 6 \\ -6 \\ 1 \end{array} $	-6 3 -3 6 8 -8 -3 6 -6	$-\frac{8}{8}$	$     \begin{array}{r}       2 \\       -2 \\       -9 \\       9   \end{array} $	:	-4 $-4$ $-3$	3 3 6	-3 -3	:	-4 4 3 -3	$-\frac{2}{2}$	:	:	2	:	:	2 -2	15 15	15 15	-3	:	-3 -3	:	:
X.64 X.65 X.66	-9 -9 -3 -3	6 -6	6 -6		-2	:	$-\frac{3}{3}$	6 6 -3	-3 -3 3 -3 -3	-3	$-\frac{3}{1}$	i	:	:	:	:	:	:	:	:		:	_9	:	:
X.67 X.68 X.69	-3		$     \begin{array}{r}       1 \\       8 \\       -1 \\       -8 \\       -9     \end{array} $	$-\frac{3}{3}$	$-\frac{1}{2}$ $-2$			$-\frac{6}{6}$	ž	-3 -3 -3	$-1 \\ -1 \\ 1$	$-1 \\ -1 \\ 1$								9	-9 -9 -9		-9 :		
$X.70 \\ X.71 \\ X.72$	-3 -3 -3	$-18 \\ -18$	-9 9	:	$-\frac{2}{2}$	3	3	-3 -3 -3	-3 -3			:	$\begin{array}{c} -3 \\ -3 \\ \end{array}$		$-3 \\ 3 \\ -1$	$\begin{array}{c} 1 \\ 1 \\ -3 \end{array}$	1 1 -3	1 -1 -1	:			9	:	:	
X.69 X.70 X.71 X.72 X.73 X.74 X.75 X.76 X.77 X.78 X.80 X.81			:		-9 -6 6 4 -4 -4	33 · · · 3 3 7 7 8 8 8 · · · 3 7 7 8 8 8 · · · 3 7 7 7 8 8 8 · · · 3 7 7 7 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8		:	:				3	-1	-3 3 -1 1	$ \begin{array}{c} 1\\1\\-3\\-3\\2\\2\\-2\\-2\end{array} $	$ \begin{array}{c} -3 \\ -3 \\ -2 \\ -2 \\ 2 \end{array} $	1	:	:		:	:		
$X.76 \\ X.77 \\ X.78$	$-1 \\ -1 \\ 4$	$-3 \\ 10$	$\begin{array}{c} 3 \\ -3 \\ 10 \end{array}$	$     \begin{array}{r}       -9 \\       9 \\       2 \\       -2     \end{array} $	$^{-6}_{\ \ 4}$	$-7 \\ -7 \\ -8$	3 3	$ \begin{array}{c} -1 \\ -1 \\ -2 \\ -2 \\ 4 \end{array} $	$-1 \\ -1 \\ 4$	$-1 \\ -1 \\ 4 \\ 4$	$\begin{array}{r} 3 \\ -3 \\ 4 \\ -4 \end{array}$	$\begin{array}{c} \cdot \\ \cdot \\ 3 \\ -3 \\ -2 \\ 2 \\ -1 \end{array}$	$\begin{array}{c} 3 \\ 3 \\ 2 \\ -2 \\ -2 \\ 4 \end{array}$	-1	:			:	$-\dot{4}$	$-\dot{4}$	5	$\frac{3}{3}$	5 - 5 - -4 -4	3 3 -7 -7 -7	3 2
X.79 X.80 X.81		$-10 \\ 14 \\ -14$	-10 -4 4	$-11 \\ 11$	$^{-4}_{-4}$	$-8 \\ -8 \\ -8$			4 4 4	$^{4}_{1}_{1}$	$-\frac{2}{2}$	$-\frac{2}{1}$	4		•	:	:	:	$-4 \\ -4 \\ -4$	-4	5 5 5 -2 -2	$\begin{array}{c} 3\\ 3\\ -4\\ -4\\ 2\\ 2\\ -8\\ -8\\ -9\\ \end{array}$	$     \begin{array}{r}       5 - \\       -4 \\       -4     \end{array} $	5 —	4
X.82 X.83 X.84 X.85			:	-8 8	4	-3		-6 -2	:		•	:	3 -3	$-1 \\ -1 \\ -\dot{2}$	:	2	2 -4	:	$-\frac{2}{-2}$	$     \begin{array}{r}       -4 \\       -2 \\       -2 \\       \end{array} $	$-2 \\ -2 \\ .$	$-8 \\ -8 \\ -9$	$-\frac{1}{2}$	-2 -2	7 7
$X.85 \\ X.86$	$-\dot{2}$		:	:	:	$-3 \\ 7$	6	$-\dot{2}$	$-\dot{2}$	$-\dot{2}$	:	:	$-3 \\ -2$	-2	:	$-4^{-4}$	$-4 \\ 4$	:	:	:		$-\dot{3}$	: -	-3 –	3

2 3	1 5	$\frac{1}{5}$	$\frac{1}{4}$	4	4	4	$\frac{7}{3}$	$_{4}^{5}$	$_2^7$	5 3	5 3	5 3	5 3	$\frac{3}{4}$	$\frac{4}{3}$	5 2	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	3 3	$\frac{4}{2}$	$\frac{4}{2}$	$\frac{3}{2}$	$\frac{3}{2}$
7 13	:	:		1	1	1	:	:	:	:	:	:	:		:	:		:	:	:	:	:	:	:	
2P	$\frac{9h}{9h}$	$\frac{9i}{9i}$	9j	$\frac{10a}{5a}$	$\frac{10b}{5a}$	$\frac{10c}{5a}$	12a 66	$\frac{12b}{65}$	$\frac{12c}{66}$	$\frac{12d}{6_{12}}$	$\frac{12e}{6_{13}}$	$\frac{12f}{6_{10}}$	$\frac{12g}{65}$	$\frac{12h}{616}$	$\frac{12i}{6_{11}}$	$\frac{12j}{622}$	$\frac{12k}{6_{22}}$ $\frac{4b}{4b}$	$\frac{12l}{6_{10}}$	$\frac{12m}{66}$	$\frac{12n}{6_{13}}$	$\frac{12o}{624}$	$\frac{12p}{6_{10}}$	$\frac{12q}{6_{11}}$	$\frac{12r}{6_{16}}$	$\frac{12s}{6_{23}}$
3 <i>P</i> 5 <i>P</i> 7 <i>P</i>	$\frac{3e}{9h}$	$     \begin{array}{c}       3e \\       9i \\       9i     \end{array} $	3f 9j 9j	2d	10b 2a	$^{2b}$	$\begin{array}{c} 4b \\ 12a \\ 12a \end{array}$	$\frac{4a}{12b}$	$\frac{4d}{12c}$	$\frac{4b}{12d}$	$\frac{4b}{12e}$	12f	$\frac{4c}{12g}$	$\frac{4b}{12h}$	$\frac{4a}{12i}$	12j	12k	12l	$\frac{4g}{12m}$	$\frac{4g}{12n}$	$\frac{4b}{12o}$	$\frac{4e}{12p}$	$\frac{4e}{12q}$	$\frac{4g}{12r}$	$\frac{4c}{12s}$
13P X.1	$\frac{9h}{9h}$	9 <i>i</i>	9j		10b		$\frac{12a}{12a}$	$\frac{12b}{12b}$	$\frac{12c}{12c}$	$\frac{12a}{12d}$	$\frac{12e}{12e}$	$\frac{12J}{12f}$	$\frac{12g}{12g}$	$\frac{12h}{12h}$	$\frac{12i}{12i}$	12j $12j$	$\frac{12k}{12k}$	12 <i>l</i>	$\frac{12m}{12m}$	$\frac{12n}{12n}$	120 120	$\frac{12p}{12p}$	$\frac{12q}{12q}$	$\frac{12r}{12r}$	12s 12s
X.2 X.3	$-1 \\ -1$	1 2	1 2	-1	-1	1 2	$\frac{1}{2}$	-1	$\frac{1}{2}$	1 2	1 2 2	-1	$-\hat{1}$	1 2	-1	$-1 \\ -1$	$-1 \\ -1$	$-\hat{1}$	-1	-1	$\frac{1}{2}$	1 2	1 2	-1	-1
X.4 X.5			:	:	:	:	5	$-1 \\ 1$	$-3 \\ -3$	2 2 2	2	$-\frac{2}{2}$	-3	$-1 \\ -1 \\ -1$	$-4 \\ 4$	3	$-1 \\ -1$	$-\frac{2}{2}$	$-1 \\ 1$	$-\frac{2}{2}$	$-1 \\ -1$			$-1 \\ 1$	:
X.6 X.7 V.8		_6 _3	3	$-1 \\ -3$	3 1	1 1	3	$-\frac{1}{3}$	$-6 \\ 3 \\ 3$	$-{}^{4}_{3}$	-3	$^{3}_{-4}$	$\overset{\cdot}{\overset{-3}{1}}$	$-2 \\ -6$	_ i	-3 ·		3	1 _3	$-\frac{1}{2}$	$-2 \\ -3$	$-\frac{1}{2}$	$-\frac{1}{2}$	i	_2
X.9 X.10		$-\frac{3}{6}$	3	3	$-\frac{1}{-3}$	1	10 3 3 3 3	$     \begin{array}{r}       -3 \\       -7 \\       7 \\       3     \end{array} $	3	-3	-3	-3	$-\frac{1}{3}$	-6	1		:	-3	$-3 \\ 3 \\ -1$	$-\frac{3}{2}$		$-\frac{5}{1}$	$-\frac{1}{2}$	-i	2
$X.11 \\ X.12 \\ Y.12$	1 1	$-2 \\ -2$	1 1					-6 6		$-3 \\ 3 \\ 3$	:	-3 3	-2	3 3 3 3		3	3 3	-3 $-1$ $-1$	$-\frac{2}{2}$	$-\frac{2}{2}$		$\frac{1}{1}$	$-2 \\ -2$	$-\frac{1}{1}$	-1 1
X.14 X.15	6	:	:	$-3 \\ 3 \\ -2$	$-3 \\ 3 \\ -2$	$-1 \\ -1$	$_{-4}^{6}$	$-2 \\ -6$	$\begin{array}{c} 6 \\ 6 \\ -4 \end{array}$	2	2	$     \begin{array}{r}       4 \\       -3 \\       -3 \\       3 \\       -2 \\       6     \end{array} $	$-\frac{2}{2}$	$\frac{3}{-1}$	$-\frac{2}{2}$	. 2	2	$-\frac{2}{2}$	$     \begin{array}{r}       -2 \\       2 \\       -2 \\       2 \\       2     \end{array} $	$-2 \\ 2 \\ -2 \\ -2$	$\begin{array}{c} 3 \\ 3 \\ -1 \end{array}$	:	:	$-1 \\ -1 \\ -1$	$-1 \\ -1 \\ -1$
$X.16 \\ X.17$	$-\frac{6}{1}$	$-\dot{4}$	2	2	2	:	-4	6	-4	2 2 6	2	-6	-2	$-\frac{1}{6}$	:	$-\frac{2}{3}$	$-\frac{2}{3}$	-2	-2	-2	-1	$\dot{2}$	$-\dot{4}$	1	1
$X.18 \\ X.19 \\ X.20$	-6 ·	3	:	:	:	:	-8	$-\frac{1}{6}$	-8	4 3 3 3 3 3	4 6 6	$-\frac{1}{3}$	2 -2	-2 ·	:	-2	-2 ·	1 -1	$-\frac{1}{2}$	$-\frac{1}{2}$	$-2 \\ 3 \\ 3$	$-\frac{1}{3}$	:	$-\frac{1}{2}$	$-\dot{1}_{1}$
X.21 X.22	6 6				:		$-3 \\ -3$	9	$-3 \\ -3$	3	3	$ \begin{array}{r} 3 \\ -3 \\ -3 \\ 2 \\ 3 \\ -2 \\ 3 \\ -2 \\ -2 \\ -2 \end{array} $	$-\frac{1}{1}$		$-\frac{3}{3}$		:	-1 1	$-\frac{2}{3}$	$-\frac{1}{3}$		1 1	1 1		$-1 \\ -1$
X.23 X.24	:	:		-3	$-\frac{1}{3}$	1	$-3 \\ -3 \\ -3$	$-9 \\ -1 \\ -1$	$-3 \\ -3 \\ -3$	-6	$^{-6}_{3}$	$-\frac{3}{2}$	$-5 \\ -1$	:	5	:	:	1 2	$\begin{array}{c} 3 \\ 3 \\ -1 \\ \end{array}$	-i	:	$-\frac{1}{2}$	-2	$\dot{2}$	$-\frac{1}{1}$
X.26 X.27	-3	:	:	-1	$-\frac{1}{3}$	1	$-3 \\ -1$	$-3 \\ 1 \\ 3$	$-3 \\ -3 \\ -1$	$^{3}_{-6}$	$-6 \\ 3 \\ -1$	$-\frac{3}{3}$	5 1 3	-i	$-\frac{.}{3}$	· 2	2	$-1 \\ -2 \\ 3 \\ -3$	$-3 \\ 1 \\ -1$	1 -1	-i	$-\frac{1}{2}$	$-2 \\ 1 \\ -1$	$-\frac{1}{2}$	1
$X.28 \\ X.29$	$-\frac{3}{2}$	$-\frac{1}{2}$	_ i	:		:	$-\frac{1}{5}$	$-\frac{3}{7}$	$-\frac{1}{5}$	-1	$-\frac{1}{2}$	$-\frac{3}{2}$	-3	$-1 \\ -1$	$-\frac{3}{2}$	2 2 2 2 2	2	$-3 \\ -2$	$\frac{1}{1}$	$\frac{1}{4}$	$-1 \\ -1$	-1	-1	1	i
X.30 X.31 X.32	2 2 2	$-\frac{2}{1}$	-1 $-1$	:	:	:	$-1 \\ -1 \\ -1$	7	$-1 \\ -1 \\ -1$	2 2 2 2	$-4 \\ 2 \\ -4$	$-\frac{2}{2}$	$-1 \\ -1 \\ 1$	$-1 \\ -1 \\ -1$	$-2 \\ -2 \\ 2$	2 2 2	2 2 2	$-2 \\ 2 \\ 2 \\ -2$	$-1 \\ -1 \\ 1$	$-4 \\ -2$	$-\frac{2}{1}$	•	:	-1 -1	$-1 \\ -1 \\ 1$
X.33 X.34	$-\frac{5}{6}$			-4	$-\dot{4}$		-6	:	$-\frac{1}{6}$	6	6	:			:		:	:		:	:	2	2		
X.35 X.36	$-3 \\ 3 \\ 3$	:	:	4	4	:	9 9	1	į		:	$-\frac{1}{2}$	5		4	i	$-\frac{3}{3}$	$-\frac{1}{2}$	$-\frac{1}{3}$	:	:	:	:	:	$-\frac{1}{1}$
X.38 X.39	-2	4	-2				$-\frac{3}{2}$	-1	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{2}$		-5	$-\frac{1}{2}$	-4	$-\frac{1}{2}$ $-2$ $-2$	$-2 \\ -2 \\ -2$				$-\frac{1}{4}$	$-\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\cdot}}}}$	$-\dot{2}$		
X.40 X.41	$-2 \\ -2 \\ 3$	-2	-2 ·	:	:	:	10 ;		10	4	4			-2	:	-2	$-2 \\ -2 \\ \vdots$				-2	:	:		
X.42 X.43 X.44	-3 -3	$-\dot{3}$		:			$-3 \\ 18$	7	$-\frac{3}{2}$	$-\frac{4}{6}$	-2	$-\frac{6}{2}$	-1	-2 6	$-\dot{2}$	-i	-2 3	$-\frac{2}{2}$	-1	2	3			2	-1
$X.45 \\ X.46$	_3	-3					$-3 \\ -5$	$-7 \\ -3$	$-\frac{1}{3}$	$-\frac{6}{4}$	-2	$-6^{2}$	1	$-\frac{6}{2}$	2	6	$-2^{-\frac{1}{2}}$	$-\frac{1}{2}$	1	$-\frac{1}{2}$	3	•		$-\frac{1}{2}$	1
X.47 X.48 Y.40		:	:	:	:	:	:	$-10^{-10}$	:	-6	$-6 \\ -6$	$-\frac{1}{2}$	8 2 -2	$-6 \\ 3 \\ 3$	4 _4	:	:	$-\frac{4}{2}$	2	$-\frac{1}{2}$	$-\frac{1}{3}$	:	:	$-\frac{1}{1}$	$-1 \\ -1 \\ 1$
X.50 X.51		6		2	$-\dot{2}$	$-\dot{2}$		-10 8		$-\dot{6}$	-0	· 2	$-2 \\ -8 \\ .$	-6	-4 $-4$			$-\frac{2}{4}$	-2	-2	-3				1
X.52 X.53		$-3 \\ -3 \\ e$	3	$-\frac{2}{2}$	$-\frac{1}{2}$	$-2 \\ -2$	:		:		:		:				:			:	:	:		:	:
X.54 X.55 X.56	3			-2		-2	$^{-10}_{-3}$	-8 -1	$_{-3}^{\dot{6}}$	$-\frac{1}{8}$	$-\frac{1}{3}$	-2 -1	-i	$-\frac{1}{4}$	-i	$-\dot{6}$	2	-2 -1	3	-3	2	1	1		-i
$X.57 \\ X.58$		:	:	:	:	:	$-3 \\ -3$	$-1 \\ -3 \\ -3$	$-3 \\ -3$	$-3 \\ 3 \\ 3$	$-3 \\ -3 \\ -3$	$-1 \\ -3 \\ -3$	$-5 \\ 5 \\$	3	$-\frac{3}{3}$	:		$-1 \\ 1 \\$	$-1 \\ 1$	$-1 \\ 1 \\ 1$	$-3 \\ -3$	1	1	$^{-1}_{1}$	$-\frac{1}{1}$
X.59 X.60 X.61	:	$-\frac{1}{3}$	$-3 \\ -3$	1 -1	i -1	1 1	-3	1 8 -8	-3 ·	-3	3	$-\frac{1}{4}$	1	-6	$\frac{1}{2}$	:	:		-3 ·	3	:	$-\frac{1}{2}$	$-\frac{1}{2}$	:	
$X.62 \\ X.63$				$-1^{\hat{1}}$	$-\frac{1}{3}$	$-1 \\ -1$	3 3 3	$-3^{\circ}$	3 3 3	$-\frac{3}{3}$		$-\frac{3}{3}$ $-6$	$-\frac{1}{5}$	3 3				$-1^{i}$	$-1 \\ 1$	$-\frac{1}{2}$	$-\frac{1}{3}$	$-1 \\ -1$	$-\frac{2}{2}$	$-\frac{1}{1}$	-1
X.64 X.65 Y.66		:	:	-3	$-1 \\ 1$	$-1 \\ -1$	$\begin{array}{c} 3 \\ 3 \\ 15 \\ -15 \end{array}$	-9 $-9$ $3$ $-3$	3	:	$-3 \\ -3$	$^{-6}_{6}$	-1 1 -1	-6	$-3 \\ 3$	:	:	$-2 \\ 2 \\ -2$	-3	-3		$\frac{2}{2}$	$-1 \\ -1$	_i	$-1 \\ 1 \\ -1$
X.67 X.68		:		:	:		$-15 \\ 15$	-3	9 -9	:	6	$-6 \\ -6$	1	3 3 3	:		:	$-\frac{2}{2}$	1 1	$-\frac{1}{4}$	3	÷	:	1 1	1 1
X.69 X.70 X.71		:	:	:	1	:	$-15 \\ -1 \\ -1$	$-\frac{3}{3}$	$-1 \\ -1$	2	$-\frac{6}{1}$	6	$-1 \\ -3 \\ -3$	3 2 2	3	$\begin{array}{c} -\dot{1} \\ -1 \end{array}$	$-\frac{1}{1}$	-2	$-1 \\ 1 \\ 1$	1	-i	$-\frac{1}{2}$	1 1	$-\frac{1}{-2}$	-1
X.72 X.73				1 -1	$-1 \\ -1$	1 1	3	$-\frac{3}{3}$	3	$-\frac{2}{3}$	$-1 \\ -1 \\ -3 \\ -3$	$-\frac{1}{3}$	$-\frac{3}{3}$		3 -3 -3 3	-1	-1	$-\frac{1}{3}$	$-3 \\ -3 \\ 3$	$     \begin{array}{c}       1 \\       -1 \\       -3 \\       3     \end{array} $	-1 -1	$     \begin{array}{r}       -2 \\       -2 \\       \hline       1 \\       1     \end{array} $	1		•
X.74 X.75			:	:	:		-9 -9 -5	3 -3 -3 -3 9 -9	$-1 \\ -1$		$-\frac{1}{2}$		$     \begin{array}{r}       -1 \\       -3 \\       3 \\       -3 \\       -3 \\       \hline       3 \\       5     \end{array} $	i		$ \begin{array}{c}     -1 \\     -1 \\     -3 \\     -3 \end{array} $	-1 -1	-2 -3 3 -4	$ \begin{array}{c} -1 \\ 1 \\ -1 \\ -3 \\ 3 \\ -3 \\ 1 \end{array} $		:		:	i i	
X.76 X.77 X 78	$\begin{array}{c} 3 \\ 3 \\ -1 \end{array}$	-1	_ i			•	-5 -5	-9	3	$\begin{array}{c} 2\\2\\-3\\-3\\-3\\ \end{array}$	$-2 \\ -2 \\ .$	6 -3 3	$-5 \\ -5 \\ .$	1 1	•	$-3 \\ -3 \\ .$	1 1	$^{-4}_{4}$	$-\frac{1}{1}$	$-\frac{1}{2}$	1 1		:	$-1 \\ -1$	$-1 \\ 1$
X.72 X.73 X.74 X.75 X.76 X.77 X.78 X.79 X.80 X.81	$-1 \\ -1 \\ -1$	$-1 \\ -1 \\ 2 \\ 2 \\ 1$	$-\frac{1}{1}$	:	:	:	:	:	:		:	:	:		:	:		:	:	:	:		:		:
X 82	4	2 1 1	$-1 \\ 1 \\ 1$		:	:	:		:			:	$-\frac{8}{8}$		:	:	:	:	:	:	:		:	:	$\frac{1}{2}$
X.83 X.84 X.85						:	$-\frac{1}{2}$		$-2 \\ -2$	4	-2	:	-0	4	:	$\frac{1}{1}$		:	:	:	-2	$-\frac{i}{\cdot}$	2	:	$\begin{array}{c} \overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\cdot$
X.86	<b> </b> -3	٠			•	•	-10	•	6	-4	-4	•	•	2		3	-1				2	•	•	•	•

2 3 5 7 13	3 2	3 2	2	1 1	1 1	3 i	2 1	1 2 1	1 2 1	3 4	2 4	3 3	3 3	3 3	3	3	1 3	1 3	1 3	1 3	3	3 i	3 1	i	3 1
$\frac{3P}{5P}$	$^{4g}_{12t}$	$\begin{array}{c} 4f \\ 12u \\ 12u \end{array}$	$     \begin{array}{r}       12v \\       6_{29} \\       4d \\       12v \\       12v \\       12v \\     \end{array} $	$13a \\ 13b \\ 13b$	13a	$14a \\ 14a \\ 2b$	$7a \\ 14b \\ 14b \\ 2a$	15a 5a 3c 15a 15a	15b 5a 3b 15b	$^{6_1}_{18a}_{18a}$	$^{9a}_{6_1}$ $^{18b}_{18b}$	$^{9d}_{6_{16}}$ $^{18c}_{18c}$	$^{9a}_{65}_{18d}$ $^{18d}_{18d}$	$^{9b}_{69}_{18e}_{18e}$	9b 6 <sub>5</sub> 18f 18f	$^{6_{16}}_{18g}$ $^{18g}$	$^{69}_{18h}$ $^{18h}$	$ 9f $ $ 6_{16} $ $ 18i $ $ 18i $	18j $18j$	9j 68 18k 18k	$^{69}_{18l}$ $^{18l}$	10c $20a$ $4a$ $20a$	10c $20b$ $4e$ $20b$	7a $21a$ $3d$	24a
X.1 X.2 X.3 X.4 X.5 X.6	-1 -1 1	-1 $-2$ $2$	-1	1 1 2 1 1 2	1 1	$\begin{array}{c} 1 \\ 1 \\ 2 \\ -1 \\ -1 \\ -2 \end{array}$	-1 -1 -1	1 2	1 2	$ \begin{array}{c} 1 \\ -1 \\ 3 \\ -3 \\ \vdots \end{array} $	$-1 \\ -3 \\ 3 \\ .$	$     \begin{array}{c}       1 \\       1 \\       -1 \\       3 \\       3 \\       -3     \end{array} $	$\begin{array}{c} 1 \\ 2 \\ -1 \\ -1 \\ -2 \end{array}$	-1 -1 1	$ \begin{array}{c} 1\\2\\-1\\-1\\-2 \end{array} $	-1 -1	-1						1 1 2	$ \begin{array}{c} 1 \\ -1 \\ -1 \\ -1 \\ 1 \end{array} $	-1 -1
1.8 1.9 10 11 12 13	$\begin{array}{c} 1 \\ \cdot \\ -1 \\ -2 \\ 2 \\ 1 \end{array}$	1 -1 1			•	-1 -1 -1 -1	$ \begin{array}{c} 1 \\ -1 \\ -1 \end{array} $	$ \begin{array}{c} -1 \\ 2 \\ 2 \\ -1 \\ \vdots \\ -2 \end{array} $	$ \begin{array}{c} 2 \\ -1 \\ -1 \\ 2 \\ -2 \end{array} $	$ \begin{array}{r}     3 \\     -2 \\     2 \\     -3 \\     1 \\     -1 \\     -3 \end{array} $	$ \begin{array}{c}     1 \\     -1 \\     -2 \\     2 \\     -3 \end{array} $	-2 -2	$     \begin{array}{r}       -2 \\       1 \\       1 \\       -2 \\       2 \\       2 \\       1     \end{array} $		$ \begin{array}{c} 1 \\ -2 \\ -2 \\ 1 \\ -1 \\ -1 \\ 1 \end{array} $	1 1	-1 $1$ $-1$ $1$ $-2$ $2$	1	-1 $-2$ $2$	1 -1 -1 -1	-1 $-1$ $1$ $1$ $1$ $-1$	1	-1 -1 -1 -1		-1
1 6 6 7 8 9	-1 -1 1	-1 -1 1	-i			1 1 2	-1 1 :	$ \begin{array}{r} -2 \\ -1 \\ -1 \\ -1 \end{array} $	$     \begin{array}{r}       -2 \\       -1 \\       -1 \\       -1 \\     \end{array} $	3	3	1 1 2 -1	1 4	3	1 -2	$ \begin{array}{c} -2 \\ -2 \\ -1 \\ 2 \end{array} $	1	1 1 -1 -1	1	-2		$     \begin{array}{c}       -1 \\       2 \\       -2 \\       \vdots    \end{array} $	-1	1 1 -1	
	-1	-1 -1 1 1 -1 -1		-1 -1	-1 -1		•	-1 2 -1 2	-1 -1 -1	-1	2	2 2	3	-1 -1 -1 i	3 3	2 2	-1	-1 -1	-1	2		-1 -1 1	-1 -1 -1		-1 -1 -1 1
	$     \begin{array}{r}       -2 \\       -1 \\       1 \\       1 \\       2 \\       -1     \end{array} $	1 -1 -1	$     \begin{array}{r}       -1 \\       -1 \\       -1 \\       -1 \\       -1 \\     \end{array} $			1 1	-1 -1	-1	2	5 4 -5	3 -1 1 1	1 1 -3	3 1 1 1	-3 -3	-1 -2 1	1 1		-2 -2 -2	-1 -2 1	-1 1 1		1	-1	1 1	-i
	-2	1	1 1	$ \begin{array}{c} -2 \\ -1 \\ -1 \\ -1 \end{array} $	$ \begin{array}{c} -2 \\ -1 \\ -1 \\ -1 \end{array} $			1 1 1	1 1	-4	-1	$ \begin{array}{r} -3 \\ -2 \\ 2 \\ 2 \\ 2 \end{array} $	1 -1 -1	-i -1	-2 -1 -1	$     \begin{array}{r}       -2 \\       -1 \\       -1 \\       -1 \\       -1     \end{array} $	-1 1	1 2 2 -1 -1	2	-1	-i				
	-1 -1		$-\frac{1}{2}$	-2 -2	-2 -2	2		2	2	-3 4		$ \begin{array}{r} -1 \\ 3 \\ -2 \\ -1 \\ -2 \end{array} $	$\begin{array}{c} 2\\2\\-2\\-3\\-2\end{array}$	i i	-4 2 1	1	i	-2 -1	1	-2	-2			-1	
	1 1 -1 1	1 -1 1 -1 -1				1 1 1 1	-1 -1 -1 -1			$     \begin{array}{r}       -4 \\       3 \\       -3 \\       \vdots \\       3    \end{array} $	-1 -3 3	-i	$ \begin{array}{r} -2 \\ -3 \\ -2 \\ 2 \\ -1 \\ -1 \\ 2 \end{array} $		$-\frac{1}{2}$	-i	-i	-1	-1	2	2				
		-2 -2 -1						1 1 1 1	1 1 1 1	-5 5	$     \begin{array}{r}       -3 \\       4 \\       -4 \\       3     \end{array}   $	1	$ \begin{array}{r}     1 \\     -2 \\     -2 \\     1 \\     -4 \\     -1 \end{array} $	-i	$-\frac{1}{1}$ $-\frac{1}{2}$ $\frac{1}{2}$	i	-1 1	1	-1	-1 -1	-1 1				
	-1 1	-1 1 1			•			1 1 1	1 1 -2	-3 3 -1 1	-3 -1 1		$ \begin{array}{c} 2 \\ -1 \\ -1 \\ -1 \end{array} $	$-3 \\ 3 \\ -1 \\ 1 \\ .$	$     \begin{array}{r}       -1 \\       -1 \\       2 \\       -1 \\       -1 \\       -1 \\    \end{array} $		-i		-1 1	-i	-i	1 -1 -1	1 1 1 1		$ \begin{array}{c} 1 \\ -1 \\ -1 \\ -1 \end{array} $
	$\begin{array}{c} 1 \\ \cdot \\ -1 \\ -2 \\ 1 \\ \end{array}$	-1 $-1$ $-1$ $-1$ $-1$ $-1$			•		•	$     \begin{array}{c}       1 \\       -2 \\       -2 \\       \vdots \\       \vdots     \end{array} $	-2 1 1	6 3 -6				$\begin{array}{c} \cdot \\ -2 \\ -1 \\ 2 \\ 1 \end{array}$	-3 -3		1 2 -1 -2				1 -1 -1	1 -1 -1	1 1 1		1
	$\begin{array}{c} 1 \\ 2 \\ 1 \\ -1 \\ \vdots \\ \vdots \\ \end{array}$		-1 -1 -1		1	-1 -1	-1 1	-1 -1	-1 -1	-3		-3 -3					-2				1	-1 -1	1	-1 -1	-1 1 :
ш	1 -1 -1	-1 1								-2 -2 -2 -2	$\begin{array}{c} \cdot \cdot \\ -2 \\ -2 \\ -2 \\ -2 \\ 2 \end{array}$	$ \begin{array}{c} -1 \\ -1 \\ 4 \\ -2 \\ -2 \end{array} $		-2 -2 -2 -2		-1 -1 1 1 1	1 -1 -2 -2	1	-1 $-1$ $-2$ $2$ $1$	-i -i 1 -1	1 -1 -1 1				
			1 1		2	$-1 \\ -1 \\ -2 \\$	-1 1			4 -4 ·	$-\frac{2}{2}$	$ \begin{array}{c} 4 \\ -2 \\ -2 \end{array} $	2 2		2 2		:		-1 -1	-1 -1				-1 -1 i	

 $Character\ table\ of\ mH\ (continued)$ 

2 3 5 7 13	3 1	3 1	1	1	3	2	1 1 1	1 1 1	2 2	2 2	i	i
	246	$\frac{24c}{12k}$	i 26a	i 26b		$\frac{1}{28a}$ $\frac{1}{14a}$	30a	306	36a	36b	39a	39 <i>b</i>
2P 3P 5P 7P	$^{12j}_{8b}_{24b}$	8c	13a 26a 26b 26b	$   \begin{array}{r}     13b \\     26b \\     26a \\     26a   \end{array} $	9c 27a 27a 27a 27a	28a	$15a$ $10a$ $6_4$ $30a$	10b	18c $12h$ $36a$ $36a$	$18d \\ 12b \\ 36b \\ 36b$	39b 13a 39b 39b	39a
7P 13P X.1 X.2	1 1	1	$\frac{2a}{-1}$	$\frac{2a}{-1}$	1	$   \begin{array}{r}     28a \\     \hline     -1 \\     \end{array} $	$30a \\ 30a \\ -1 \\ -1$	$     \begin{array}{r}       30b \\       30b \\       \hline       1 \\       -1     \end{array} $	36a 1 1	$\frac{36b}{-1}$	$\frac{3d}{1}$	3d 1 1
X.4 X.5	$-1 \\ 1 \\ 1 \\ -1$	$-1 \\ -1 \\ -1 \\ 1$	-1	1 -1	-1 :	1 -1			$-1 \\ -1 \\ -1 \\ 1$	$-\frac{1}{1}$	$-1 \\ 1 \\ 1 \\ -1$	$-1 \\ 1 \\ 1 \\ -1$
X.7 X.8 X.9						$-1 \\ -1 \\ 1$	-i	1 -1		-1 1		:
$X.10 \\ X.11 \\ X.12 \\ X.13$	$-1 \\ -1$	$-1 \\ -1$	:		i 1 1	1	1	:	:	i		•
$X.14 \\ X.15 \\ X.16$						-1 1	1 -1	1 -1	-1 -1	-1		•
$X.17 \\ X.18 \\ X.19$	1	1	:	:	-1 :			:	i	:		:
X.20 X.21 X.22 X.23 X.24			$-1 \\ 1 \\ 1$	-i	•		•	i	•	•	$-1 \\ -1$	$-1 \\ -1$
$X.24 \\ X.25 \\ X.26 \\ X.27$		:	:	:			1 - i	−i		-1 i		
$X.28 \\ X.29$				•	-1 -1	$-1 \\ -1 \\ \vdots$			$-1 \\ -1 \\ 2 \\ -1$	-i		:
X.30 X.31 X.32 X.33					$-1 \\ -1 \\ -1$				$-\frac{2}{1}$ $-1$ $-1$	$-\frac{1}{\cdot}$	1	i
X.34 X.35 X.36 X.37 X.38	$-1 \\ -1 \\ -1$	1 1	-1	-1		:	-1 1	-1 1	:	i -1	$-1 \\ -1 \\ .$	$-1 \\ -1 \\ .$
X.40					1 1				$\begin{array}{c} 1 \\ 1 \\ -2 \end{array}$			
$X.41 \\ X.42 \\ X.43 \\ X.44$	· · i	-i	:					:	i	1	1	1
$X.45 \\ X.46 \\ X.47$						1			i	-1		:
$X.48 \\ X.49 \\ X.50 \\ X.51$		:	:	:		$-1 \\ 1 \\ -1$			:	$-1 \\ -1 \\ -i$		:
X.51 X.52 X.53 X.54 X.55							$-1 \\ -1 \\ 1 \\ 1$	$\begin{array}{c} 1 \\ 1 \\ -1 \\ -1 \end{array}$		-1 i		
X.55 X.56 X.57 X.58 X.59									-1	-i		
$X.58 \\ X.59 \\ X.60 \\ X.61$				:			1 -1	1 -1		$-\overset{\cdot}{\overset{\cdot}{\underset{1}{1}}}$		:
X.62 X.63 X.64							-1 -1	-i				
X.65 X.66 X.67 Y.68								1	:	:		:
X.69 X.70 X.71	1 1	1 1							-1 -1 -1			
$X.72 \\ X.73 \\ X.74$	-1	1	1	1	:	-1	-1 -1	$-1 \\ -1 \\ .$			1	1
X.76 X.77 X.78	$-1 \\ 1 \\ 1 \\ .$	$     \begin{array}{c}       1 \\       -1 \\       -1     \end{array} $	-1 :	-1 :	- i	1		-1	1 1	•	1	1
X.79 X.80 X.81				-	$-1 \\ -1 \\ -1$							
X.68 X.69 X.70 X.71 X.72 X.73 X.74 X.75 X.76 X.77 X.80 X.81 X.82 X.83 X.84 X.85 X.86	$-\frac{1}{1}$	-1 -1	:	:	1 1	$-1 \\ 1 \\ .$		:	i	:	-i	
X.86	-1	1				÷			-1	:		

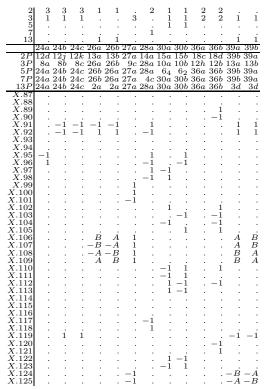
2 3 5	$^{13}_{13}_{2}$	10 9 1	11 6 1	13 5	10 5 1	8 13	$\begin{smallmatrix} 7\\10\\1\end{smallmatrix}$	$^{7}_{10}$	$\frac{6}{7}$	$^{4}_{11}$	$\begin{smallmatrix} 4\\10\end{smallmatrix}$	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{3}{7}$	9 4 1	$^{10}_{\ 4}$
7 13	1 1	1	î	:					i				:	÷		÷
13	1a	2a	2 <i>b</i>	2c	2d	3a	3b	3c	$\frac{1}{3d}$	3e	3 f	3 q	3h	3i	4a	4b
2P	1 <i>a</i>	1a	1a	1a	1a	3a	36	3c	$\frac{3d}{3d}$	3e	3f	3g	3h	3i	2b	2c
3P	1a	$\frac{2a}{a}$	$\frac{2b}{a}$	$\frac{2c}{c}$	$\frac{2d}{d}$	$\frac{1}{a}$	1a	$\frac{1}{a}$	$\frac{1}{a}$	$\frac{1}{a}$	$\frac{1a}{a}$	$\frac{1}{a}$	$\frac{1}{a}$	1a	4a	4b
$\frac{5P}{7P}$	1a $1a$	$\frac{2a}{2a}$	2b 2b	$\frac{2c}{2c}$	$\frac{2d}{2d}$	$\frac{3a}{3a}$	$\frac{3b}{3b}$	$\frac{3c}{3c}$	$\frac{3d}{3d}$	$\frac{3e}{3e}$	3 f 3 f	$\frac{3g}{3g}$	$\frac{3h}{3h}$	$\frac{3i}{3i}$	$\frac{4a}{4a}$	$\frac{4b}{4b}$
13P	$\frac{1a}{1a}$	$\frac{2a}{2a}$	$\frac{2b}{2b}$	$\frac{2c}{2c}$	$\frac{2a}{2d}$	$\frac{3a}{3a}$	3b	3c	$\frac{3a}{3d}$	3e	3 f	3g 3a	$\frac{3h}{3h}$	$\frac{3i}{3i}$	$\frac{4a}{4a}$	$\frac{4b}{4b}$
X.87	491400	-16380	-840	200	-60	-675	540	540	546	54	54	-27	-27	-21	60	<del></del>
X.88	491400	16380	-840	200	60	-675	540	540	546	54	54		-27 - 27			-:
$X.89 \\ X.90$	491400 491400	$5460 \\ -5460$	$-280 \\ -280$	264	$^{180}_{-180}$	3699 3699	$-270 \\ -270$	$-270 \\ -270$	:	54 54	54 54	$\frac{135}{135}$			$-20 \\ 20$	$\frac{72}{72}$
X.91	531441		729		-243	3033	-210	-210				100	-21			-27
X.92	531441	19683	729	81	243				729			:			-81 - 81	
X.93	552825	12285	945	153	45	1701				243						-75
$X.94 \\ X.95$	552825 $568620$	-12285 $2106$	$945 \\ -36$	153	-45	-2187	-729	$\frac{1215}{1458}$		243					-6	$-75 \\ 36$
X.96	568620	-2106	-36	108		-2187	-729	1458				:			-6	36
X.97	568620	16848	-36	108	144	-2187	1458	-729							96	36
X.98	568620	-16848	-36	108		-2187	1458	-729			150				-96	36
$X.99 \\ X.100$	582400 582400	:		-256		$3088 \\ -4688$	$\frac{1360}{1360}$		$-728 \\ -728$	-44	$-152 \\ -44$	-80	$^{-26}_{-8}$	28 - 26	•	
$X.100 \\ X.101$	665600		2560	-250		-3136	320	320	-832	-112	-112	68	32	32		
X.102	698880	17472	896			-960		-1176			-96	12	12	٠.		
X.103	698880	5824	896		-192		-1176	552		-312	120	12	12		-64	
$X.104 \\ X.105$	698880 698880	-17472 $-5824$	896 896	:	$\frac{64}{192}$	-960	-1176	$-1176 \\ 552$		$336 \\ -312$	$-96 \\ 120$	$\frac{12}{12}$	$\frac{12}{12}$		$-64 \\ 64$	
$X.105 \\ X.106$	716800	$\frac{-3624}{17920}$			192		-320	-320		-312 $-104$		-32		16	04	•
X.107	716800			:		1408	-320	-320	448	-104	-104	-32	-14	16		
X.108	716800					1408	-320	-320		-104		-32		16		
$X.109 \\ X.110$	$716800 \\ 786240$	$\frac{17920}{4368}$	110	102	10	$-1408 \\ -1080$	$-320 \\ -756$	$-320 \\ 864$	448	$-104 \\ 378$	$-104 \\ 54$	-32	$-14 \\ -27$	16	-80	
$X.110 \\ X.111$	786240		-448			-1080	864	-756		-108	216		$-27 \\ -27$		-80	•
X.112	786240	-4368	-448			-1080	-756	864			54		-27		80	
X.113	786240	17472	-448	-192	-192	-1080	864	-756		$-108 \\ -228$	216		-27			
$X.114 \\ X.115$	873600 873600	-14560 $14560$		-384 $-384$	-160	$\frac{4632}{4632}$	$-120 \\ -120$	960 960		$-228 \\ -228$	42 42	$-12 \\ -12$	-39 -30			•
X.116	982800		-1680	400	100	-1350	1080		-546		108		-54	21	:	:
X.117	998400	16640	-1280			-4704	480	480		-168		102	48			
X.118	998400			100		-4704	480	480		-168	-168	102	48			- i
	1062882 $1257984$	17472	1458 896	162	102	_1728	-1080	_1080	-729	216	216	-108	$5\dot{4}$		_6i	-54
	1257984		896		-192	-1728	-1080	-1080	:	216		-108	54		-64	
X.122	1397760	-11648	-1792		128	$-1920 \\ -1920$	-624	-624		24	24	24	24			
	1397760	11648	-1792		-128	-1920	-624	-624	440	24	24	24	24	16		
	1433600 1433600					$\frac{2816}{2816}$	-640	$-640 \\ -640$	-448			-64			•	
21.120	1 200000					2010	0-10	0-20	440	200	200	04	20	10	•	•

2 8 3 3	$^{10}_{\ 2}$	$\frac{7}{2}$	9 8	3 4 2 2 2 2	6 9	5 7 1	7 7	7 5 1	$_{6}^{7}$	8 5	$\frac{4}{7}$	$\frac{4}{7}$	6 5	6 5	6 5	$_{4}^{7}$	$_{4}^{7}$	3 6	$^6_4$
7 i		:					:		:	÷			:		÷	:	÷		
2P 2b			$\frac{1f}{2b} = \frac{4g}{2b}$		$\frac{6_1}{3a}$	$\frac{6_2}{3b}$	$\frac{6_3}{3c}$	$\frac{6_4}{3c}$	$\frac{65}{3a}$	$\frac{6_6}{3a}$	$\frac{67}{3e}$	6 <sub>8</sub>	$\frac{69}{3a}$	$\frac{6_{10}}{3b}$	$\frac{6_{11}}{3c}$	$\frac{6_{12}}{3b}$	$\frac{6_{13}}{3c}$	$\frac{6_{14}}{3q}$	$\frac{6_{15}}{3c}$
$\begin{array}{c c} 2P & 20 \\ 3P & 4c \\ 5P & 4c \end{array}$	4d	4e	4f = 4g	5a	2a	2a	2a	2d	$^{2b}$	2c	2a	2a	2d	$^{2b}$	$^{2b}$	2c	2c	2a	2d
7P - 4c	4d	4e 4	$\begin{array}{ccc} 1f & 4g \\ 1f & 4g \end{array}$	5a	$\frac{6_1}{6_1}$	$\frac{6_2}{6_2}$	$\frac{6_3}{6_3}$	$\frac{6_4}{6_4}$	$\frac{6_{5}}{6_{5}}$		$\frac{6_{7}}{6_{7}}$	$\frac{6_8}{6_8}$	69	$\frac{6_{10}}{6_{10}}$	$6_{11}$	$6_{12}$	$6_{13}$		$6_{15}$
$\begin{array}{c cc}  & 13P & 4c \\ \hline  & X.87 & 28 \end{array}$	16		$\frac{1f}{-4} - \frac{4c}{-12}$		$\frac{6_1}{-99}$	$\frac{6_2}{90}$					$\frac{67}{-18}$		21	$\frac{6_{10}}{-12}$	-12	-4	-4	$\frac{6_{14}}{9}$	12
	-24	: -	$\begin{array}{ccc} 4 & 12 \\ -4 & 12 \end{array}$	2 .	-129	-90 60	60 -	-60 -		3	18 6	6	-9	$-12 \\ -10$	-10	$-4 \\ -6$	$-4 \\ -6$	15	-12
$X.90 28 \\ X.91 -27$		9	$\frac{4-12}{9}$	-9	129	-60 ·	-60	60 -	-37	3	-6	-6	9	-10	-10	-6	-6	-15	
$X.92  27 \\ X.93  21$			9 -9	9 -9	135		27	-45 ·	-27·	_27	$-27^{-2}$	54	_9		2 <sup>.</sup>	:	_ <u>.</u>		_ ·
X.94 - 21	-3 -	-15 -	-9 -6 -6	· . ·	$-135 \\ -81$		$-\bar{2}\dot{7}$	45 -	-27 -		27	-54	9 -9	-9	27 18	-9	-9 18		9
X.96 6	-12	$-\frac{1}{4}$	6 6		81 81	-81	-81 ·		45	$-\frac{5}{27}$			9	$-9 \\ 18$	18	$-9 \\ 18$	18		_9
		$-4^{-1}$	:		-81		81	81	45	-27			$-\tilde{9}$	18	$-\tilde{9}$	18	-9	:	9
X.100 X.101	:	:	:			:				80			:	-32	20	-16	-16		
X.101 X.102 X.103 .	: -	-16	:	. 5 . 5	$-24^{\circ}$		-24 -				-24	-24	8	-32 $-4$ $-4$	-32 $-4$ $-4$	:	:	12	$-\frac{1}{4}$
X.104 .	: =	-16	:	. 5	$\frac{-8}{24}$		24		32		24	24	$\frac{24}{-8}$	-4	-4	:		-12	4
X.105  X.106 .	: -	-16	:		-224	$-154 \\ -80$	$\frac{8}{64}$		32		$^{8}_{-8}$	-8	-24		-4	:	:	$^{8}_{-8}$	12
X.107 X.108	:	:	:		224	80 -	$-64 \\ -64$	:	:		8	8		:		:	:	8	
X.109 X.110	:		16	-10	$-224 \\ 156$	$-80 \\ 66$	$^{64}_{-24}$	24 -	-16	24	-6		-12		-16	12	:	$^{-8}_{-6}$	
X.111 X.112		:		-10		$^{96}_{-66}$		-36 - -24 -		$\frac{24}{24}$	24 6	24 6	12 12	$-16_{20}$	$^{20}_{-16}$	12	12	6	-12
X.113 X.114				-10	300 20	-96 ·		36 · 40	-16		$-24 \\ 20$	$-24 \\ -34$	$-12 \\ -20$	-16	20	24	12	$-\frac{5}{2}$	12 -8
X.115 X.116	32						88			-24	-20	34	20	$-24^{\circ}$			-8	$-\overline{2}$	8
X.110 X.117 - 64 X.118 - 64		:	:	: :	$-208 \\ 208$		$-64 \\ 64$			-30	8	8 -8		16	16 16	-6		$-10^{\circ}_{10}$	:
X.119 .	$-\dot{6}$	18	:	-18	-24		-24	:	32		-8 -24		-24			:	:		
$X.120 \\ X.121 \\ X.121$	:	16 16	:	. 9	24	-30	24		32		$^{-24}$	$^{-24}$	24	$-4 \\ -4$	$-4 \\ -4$	:	:	-12	$-12 \\ -12$
X.122 X.123	:	:	:	. 10 . 10	-16 - 16	$^{124}_{-124}$		-64 -64	$-64 \\ -64$	:	$^{16}_{-16}$		$^{-16}_{16}$	8	8	:	:	$^{-20}_{20}$	$^{8}_{-8}$
X.124 X.125	:	:			:	:	:	:	:	:	:	:	:	:		:	:	:	

3	4 5	4 5		4 5	2 6	5 4	6	4	4	4	2 5	3	2	3	3	6 1	6	6	6	3	3	$\frac{1}{7}$	3	$\frac{1}{6}$	1
5															i				÷			÷			
13			<u>:</u>	:	_ :	:	<u>:</u>	:	:	:	<u>:</u>	:	<u>:</u>	<u>:</u>		_ :		_ :	:	. :		:		: .	:
2P	$\frac{6_{16}}{3e}$	$\frac{6_{17}}{3g}$	3e	$\frac{6_{19}}{3f}$	$\frac{6_{20}}{3h}$	$\frac{621}{3b}$	3d	$\frac{6_{23}}{3g}$	3f	3g	3h	$\frac{6_{27}}{3g}$	$\frac{6_{28}}{3h}$	$\frac{6_{29}}{3i}$	$\frac{7a}{7a}$	$\frac{8a}{4b}$	$\frac{8b}{4d}$	$\frac{8c}{4b}$	$\frac{8d}{4d}$	9a 9a	9b 9b			9e 9 9e 9	<u>f</u>
$^{3P}_{5P}$	$\frac{2c}{6_{16}}$	2c	$^{2d}_{6_{18}}$	$6_{19}^{2d}$	$\frac{2a}{6aa}$		$\frac{2c}{6aa}$	$\frac{2b}{6aa}$	2c	2c		2d	$^{2d}_{6_{28}}$			$_{8a}^{8a}$		$\frac{8c}{8c}$		$\frac{3a}{9a}$	$\frac{3a}{9b}$		$\frac{3e}{9d}$		se f
7P $13P$	616	617	618	$6_{19}$	620	621	622	623	624	625	626	627	628	629	1a	8a	8b	8c	8d	9a	9b	9c		9e 9	f
X.87	$\frac{6_{16}}{2}$	-1	-6	$\frac{6_{19}}{-6}$	- 9	-6	2	-3	- 2	-1	-1	3	$\frac{628}{3}$	-1	1 a	8a	80	8c	8a	9a	9b	9c	6	9e 9 . –	3
$X.88 \\ X.89$	$^{2}_{-6}$	$-\frac{1}{3}$		$^{6}_{-6}$	$-9 \\ -3$	6	2	$-3 \\ -1$	$-\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{3}{3}$	$-3 \\ 3$	-1	:			:		_ <u>.</u>	_ <u>.</u>		6	. –	3
X.90 X.91	-6	3		6	3			-1	-6	3	3	$-\frac{3}{3}$	-3							-9	$-\tilde{9}$				
X.92							9				:			:	1	$-\frac{3}{2}$	$-1 \\ -1$	-1	1	:	:	:		:	:
$X.93 \\ X.94$	$-9 \\ -9$		-9	$^{-18}_{18}$		:	:		:		:		:	:	:	$-\frac{3}{3}$	3	3	$^{-1}_{1}$		:	:	:	:	:
$X.95 \\ X.96$						_9 _9									3	$-\frac{2}{2}$		٠	$-\frac{2}{2}$				٠		٠
X.97 X.98				·			÷	÷	÷		÷	:		÷	3	-	÷	:	-	÷	÷	÷	÷	÷	:
X.99	-4	$-16^{\circ}$		:			8	:	8	8	2			$-\dot{4}$				:		$-\dot{8}$	$-\dot{8}$	10	- <u>i</u> -	-ė-	5
$X.100 \\ X.101$	-4	8		:		:	8	$\dot{4}$	-4	8	8		:	2	$-\dot{2}$	:	:	:	:	$^{-8}_{-4}$	$^{-8}_{-4}$	$^{10}_{-4}$		$^{10}_{-4}$	$\frac{7}{2}$
$X.102 \\ X.103$			8	8	$-6 \\ -8$	2 6		-4				-4	2							3	3	$-15 \\ 12$	٠.	3 -6	
X.104			-8	$-\dot{8}$	6	-2	:	$-\frac{1}{4}$		:	:	$\dot{4}$	$-\dot{2}$	:			:	:	:	3	3 -	$-15 \\ 12$	÷	š	:
$X.105 \\ X.106$				:	8 10	-6	:	-4	:	:	:	:	:	:	:		:	:	:	4	$\frac{3}{4}$	4	$-8^{-3}$	$^{-6}_{4}$	4
$X.107 \\ X.108$					$-10 \\ -10$															4	4	4	$-8 \\ -8$	4	4
X.109 X.110	6	-12	6	6	10	-6			-6											4 -9	4 18		-8	4	4
X.111	-12	6			-3		:	2		$-\dot{6}$	-3	$-\dot{6}$	-3	:	:	:	:	:	:	18	-9	:	:	:	:
$X.112 \\ X.113$	$^{6}_{-12}$	$^{-12}_{6}$		-6	$-\frac{3}{3}$	6		2 2	-6	$-\dot{6}$	$-3 \\ -3$	6	-3 3	:	:					$\frac{-9}{18}$	$^{18}_{-9}$	:		:	:
$X.114 \\ X.115$	12 12	12 12		$-14 \\ 14$	-11	-4			6 6		3	$-\frac{2}{2}$	-1		:			:		-12 - 12		-12		-3 -3	:
$X.116 \\ X.117$	4	-2			. 8		-2	-6	4	-2	$-\tilde{2}$	-		1						- <u>6</u>	- <u>6</u>	-6	-6	ė	3
X.118				:	-8			$-\frac{2}{2}$	:	:	:		:	:	-3		÷	j	:	-6	-6	-6		-6	:
$X.119 \\ X.120$	:	:	:	:	$-\dot{6}$	$-\dot{6}$	-9	$-\dot{4}$	:	:	:	:	:	:	2	:	-2	-2	:	$-\dot{9}$	$-\dot{9}$	:	:	:	:
$X.121 \\ X.122$			8	8	$-\frac{6}{2}$	$^{6}_{-4}$		$-\frac{4}{8}$				$-\dot{4}$	·					٠	٠	$-9 \\ 6$	$-9 \\ 6$	$-\dot{3}$	٠.	_ <u>.</u>	٠
X.123 X.124			-8	-8	2		÷	8	:	:	:	4	$-\overline{2}$	:	:		:	:	:	6	6	$-\frac{3}{8}$		$-\frac{3}{8}$ -	
$X.124 \\ X.125$	:		:	:	:	:	:		:	:	:	:	:	:	:	:	:	:	:	8	8	8	8	8 –	4

	3	6	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{4}{1}$	4 1 1	4 i	$\frac{7}{3}$	$\frac{5}{4}$	$_2^7$	5 3	$\frac{5}{3}$	5 3	5 3	$\frac{3}{4}$	$\frac{4}{3}$	$\frac{5}{2}$	$\frac{5}{2}$	5 2	$\frac{5}{2}$	5 2	3	$\frac{4}{2}$	$\frac{4}{2}$	$\frac{3}{2}$
	7 13	:	:	:	:				:	:		:	:	:	:	:	:	:	:	:	:	:		:	<u>:</u>	<u>:</u>
	2P	9g 9a		$\frac{9i}{9i}$	9 <i>j</i>	$\frac{10a}{5a}$	$\frac{10b}{5a}$	$\frac{10c}{5a}$	$\frac{12a}{66}$	$\frac{12b}{65}$	12c	$\frac{12d}{6_{12}}$	$\frac{12e}{6_{12}}$	$\frac{12f}{6_{10}}$	12g	$\frac{12h}{6_{16}}$	$\frac{12i}{611}$	$\frac{12j}{622}$	$\frac{12k}{6_{22}}$	12l	12m	$\frac{12n}{612}$	624	$\frac{12p}{6_{10}}$	$\frac{12q}{611}$	$\frac{12r}{6_{16}}$
	$\frac{5}{5}P$	3e	3e	3e		10a	10b	10c	4b	4a	4d	4b	4b	4a	4c	4b	4a	$\overline{4}d$	$\overline{4b}$	4f	4g	4g	4b	4e	4e	4g
	7P	9g	9h 9h	9i	9j	10a	10b	10c	12a	12b	12c	12d	12e	12f	12g	12h	12i	12j	12k	12l	12m $12m$	12n	12o	12p	12q	12r
	$\frac{3P}{.87}$		$\frac{9h}{-3}$		9j	10a	10b	10c	$\frac{12a}{9}$	$\frac{12b}{-3}$	$\frac{12c}{1}$	$\frac{12d}{\cdot}$	12e	$\frac{12f}{-6}$	$\frac{12g}{1}$	12h	12 <i>i</i>	$\frac{12j}{-2}$	$\frac{12k}{6}$	$\frac{12l}{2}$	$\frac{12m}{-3}$	12n	120	12p	12q	$\frac{12r}{}$
X	.88 .89	$-\ddot{3}$	$-\ddot{3}$	÷			÷	÷	9	3 7	1 3	6	6	$-\frac{6}{2}$	$-1 \\ -1$			$-\overline{2}$	6	$-\frac{5}{2}$	3 3				÷	÷
X	.90		:	:					3	-7	3	6	6	$-\frac{2}{2}$	1		$-\frac{2}{2}$			$-\frac{2}{2}$	-3				:	:
	.91 .92		:			$-\frac{3}{3}$	$-\frac{3}{3}$	$-1 \\ -1$		:	:		:	:	:		:	$-3 \\ -3$	$-3 \\ -3$	:	:			:	:	
	.93 .94								-3	$-\frac{3}{3}$	$-3 \\ -3$		3		$-3 \\ -3$	$-3 \\ -3$	$-\frac{3}{3}$				$-\frac{1}{1}$	$-\frac{1}{1}$			$-3 \\ -3$	$-\frac{1}{1}$
X	.95		:			$-\frac{1}{3}$	1	-1	-3	3	-3	3	-6	$-\frac{1}{3}$	3	-3				$-\frac{1}{3}$	3			-1	$\frac{-3}{2}$	
X	.96 .97			:		-1	$-\frac{1}{3}$	$-1 \\ -1$	$-3 \\ -3$	$-3 \\ -3$	$-3 \\ -3$	$-\tilde{6}$	$-\frac{6}{3}$	3	$-3 \\ -3$		3	:		3	$-3 \\ -3$	3		$-\frac{1}{2}$	$-1^{2}$	
	.98 .99	4	i	4	$-\dot{2}$	1	-3	-1	-3	3	-3	-6	3		3		-3				3	-3		2	-1	
X.1		$-2 \\ -7$	$-\frac{1}{4}$	$-\frac{2}{2}$	$-\frac{2}{2}$																					
X.1	102	_ '.	-4	$-\overline{6}$		i	-3	1		$-\frac{1}{8}$	:		:	4	:		$-\frac{\dot{2}}{2}$			:				2 2	2	:
X.1	104			$^{3}_{-6}$	-3	$-1^{3}$	$-\frac{1}{3}$	$\frac{1}{1}$		8		:	:	$-4 \\ -4$			$\frac{2}{2}$	:						2	2	
X.1		-5	_ ·	3	-3	-3	1	1		-8			٠	4			-2							2	2	•
X.:		$-\frac{5}{5}$	$-\frac{1}{2}$	1	1																					
X.1	109	-5 - 5	$-\frac{2}{2}$	1	1	:			:		:	:	:		:	:	:	:	:			:		:	:	:
X.	$11\overline{1}$			:		2	$-2 \\ -2$	$\frac{2}{2}$		-8		:	:	-2			4			-2					:	:
X.1						$-2 \\ -2$	2 2	2 2		8				2	:		-4			2						
X.				$-3 \\ -3$		-	-	-					•				•									
X.	116	3	3						18		$\dot{2}$		:	:			:	2	$-\dot{6}$	:					:	:
X.1	118		:	3	3	:	:	:	:	:	:	:	:	:	-8	:	:	:	:	:		:	:	:	:	
X.1						-3	-3	$-2 \\ 1$		8				$-\dot{4}$	:		·	3	3					$-\dot{2}$	$-\dot{2}$	
X.:	121			_3	-3	_3	3 2	$-\frac{1}{2}$		-8			•	4			$-\overline{2}$							$-\overline{2}$	$-\overline{2}$	
X.1	123			-3	-3	$\frac{-2}{2}$	$-\frac{2}{2}$	$-\frac{2}{2}$	:	:	:	:	:		:	:	:	:	:			:		:	:	:
X. I		5 5	$\frac{2}{2}$	$\frac{2}{2}$	$\frac{2}{2}$	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:

2 3	3 2	$\frac{3}{2}$	3 2	$\frac{2}{2}$	$\frac{1}{1}$	1	3	2	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{2}{4}$	3	3	3	3	1 3	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	3	3	i
5							i	í	1	1													1	1	i
13	:	:	:	:	i	1			:	:	:	:	:	:	:		:	:			:		:		
	12s	$\frac{12t}{2}$	12u	$\frac{12v}{c}$	13a	$\frac{13b}{10}$	$\frac{14a}{2}$	$\frac{14b}{2}$	15a	15b	18a	18b	18c	18d	18e	18f	18g	18h	18i	18j	18k	18l	$\frac{20a}{10}$	$\frac{20b}{10}$	21a
$\frac{2P}{3P}$		$\frac{624}{4q}$	$6_{23}_{4f}$		$\frac{13b}{13a}$		7a $14a$		$\frac{15a}{5a}$	15b	9b	9a	9d	9a 65	9b				$^{9f}_{6_{16}}$	$\frac{9i}{67}$	$\frac{9j}{6a}$	9c	$\frac{10c}{20a}$		$\frac{21a}{7a}$
	12s	12t	12u	12v	13b	13a	14a	14b	3c	3b	18a	18b	18c	18d	18e	187	18q	18h	18i	18i	18k	18l	$\frac{20a}{4a}$		
7P	12s	12t	12u	12v	13b	13a	$^{2b}$	2a	15a	15b	18a	18b	18c	18d	18e	18 f	18q	18h	18i	18i	18k	18l	20a	20b	3d
$\frac{13P}{X.87}$	12s	12t	$\frac{12u}{-1}$	$\frac{12v}{1}$	1a	1a	14a	146	15a	156	18a	186	$\frac{18c}{2}$	18d	18e	18 <i>f</i>	18g	18h	181	18 <i>j</i>	18 <i>k</i>	181	20a	206	21a
X.88	-1	:	1	1		- :		:	:	- :			2	:	:		-1	:	-1			:			
X.89	-1		-1								-3	-3		-1	-3	-1									
$X.90 \\ X.91$	1		1		i	i	i	i			3	3		-1	3	-1							i	_1	i
X.92			:		1	1	1	-1															-1	$-1 \\ -1$	i
X.93		2																							
$X.94 \\ X.95$		-2					-i	_ i	_ 2	i													_i	i	
X.96			:				$-1 \\ -1$	1	$-\frac{2}{2}$	1													1	1	
X.97							-1	-1	1	-2													1	1	
$X.98 \\ X.99$							-1	1	1	-2							- 4		i				-1	1	
X.100			:										$-\frac{2}{4}$				-1		-1						
X.101							-2			:				4	- 3	4							- 2	- 2	1
$X.102 \\ X.103$									$-\frac{1}{2}$	_1	3	3		-1	-1	-1		-1		í	i	-1	-1	-1	
X.103 X.104		:	:	:	:	:	:	:	$-1^{2}$	-1	$-\frac{1}{3}$	$-\frac{1}{3}$	:	$-1 \\ -1$	$^{-3}_{1}$	-1	:	i	:			i	1	$-1 \\ -1$	:
X.105									2	-1	-1	-1		-1	3	-1				-1	-1		-1	-1	
$X.106 \\ X.107$					A	$_{B}^{B}$					4	$-\frac{2}{2}$								1	1				
$X.107 \\ X.108$	:	:	:	:	$\vec{B}$	$\stackrel{D}{A}$	:	:	:	:	$-4 \\ -4$	2	:	:	:	:	:	:	:	-1	-1	- :		:	
X.109					B	A					4	-2								1	1				
$X.110 \\ X.111$			-2						-1	-1	3	-3		$-\frac{1}{2}$	3	2									
X.1112	:	:	2	:	:	:	:	:	-1	$-1 \\ -1$		3	:	$-1^{2}$		-1	:	:	:	:	:	:		:	
X.113									-1	-1	-3	-		2	-3	-1									
$X.114 \\ X.115$											$-\frac{2}{2}$	$-\frac{2}{2}$			$-\frac{2}{2}$			1		-1	2	$-\frac{2}{2}$			
X.116	:	:	:	-i	:	:	:	:	:	:	-2	-2	$-\dot{2}$	:	-	:	i	-1	i		-2	-		:	
X.117	2						1	1			-4	2		-2		-2				-1	-1				
$X.118 \\ X.119$	-2				2	2	1 2	-1			4	-2		-2		-2				1	1				4
$X.119 \\ X.120$		:	:	:				:	:	- :	3	3	:	-i	3	-i	:					:	i	- <sub>2</sub>	-1
X.121										÷	-3	-3		-1	-3	-1						j	-1	1	
$X.122 \\ X.123$									1	1	$-\frac{2}{2}$	$-\frac{2}{2}$		2	$-\frac{2}{2}$	2		$-\frac{1}{1}$		1	1	-1			•
$X.123 \\ X.124$	:	:	:	:	$\dot{C}$	$\dot{D}$		:	1	1			:		-2	۷.				-1	-1	1	:		
X.125		:			$\check{D}$	$\overline{C}$									÷							÷	:		



, where  $A=-\zeta(13)^{11}-\zeta(13)^8-\zeta(13)^7-\zeta(13)^6-\zeta(13)^5-\zeta(13)^2-1,\ B=-A-1,\ C=2B,\ D=2A.$ 

**B.3.** Character table of  $mE = \langle u, v, s \rangle$ 

2 3 5	10 10 1	10 9 1	10 7 1	10 6 1	10 5 1	$\begin{array}{c} 10 \\ 4 \\ 1 \end{array}$	10 4	10 3	9 10 1	6 10	5 10	5 8 1	7 8	$\frac{4}{8}$	6 8	4 8	4 8	3 8	3 8	3 7
$\frac{13}{2P}$	$\begin{array}{c} 1\\1\\1a\\1a\end{array}$	$\begin{array}{r} 1\\1\\2a\\1a\end{array}$	$\frac{1}{2b}$	$\frac{1}{2c}$	$\frac{2d}{1a}$	2e 1a	$\frac{2f}{1a}$	$\frac{2g}{1a}$	$ \begin{array}{r} 1\\ 3a\\ 3a \end{array} $	3b 3b	$\frac{3c}{3c}$	$\frac{3d}{3d}$	3e 3e	3 f 3 f	3g 3g	$\frac{3h}{3h}$	$\frac{3i}{3i}$	$\frac{3j}{3j}$	$\frac{3k}{3k}$	3 <i>l</i> 3 <i>l</i>
3P 5P 7P 13P	1a $1a$ $1a$ $1a$	2a $2a$ $2a$ $2a$	$2b \\ 2b \\ 2b \\ 2b \\ 2b$	$\begin{array}{c} 2c \\ 2c \\ 2c \\ 2c \end{array}$	2d $2d$ $2d$ $2d$	$\begin{array}{c} 2e \\ 2e \\ 2e \\ 2e \end{array}$	2f 2f 2f 2f	$\begin{array}{c} 2g \\ 2g \\ 2g \\ 2g \\ 2g \end{array}$	1a $3a$ $3a$ $3a$	1a $3b$ $3b$ $3b$	$ \begin{array}{c} 1a \\ 3c \\ 3c \\ 3c \end{array} $	1a $3d$ $3d$ $3d$	$     \begin{array}{c}       1a \\       3e \\       3e \\       3e     \end{array} $	$ \begin{array}{c} 1a \\ 3f \\ 3f \\ 3f \\ 3f \end{array} $	$ \begin{array}{c} 1a \\ 3g \\ 3g \\ 3g \\ 3g \end{array} $	1a $3h$ $3h$ $3h$	1a $3i$ $3i$ $3i$	1a $3j$ $3j$ $3j$	$\begin{array}{c} 1a \\ 3k \\ 3k \\ 3k \end{array}$	$\begin{array}{c} 1a \\ 3l \\ 3l \\ 3l \end{array}$
X.1 X.2 X.3 X.4	1 1 2 78	-1 -1 78	1 1 2 -34	-1 -34	1 1 2 14	1 -1 14	1 1 2 -2	$-\frac{1}{1}$ $-1$ $-\dot{2}$	1 1 -1 78	1 1 2 -3	1 1 -1 -3	1 1 2 15	1 1 2 6	1 1 -1 15	$\begin{array}{c} 1 \\ 1 \\ -1 \\ 6 \end{array}$	1 1 2 -3	1 1 2 -3	1 1 -1 -3	1 1 -1 -3	1 1 2 6
$X.5 \\ X.6 \\ X.7$	78 91 91	$^{-78}_{-91}$	$     \begin{array}{r}       -34 \\       -21 \\       -21     \end{array} $	$^{34}_{21}_{-21}$	$\frac{14}{11}$	$-14 \\ -11 \\ 11$	$     \begin{array}{r}       -2 \\       -5 \\       -5     \end{array} $	$\begin{array}{c} 2\\5\\-5\end{array}$	78 91 91	$^{-3}_{10}$	$^{-3}_{10}_{10}$	$^{15}_{1}$	6 19 19	$^{15}_{1}_{1}$	6 19 19	$^{-3}_{10}$	$^{-3}_{10}$	$^{-3}_{10}_{10}$	$^{-3}_{10}_{10}$	6 1 1
$X.8 \\ X.9 \\ X.10 \\ X.11$	105 105 156 168	$     \begin{array}{r}       105 \\       -105 \\       \hline       168     \end{array} $	$-35 \\ -35 \\ -68 \\ 56$	-35 35	5 5 28 24	$-5 \\ -5 \\ 24$	$\begin{array}{c} 1 \\ 1 \\ -4 \\ 8 \end{array}$	$-\frac{1}{8}$	$105 \\ 105 \\ -78 \\ 168$	$     \begin{array}{r}       24 \\       24 \\       -6 \\       6     \end{array} $	24 24 3 6	15 15 30 15	6 12 24	15 15 -15 15	$\begin{array}{c} 6 \\ 6 \\ -6 \\ 24 \end{array}$	$     \begin{array}{r}       -3 \\       -3 \\       -6 \\       6     \end{array} $	$     \begin{array}{r}       -3 \\       -3 \\       -6 \\       6     \end{array} $	-3 -3 3 6	$-3 \\ -3 \\ 3 \\ 6$	$     \begin{array}{r}       -3 \\       -3 \\       12 \\       \hline       6     \end{array} $
$X.12 \\ X.13 \\ X.14 \\ X.15$	168 182 182 182	$-168 \\ 182 \\ -182$	$   \begin{array}{r}     56 \\     70 \\     70 \\     -42   \end{array} $	-56 $70$ $-70$	24 22 22 22	-24 $-22$ $-22$	8 6 6 -10	$^{-8}_{-6}$	$     \begin{array}{r}       168 \\       182 \\       182 \\       -91     \end{array} $	6 20 20 20	$\begin{array}{c} 6 \\ 20 \\ 20 \\ -10 \end{array}$	15 29 29 2	24 11 11 38	$   \begin{array}{c}     15 \\     29 \\     29 \\     -1   \end{array} $	$     \begin{array}{c}       24 \\       11 \\       11 \\       -19     \end{array} $	$\begin{array}{r} 6 \\ -7 \\ -7 \\ 20 \end{array}$	$\begin{array}{c} 6 \\ -7 \\ -7 \\ 20 \end{array}$	$ \begin{array}{r}     6 \\     -7 \\     -7 \\     -10 \end{array} $	$     \begin{array}{r}       6 \\       -7 \\       -7 \\       -10     \end{array} $	6 2 2 2
X.16 X.17 X.18 X.19	195 195 210 260	-195 195 260	55 55 -70 20	-55 55	15 15 10 20	-15 15 20		-11 11 4	$     \begin{array}{r}       195 \\       195 \\       -105 \\       260     \end{array} $	33 33 48 17	$\begin{array}{c} 33 \\ 33 \\ -24 \\ 17 \end{array}$	$15 \\ 15 \\ 30 \\ -10$	$\frac{24}{24}$ $12$	$^{15}_{15}$ $^{-15}$	$ \begin{array}{r} 24 \\ 24 \\ -6 \\ -10 \end{array} $	6 6 -6 -10	6 6 -6 17	6 6 3 -10	6 6 3 17	$-\frac{1}{3}$ $-\frac{1}{6}$ $-\frac{1}{8}$
$X.20 \\ X.21 \\ X.22$	$\frac{260}{260}$	$^{-260}_{260}$ $^{-260}$	20 20 20	$-20 \\ 20 \\ -20$	$\frac{20}{20}$	$-20 \\ 20 \\ -20$	$\frac{4}{4}$	$^{-4}_{-4}$	$\frac{260}{260}$	$17 \\ 17 \\ 17$	$\frac{17}{17}$	$-10 \\ -10 \\ -10$	-10 - 10 - 10 - 10	-10 - 10 - 10	$-10 \\ -10 \\ -10$	$^{17}_{17}_{-10}$	$^{-10}_{-10}$	$^{17}_{17}$	$-10 \\ -10 \\ 17$	8
$X.23 \\ X.24 \\ X.25 \\ X.26$	273 273 336 364	273 -273 :	-91 $-91$ $112$ $140$	-91 91	29 29 48 44	-29 -29 :	-7 $-7$ $16$ $12$	$-7 \\ 7 \\ \vdots \\$	$   \begin{array}{r}     273 \\     273 \\     -168 \\     -182   \end{array} $		$\begin{array}{r} 30 \\ 30 \\ -6 \\ -20 \end{array}$	30 30 30 58			$     \begin{array}{r}       30 \\       30 \\       -24 \\       -11    \end{array} $	$\begin{array}{c} 3 \\ 3 \\ 12 \\ -14 \end{array}$	$\begin{array}{c} 3 \\ 3 \\ 12 \\ -14 \end{array}$	$\begin{array}{c} 3 \\ 3 \\ -6 \\ 7 \end{array}$	$\begin{array}{c} 3 \\ 3 \\ -6 \\ 7 \end{array}$	8 3 12 4
$X.27 \\ X.28 \\ X.29 \\ X.30$	390 520 520 546		$     \begin{array}{r}       110 \\       40 \\       40 \\       -182     \end{array} $	:	30 40 40 58		$\begin{array}{c} 22 \\ 8 \\ 8 \\ -14 \end{array}$		-195 $-260$ $-260$ $-273$	34 34 60	$-33 \\ -17 \\ -17 \\ -30$	$     \begin{array}{r}       30 \\       -20 \\       -20 \\       \hline       60     \end{array} $	$ \begin{array}{r} 48 \\ -20 \\ -20 \\ 60 \end{array} $	-15 $10$ $10$ $-30$	$-24 \\ 10 \\ 10 \\ -30$	$-{12 \atop -20 \atop 34}$	$^{12}_{34}_{-20}$	$     \begin{array}{r}       -6 \\       10 \\       -17 \\       -3     \end{array} $	$     \begin{array}{r}     -6 \\     -17 \\     10 \\     -3     \end{array} $	$     \begin{array}{r}     -6 \\     16 \\     16 \\     6     \end{array} $
$X.31 \\ X.32 \\ X.33 \\ X.34$	546 546 819 819	546 $-546$ $-819$ $819$	154 $154$ $-21$ $-21$	$     \begin{array}{r}       154 \\       -154 \\       \hline       21 \\       -21     \end{array} $	$ \begin{array}{r} 26 \\ 26 \\ -21 \\ -21 \end{array} $	$ \begin{array}{r} 26 \\ -26 \\ 21 \\ -21 \end{array} $	2 19 19	$\begin{array}{c} 2 \\ -2 \\ -19 \\ 19 \end{array}$	546 546 819 819	$     \begin{array}{r}       -21 \\       -21 \\       90 \\       90   \end{array} $	$     \begin{array}{r}       -21 \\       -21 \\       90 \\       90     \end{array} $		$     \begin{array}{r}       -12 \\       -12 \\       9 \\       9   \end{array} $		$     \begin{array}{r}       -12 \\       -12 \\       9 \\       9   \end{array} $	6 6 9	6 6 9	6 6 9	6 6 9	$     \begin{array}{r}       -3 \\       -3 \\       9 \\       9   \end{array} $
X.35 X.36 X.37 X.38	910 910 910 910	910 -910 910 -910	$ \begin{array}{r} -210 \\ -210 \\ -210 \\ -210 \\ -210 \end{array} $	$     \begin{array}{r}       -2\overline{10} \\       210 \\       -210 \\       210 \\    \end{array} $	30 30 30 30	$ \begin{array}{r}     30 \\     -30 \\     30 \\     -30 \end{array} $	$ \begin{array}{r} -2 \\ -2 \\ -2 \\ -2 \\ -2 \end{array} $	$-\frac{2}{2}$ $-\frac{2}{2}$	910 910 910 910	19 19 19 19	19 19 19 19	55 55 55	$-17 \\ -17 \\ -17 \\ -17 \\ -17$	55 55 55	-17 $-17$ $-17$ $-17$	19 -8 -8 19	-8 19 19 -8	19 -8 -8 19	-8 19 19 -8	1 1 1 1
$X.39 \\ X.40 \\ X.41$	1092 1092 1092 -	1092	$     \begin{array}{r}       308 \\       -140 \\       -140 \\       -35     \end{array} $	$-140 \\ 140$	52 52 52 45	52 -52 45	4 4 4 5	$\begin{array}{c} 1 \\ 4 \\ -4 \\ 5 \end{array}$	-546 $1092$ $1092$	$-42 \\ -42 \\ -42$	$ \begin{array}{r}     21 \\     -42 \\     -42 \\     -12 \end{array} $	$     \begin{array}{r}       102 \\       -6 \\       -6 \\       -30     \end{array} $	-24 30 30 87	$ \begin{array}{r} -51 \\ -6 \\ -6 \\ -30 \end{array} $	12 30 30 87	$^{12}_{-15}$	$ \begin{array}{c} 12 \\ -15 \\ -15 \\ 15 \end{array} $	$-6 \\ -15 \\ -15$	$-6 \\ -15 \\ -15$	$ \begin{array}{c} -6 \\ 12 \\ 12 \\ -3 \end{array} $
$X.43 \\ X.44 \\ X.45$	1365 - 1365	-1365 -1365 1365	$     \begin{array}{r}       245 \\       -35 \\       245     \end{array} $	$     \begin{array}{r}       -35 \\       -245 \\       \hline       35 \\       245 \\    \end{array} $	5 45 5	$-5 \\ -45 \\ 5$	$-27 \\ -27 \\ -27$	$\begin{array}{r} 27 \\ -5 \\ -27 \end{array}$	1365 1365 1365 1365	$^{69}_{-12}$	$-{}^{69}_{-69}$	$-{}^{60}_{60}$	$     \begin{array}{r}       -3 \\       87 \\       -3     \end{array} $	$^{60}_{-30}$	$     \begin{array}{r}       -3 \\       87 \\       -3     \end{array} $	15 15 15 15	15 15 15	15 15 15 15	15 15 15 15	$^{6}_{-3}$
$X.46 \\ X.47 \\ X.48 \\ X.49$	1560 - 1560 - 1560 -	$     \begin{array}{r}       1560 \\       -1560 \\       1560 \\       -1560     \end{array} $	120 $120$ $120$ $120$	$     \begin{array}{r}       120 \\       -120 \\       120 \\       -120     \end{array} $	$-40 \\ -40$	$     \begin{array}{r}     -40 \\     40 \\     -40 \\     40     \end{array} $	$     \begin{array}{r}       -8 \\       -8 \\       -8 \\       -8     \end{array} $	$-8 \\ -8 \\ -8$	$\begin{array}{c} 1560 \\ 1560 \\ 1560 \\ 1560 \end{array}$	-60 -60	$     \begin{array}{r}     -60 \\     -60 \\     -60 \\     -60     \end{array} $	30 30 30 30	$     \begin{array}{c}       12 \\       12 \\       12 \\       12     \end{array} $	30 30 30 30	$     \begin{array}{c}       12 \\       12 \\       12 \\       12     \end{array} $	$     \begin{array}{r}     -6 \\     -6 \\     -6     \end{array} $	$     \begin{array}{r}     -6 \\     -6 \\     -6 \\     \end{array} $	$     \begin{array}{r}     -6 \\     -6 \\     -6     \end{array} $	$     \begin{array}{r}     -6 \\     -6 \\     -6     \end{array} $	3 3 3
$X.50 \ X.51 \ X.52 \ X.53$	1638	1638 -1638	-42 $294$ $294$ $-420$	294 -294	-42 54 54 60		$     \begin{array}{r}       38 \\       -10 \\       -10 \\       -4     \end{array} $	$-10_{10}$	-819 $1638$ $1638$ $-910$	-63 - 63	$-90 \\ -63 \\ -63 \\ -19$	$   \begin{array}{r}     18 \\     45 \\     45 \\     110   \end{array} $	$     \begin{array}{r}       18 \\       18 \\       18 \\       -34     \end{array} $	-9 45 45 -55	$-9 \\ 18 \\ 18 \\ 17$	$     \begin{array}{r}       18 \\       -9 \\       -9 \\       38     \end{array} $	$ \begin{array}{r} 18 \\ -9 \\ -9 \\ -16 \end{array} $	$     \begin{array}{r}       -9 \\       -9 \\       -9 \\       -19     \end{array} $	-9 -9 -9 8	18 9 9 2
$X.54 \\ X.55 \\ X.56 \\ X.57$	1820 -	1820 -1820 2106		$140 \\ -140 \\ -414$	$     \begin{array}{r}       60 \\       -20 \\       -20 \\       66     \end{array} $	-20 20 66	$     \begin{array}{r}     -4 \\     -4 \\     -4 \\     -6     \end{array} $	$-\frac{1}{4}$ $-6$	-910 $1820$ $1820$ $2106$	-43	$     \begin{array}{r}       -19 \\       -43 \\       -43 \\       -81     \end{array} $	110 20 20 81	-34 56 56	-55 20 20 81	17 56 56	-16 11 11	38 11 11	8 11 11	-19 11 11	$     \begin{array}{r}       2 \\       -7 \\       -7     \end{array} $
$X.58 \\ X.59 \\ X.60$	2106 - 2184 - 2184	-2106 $-2184$ $2184$	$-414 \\ 56 \\ 56 \\ -280$	$^{414}_{-56}$ $^{56}$	66 24 24 104	$     \begin{array}{r}       -66 \\       -24 \\       24   \end{array} $	$ \begin{array}{r} -6 \\ -24 \\ -24 \\ 8 \end{array} $	$\begin{array}{c} & & 6 \\ 24 \\ -24 \end{array}$	2106 2184 2184 -1092	-81 78 78 -84	-81 78 78	$ \begin{array}{r} 81 \\ -21 \\ -21 \\ -12 \end{array} $	96 96 60	$ \begin{array}{r} 81 \\ -21 \\ -21 \\ 6 \end{array} $	96 96 –30	24 24 -30	24 24 -30	24 24 15	24 24 15	6 6 24
X.61 X.62 X.63 X.64 X.65	2457 - 2730 -	$2457 \\ -2457 \\ -2730$	189 189	189 $-189$ $-490$	21 21 90 10	$-21 \\ -21 \\ -90$	$\begin{array}{c} 33 \\ 33 \\ 26 \\ -54 \end{array}$	$     \begin{array}{r}       33 \\       -33 \\       -26     \end{array} $	2457 $2457$ $2730$ $-1365$	$\begin{array}{c} 27 \\ 27 \\ -24 \end{array}$	27 27 -24 -69	:	54 54 -15 -6	75 -60	54 54 -15 3	27 27 3 30	27 27 3 30	$\begin{array}{c} 27 \\ 27 \\ 27 \\ 3 \\ -15 \end{array}$	$ \begin{array}{r}     27 \\     27 \\     3 \\     -15 \end{array} $	12 12
X.66 X.67	2730	2730 2730	$   \begin{array}{r}     490 \\     -70 \\     -70   \end{array} $	490 -70	90 10 90	90	$\frac{26}{-6}$		$2730 \\ 2730 \\ -1365$	$     \begin{array}{r}       -24 \\       138 \\       -24     \end{array} $	$^{-24}_{138}$	$     \begin{array}{r}       75 \\       -15 \\       -60     \end{array} $	$^{-15}_{21}$ $^{174}$	$\begin{array}{r} 75 \\ -15 \\ 30 \end{array}$	$^{-15}_{21}$ $^{-87}$	$-24 \\ 30$	$-{24\atop 30}$	$^{3}_{-24}$ $^{-15}$	$     \begin{array}{r}       3 \\       -24 \\       -15     \end{array} $	$^{12}_{-6}$
X.70 X.71 X.72	2835 - 2835 - 2835 3120	-2730 -2730 -2835 2835 	-70 315 315 240		$75 \\ 75 \\ -80$	$-10 \\ -75 \\ 75 \\ .$	$-6 \\ 3 \\ 3 \\ -16$	$-\frac{6}{3}$	2730 $2835$ $2835$ $-1560$ $-1560$	$ \begin{array}{r} 138 \\ -81 \\ -81 \\ -120 \end{array} $	$-81 \\ -81 \\ 60$	60	81 81 24	-15 -30	$   \begin{array}{r}     81 \\     81 \\     -12   \end{array} $	-12	-24 - -12	6	6	-6
21.70	4030	4030	240 588 280 315	315		-45	-16 $-20$ $-8$ $-25$	-25	$-1638 \\ -1820 \\ 4095$	$-126 \\ -86 \\ 207$	60 63 43 207	45	36 112 18	$^{-20}_{45}$	$^{-18}_{-56}$	$-18 \\ 22 \\ -9$	$     \begin{array}{r}       -12 \\       -18 \\       22 \\       -9     \end{array} $	-9	$     \begin{array}{r}       6 \\       9 \\       -11 \\       -9     \end{array} $	6 18 -14
$X.79 \\ X.80$	4095 - 4095 - 4212	-4095 -4095	$     \begin{array}{r}       315 \\       -525 \\       -828     \end{array} $	525	$75 \\ -45 \\ 75 \\ 132$	-75	$     \begin{array}{r}       7 \\       -25 \\       7 \\       -12     \end{array} $	$\begin{array}{c} 7 \\ 25 \\ -7 \\ \end{array}$		-36 $207$ $-36$ $-162$	$-36 \\ 207 \\ -36 \\ 81$	45 $45$ $45$ $162$	72 18 72	$\begin{array}{r} 45 \\ 45 \\ 45 \\ -81 \end{array}$	72 18 72	18 -9 18	18 -9 18	18 -9 18	18 -9 18	9 9
X.81 X.82 X.83 X.84	4368 - 4368 4368	-4368 -4368 -4536	560 $112$ $560$ $-504$	-560 560 504	48 48 48	-48		-16 $16$ $-24$	$^{4368}_{-2184}$ $^{4368}_{4536}$	156 $156$ $156$ $162$	$^{156}_{-78}$ $^{156}$	-42	$     \begin{array}{r}       -24 \\       192 \\       -24 \\       \end{array} $	21	-24 -96 -24	$^{-6}_{48}$ $^{-6}$	$^{-6}_{48}$	$^{-6}_{-24}$	$     \begin{array}{r}     -6 \\     -24 \\     -6 \\     \end{array} $	$     \begin{array}{r}     -6 \\     12 \\     -6   \end{array} $
X.85 X.86 X.87	$\frac{4536}{4914}$	4536 5265		-504		-24	24 66 33	$^{24}$	$ \begin{array}{r} 4536 \\ -2457 \\ 5265 \end{array} $	162	$\frac{162}{-27}$	81	108 81	81	-54 81	54	54	-27	-27	

Ch	arac	eter ta	ble	of r	nE	(ca)	nt	ini	uea	!)												
2 3 5 7	2 7	$\begin{array}{ccc}2&1\\7&7\\\vdots&\vdots\end{array}$	7 3 1	8 3	7 2 1	8 2	8 2	7 2	8	7 1	$\begin{array}{c} 4 \\ 2 \\ 1 \\ \cdot \end{array}$	9 7 1 1	6 9 :	9 5 1	5 7 1	7 7	6 7	5 7	6 6	9 4 :	$\begin{array}{c} 4\\7\\ \vdots\\ \end{array}$	4 7 :
$\frac{13}{2P}$	$\frac{3m}{3m}$	$\begin{array}{ccc} 3n & 3o \\ 3n & 3o \end{array}$	$\frac{4a}{2d}$	$\frac{4b}{2f}$	$\frac{4c}{2d}$	$\frac{4d}{2f}$	$\frac{4e}{2f}$	$\frac{4f}{2d}$	$\frac{4g}{2f}$	$\frac{4h}{2d}$	$\frac{5a}{5a}$	$\frac{6_1}{3a}$	$\frac{6_{2}}{3b}$	$\frac{6_3}{3a}$	$\frac{6_4}{3d}$	$\frac{6_5}{3e}$	$\frac{6_{6}}{3b}$	$\frac{67}{3c}$	$\frac{6_{8}}{3b}$	$\frac{69}{3a}$	$\frac{6_{10}}{3h}$	$\frac{6}{3i}$
3P 5P 7P 13P	$\begin{array}{c} 1a \\ 3m \\ 3m \\ 3m \end{array}$	1a 1a 3n 3o 3n 3o 3n 3o	4a $4a$ $4a$ $4a$	$\begin{array}{c} 4b \\ 4b \\ 4b \\ 4b \end{array}$	4c $4c$ $4c$ $4c$	$\begin{array}{c} 4d \\ 4d \\ 4d \\ 4d \end{array}$	$\frac{4e}{4e}$		$\frac{4g}{4g}$	$\begin{array}{c} 4h \\ 4h \\ 4h \\ 4h \end{array}$	5a 1a 5a 5a	$\begin{array}{c} 2b \\ 6_1 \\ 6_1 \\ 6_1 \end{array}$	$\begin{array}{c} 2a \\ 6_2 \\ 6_2 \\ 6_2 \end{array}$	$\begin{array}{c} 2d \\ 6_{3} \\ 6_{3} \end{array}$	2a 6 <sub>4</sub> 6 <sub>4</sub>	2a 65 65	2b 66 66	$\begin{array}{c} 2b \\ 67 \\ 67 \\ 67 \end{array}$	$\frac{2c}{68}$	2f 69 69	$\begin{array}{c} 2a \\ 6_{10} \\ 6_{10} \\ \end{array}$	$ \begin{array}{c} 2a \\ 6_{11} \\ 6_{11} \\ 6_{11} \end{array} $
X.1 $X.2$ $X.3$	1 1	$\begin{array}{ccc} 1 & 1 \\ 1 & 1 \end{array}$	1 1	1 1	1 -1	1 -1	1	1 1	1 -1	1	1	6 <sub>1</sub>	$\frac{62}{1}$	6 <sub>3</sub>	$\frac{6_4}{1}$	$\frac{6_5}{1}$	6 <sub>6</sub>	1	$\frac{68}{1}$	69 1 1	$6_{10}$ $1$ $-1$	$\frac{6_{11}}{1}$
$X.4 \\ X.5$	$\begin{array}{c} 2 \\ -3 \\ -3 \end{array}$	$     \begin{array}{r}       -1 & -1 \\       6 & -3 \\       6 & -3     \end{array} $	$\begin{array}{c} 2 \\ -6 \\ -6 \end{array}$	2 2 2	-6 6	$-\frac{1}{2}$	2 2 2	2 2 2	$-\frac{1}{2}$	$-\frac{1}{2}$	3 3	$-1 \\ -34 \\ -34$	-3 3	$-1 \\ 14 \\ 14$	$^{15}_{-15}$	6 -6	$-\frac{2}{7}$	$     \begin{array}{r}       -1 \\       -7 \\       -7     \end{array} $	$-\frac{1}{7}$	$-1 \\ -2 \\ -2$	-3 3	$-\frac{3}{3}$
X.6 X.7 X.8	1 1 6	$ \begin{array}{cccc} 1 & 1 \\ 1 & 1 \\ -3 & 6 \\ -3 & 6 \end{array} $	$     \begin{array}{r}       -1 \\       -1 \\       -5 \\       -5     \end{array} $	3 5 5	$-\frac{1}{-5}$	-3 5	3	$-1 \\ -1 \\ -1$	$-3 \\ 3 \\ 1 \\ -1$	$-1 \\ -1 \\ -1$	1	$     \begin{array}{r}       -21 \\       -21 \\       -35 \\     \end{array} $	-10 $10$ $24$ $-24$	11 11 5 5	$-1 \\ 1 \\ 15$	-19 19 6	6 -8	6 -8	$-6 \\ -8 \\ -8$	$-5 \\ -5 \\ 1 \\ 1$	-10 - 10 - 3 - 3	$-10 \\ 10 \\ -3$
X.9 X.10 X.11	$\begin{array}{c} 6 \\ -6 \\ -3 \end{array}$	$^{-6}_{6}$ $^{3}_{-3}$	-12 4	5 4	5 4	-5 :	4	-1 $4$ $4$	-1	1 4	6 3	-35 34 56	6	$^{-14}_{24}$	-15 15	-6 24	$^{-8}_{-14}$	$-8 \\ 7 \\ 2 \\ 2$	8 -2	2 8	6	3 6
$X.12 \\ X.13 \\ X.14 \\ X.15$	$-3 \\ 2 \\ 2 \\ 2$	$     \begin{array}{ccc}       6 & -3 \\       2 & 2 \\       2 & 2 \\       -1 & -1     \end{array} $	$\begin{array}{c} 4 \\ 10 \\ 10 \\ -2 \end{array}$	2 2	$^{-4}_{-10}$	$-\overset{\cdot}{\overset{\cdot}{2}}$	2 2 6	$\begin{array}{c} 4 \\ 2 \\ 2 \\ -2 \end{array}$	$-\frac{1}{2}$	$-4 \\ -2 \\ -2$	3 2 2 2	56 70 70 21	$^{-6}_{20}_{-20}$	$\begin{array}{c} 24 \\ 22 \\ 22 \\ -11 \end{array}$	$-15 \\ 29 \\ -29$	$-24 \\ -11 \\ -11$	$\begin{array}{c} 16 \\ 16 \\ 12 \end{array}$	16 16 -6	$^{-2}_{-16}$	8 6 6 5	$^{-6}_{-7}$	$^{-6}_{-7}$
X.16 X.17 X.18	6 6 12	$ \begin{array}{ccc} -3 & 6 \\ -3 & 6 \end{array} $	5 5 -10	6 3 3 10	$-\frac{.}{5}$	$-\frac{.}{3}$	$-1 \\ -1 \\ 2$	$-\frac{2}{1}$ $1$ $-2$	_1 _1	$-\overset{\cdot}{\underset{1}{1}}$		55 55 35	$-33\\33$		$-15 \\ 15$	$-24^{\circ}_{24}$	1 1 -16	1 1 8	$-\overset{\cdot}{\underset{1}{1}}$	11 11 -1	$-\frac{.}{6}$	$-\frac{\dot{6}}{6}$
X.19 X.20 X.21	-1 $-1$ $-1$	$     \begin{array}{r}       3 - 0 \\       8 - 1 \\       8 - 1 \\       8 - 1   \end{array} $	-10	4 4 4	:	$-\frac{4}{4}$	4 4 4	- <u>-</u> -	-4 $4$		:	20 20 20 20	$-{}^{17}_{17}$	20 20 20 20	$-10 \\ 10 \\ -10$	$-10 \\ 10 \\ -10$	$-7 \\ -7$	$-7 \\ -7 \\ -7 \\ -7$	$-\frac{7}{7} \\ -7$	4	$-10 \\ -17 \\ 17$	$\begin{array}{c} 17 \\ 10 \\ -10 \end{array}$
X.22 X.23 X.24	$-\frac{1}{3}$	$     \begin{array}{ccccccccccccccccccccccccccccccccc$	-9 -9	4 1 1	-9 9	$-\frac{1}{1}$	$\begin{array}{r} 4 \\ -3 \\ -3 \end{array}$	3	$-\frac{4}{3}$	3 -3	3	$   \begin{array}{c}     20 \\     -91 \\     -91   \end{array} $	-17 $30$ $-30$	20 29 29	10 30 -30	10 30 -30	-7 $-7$ $-10$ $-10$	$-7 \\ -10 \\ -10$	$-10 \\ -10 \\ 10$	$\begin{array}{c} 4 \\ -7 \\ -7 \end{array}$	10 · 3 · -3	$-17 \\ -3 \\ -3$
X.25 X.26 X.27	$-\frac{6}{4}$	$ \begin{array}{rrr} -6 & 3 \\ -2 & -2 \\ 3 & -6 \end{array} $	8 20 10	4			4 -2	8 4 2			6	$-56 \\ -70 \\ -55$		$     \begin{array}{r}       -24 \\       -22 \\       -15     \end{array} $			32 2	$-\frac{10}{2}$ $-16$ $-1$		$     \begin{array}{r}       -8 \\       -6 \\       -11     \end{array} $		
$X.28 \\ X.29 \\ X.30$	$-2 \\ -2 \\ 6$	$ \begin{array}{cccc} -8 & 1 \\ -8 & 1 \\ -3 & -3 \end{array} $	-18	8 8 2			$^{8}_{-6}$	6			6	$-20 \\ -20 \\ 91$		$-20 \\ -20 \\ -29$			$-14 \\ -14 \\ -20$	7 7 10		$^{-4}_{-4}$		:
X.31 X.32 X.33	-3 -3 9	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	14 14 -1	6 6 7	$^{14}_{-14}$	$-6 \\ -7$	$-\frac{5}{2}$	$-\frac{5}{2}$ $-2$ $-1$	$-\frac{1}{2}$	$-\frac{1}{2}$	1 1 -1	$154 \\ 154 \\ -21$	$-21 \\ 21 \\ -90$	$ \begin{array}{r}     \hline     26 \\     26 \\     -21 \end{array} $	$-51 \\ -51 \\ -9$	$-12 \\ 12 \\ -9$	19 19 6	19	19 -19 -6	2 2 19	6 -6 -9	6 -6 -9
$X.34 \\ X.35 \\ X.36$	9 1 1		$-10 \\ -10 \\ -10$	$\begin{array}{c} 7 \\ 2 \\ 2 \end{array}$	$-10 \\ -10 \\ 10$	$\begin{array}{c} 7 \\ 2 \\ -2 \end{array}$	$\frac{2}{2}$	$-1 \\ -2 \\ -2$	$-1 \\ -2 \\ -2$	$-\frac{2}{2}$	-1 :	$-21 \\ -210 \\ -210$	$   \begin{array}{r}     90 \\     19 \\     -19   \end{array} $	$-21 \\ 30 \\ 30$	$\begin{array}{c} 9 \\ 55 \\ -55 \end{array}$	$-{}^{9}_{17}$	$^{6}_{-21}$ $^{-21}$	$^{6}_{-21}$ $^{-21}$	$-{21\atop 21}$	$     \begin{array}{r}       19 \\       -2 \\       -2     \end{array} $	9 19 8	$_{-8}^{9}$ $_{-19}$
$X.37 \\ X.38 \\ X.39$	$\begin{array}{c} 1 \\ 1 \\ -6 \end{array}$	$\begin{array}{cc} 1 & 1 \\ 3 & 3 \end{array}$	$-10 \\ -10 \\ 28$	$\frac{2}{2}$ 12	$^{-10}_{10}$	$-\frac{2}{2}$	$\frac{2}{2} -4$	$-2 \\ -2 \\ -4$	$-\frac{2}{\cdot}$	$-\frac{2}{2}$		$-210 \\ -210 \\ -154$	$^{19}_{-19}$	$\begin{array}{c} 30 \\ 30 \\ -26 \end{array}$	$-55 \\ -55 \\ .$	$^{-17}_{17}$	$-21 \\ -21 \\ 38$	$-21 \\ -21 \\ -19$	$^{-21}_{21}$	$-2 \\ -2 \\ -2$	$^{-8}_{-19}$	19 8
$X.40 \\ X.41 \\ X.42$	3 -3	$\begin{array}{ccc} 12 & 3 \\ 12 & 3 \\ -3 & -3 \end{array}$	5	4 4 5	5	$-{4\atop 5}$		-3	$-4 \\ -3 \\ -3$	-3		$-140 \\ -140 \\ -35$	$     \begin{array}{r}     -42 \\     42 \\     -12     \end{array} $		$-6 \\ -30$	$-{30}\atop {87}$	$\frac{22}{22} - 8$	$\frac{22}{22} - 8$	$^{22}_{-22}$	4 4 5	$-15 \\ 15 \\ 15$	$-15 \\ 15 \\ 15$
$X.43 \\ X.44 \\ X.45$	$-\frac{6}{6}$	$\begin{array}{ccc}  & 6 & 6 \\  -3 & -3 & 6 \\  & 6 & 6 \end{array}$	5 5 5	1 5 1	$-5 \\ -5 \\ 5$	$-1 \\ -5 \\ 1$		$-3 \\ -3 \\ -3$	$-1 \\ 3 \\ 1$	$\begin{array}{c} 3 \\ 3 \\ -3 \end{array}$	:	$     \begin{array}{r}       245 \\       -35 \\       245     \end{array} $	$     \begin{array}{r}       -69 \\       12 \\       69     \end{array} $	5 45 5	$-60 \\ 30 \\ 60$	$-87 \\ -3$	$     \begin{array}{r}       29 \\       -8 \\       29     \end{array} $	$     \begin{array}{r}       29 \\       -8 \\       29     \end{array} $	$-29 \\ 8 \\ 29$	$-27^{5}$	$^{-15}_{15}$	$-15 \\ -15 \\ 15$
X.46 X.47 X.48	3 3	3 3 3 3	:	•	:	:	:				:	120 120 120	$^{60}_{-60}$	$-40 \\ -40 \\ -40$	$-\frac{30}{30}$	$-12 \\ -12 \\ 12$	12 12 12	12	$-12 \\ -12 \\ 12$	$-8 \\ -8 \\ -8$	$-6 \\ -6 \\ -6$	$-6 \\ -6 \\ -6$
X.49 X.50 X.51	3 18	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$-\frac{1}{6}$	$1\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}$	6	2		$-\frac{1}{2}$	2	-2	$-\frac{1}{2}$	120 21 294	-63	-40 21 54	-30 45	-12 18	$\frac{12}{12}$	$-6 \\ -3$	-3	$     \begin{array}{r}       -8 \\       -19 \\       -10 \\     \end{array} $	6 -9	6 -9
X.52 X.53 X.54	$\begin{array}{c} \dot{2} \\ 2 \\ -7 \end{array}$	-1 - 1	$     \begin{array}{r}       6 \\       -20 \\       -20     \end{array} $	$\frac{4}{4}$	-6 :	-2	$\frac{4}{4}$	$-2 \\ -4 \\ -4$	-2	2	3	294 210 210	63	$     \begin{array}{r}       54 \\       -30 \\       -30 \\       \hline     \end{array} $	-45	-18	$-3 \\ -42 \\ -42$	$-3 \\ 21 \\ 21 \\ 2 \\ 1$	:	$-10$ $\frac{2}{2}$	9	9
$X.55 \ X.56 \ X.57 \ X.58$	-7 -7	$     \begin{array}{r}       -7 - 7 \\       -7 - 7 \\       \hline       \cdot & \cdot \\     \end{array} $	$-14 \\ -14$	4 4 6 6	$-14 \\ 14$	$     \begin{array}{r}       4 \\       -4 \\       6 \\       -6     \end{array} $	$\begin{array}{r} 4 \\ 4 \\ -2 \\ -2 \end{array}$	2 2	$-4 \\ -2 \\ 2$	$-\frac{1}{2}$	1 1	140 $140$ $-414$ $-414$	$     \begin{array}{r}     -43 \\     43 \\     -81 \\     81     \end{array} $	$     \begin{array}{r}     -20 \\     -20 \\     66 \\     66     \end{array} $	$^{20}_{-20}_{81}_{-81}$	$^{56}_{-56}$	5 -9 -9	5 -9 -9	$     \begin{array}{r}       5 \\       -5 \\       -9 \\       9   \end{array} $	$     \begin{array}{r}     -4 \\     -4 \\     -6 \\     -6     \end{array} $	$-^{11}_{-11}$	$-11 \\ -11 \\ \cdot$
X.59 X.60 X.61	6 6	$\begin{array}{ccc}  & 6 & 6 \\  & 6 & 6 \\  & -12 & -3 \end{array}$	$-4 \\ -4$	8	$\begin{array}{c} 14 \\ 4 \\ -4 \end{array}$	-0	-2 8	$-\frac{2}{4}$		$-\frac{2}{4}$ $-4$	$-\frac{1}{1}$	56 56 140	$-78 \\ 78$	$24 \\ 24 \\ -52$	$-31 \\ -21 \\ -21$	-96 96	2 2 44	$-\frac{9}{2}$ $-22$	-2	-24 $-24$ $-4$	$-24 \\ 24$	$-24^{\circ}_{24}$
X.62 X.63 X.64	3	12 3	$     \begin{array}{r}       -9 \\       -9 \\       10     \end{array} $	$-7 \\ -7 \\ -2$	$-\frac{1}{9}$	$-\frac{1}{7}$	5 -2	3 3 2	-52	$\begin{array}{c} 3 \\ -3 \\ -2 \end{array}$	$-3 \\ -3$	189 189 490	$-27 \\ -27 \\ 24$	21 21 90	-75	$_{-54}^{54}$ $_{15}^{-34}$	27 27 4	27 27 4	$\begin{array}{r} 27 \\ -27 \\ -4 \end{array}$	33 33 26	$^{27}_{-27}$	$^{27}_{-27}$
X.65 X.66 X.67	12	$\begin{array}{cc} -6 & -6 \\ 12 & 3 \end{array}$	$10 \\ 10 \\ -10$	$-\frac{2}{2}$	10 -10	$-\dot{2}$	$-\frac{5}{6}$	$^{-6}_{2}$	$-\dot{2}$			-245	$-24 \\ 138$	$-5 \\ 90$	75		58 4 38	-29 $-4$ $-38$	4 38	27 26 -6	-24	$\dot{3}$
X.68 X.69	$-6 \\ 3$	3 3	$-10 \\ -10 \\ -5$	10 6 3	10 5	-6 -3	$^{-6}_{6}$	$-6 \\ -2$	$-\dot{6}$	-2 -3		-70		-45	15		$^{-16}_{38}$	8	-38 9	$-\frac{5}{6}$	24	24
X.70 X.71 X.72 X.73 X.74	6 6	$\begin{array}{c} -3 & -3 \\ -3 & -3 \\ -3 & -3 \end{array}$	-5 :	3	-5 :		-5 :	3	-5 :	3		$ \begin{array}{r} 315 \\ -120 \\ -120 \end{array} $	-81 :	75 40 40	:	81	$     \begin{array}{r}       -9 \\       -9 \\       24 \\       24   \end{array} $	$-9 \\ -12 \\ -12$	-9 :	3 8 8	:	:
X.76	$-14^{\circ}_{9}$	-9 . 7 7 . 9	12 5	$\frac{4}{8}$	5		$-{}^{8}_{1}$	$-4$ $\dot{i}$	-i	i	6	$-294 \\ -140 \\ 315$	207	$-54 \\ 20 \\ -45$	45	18	$^{-6}_{10}_{-9}$	$^{3}_{-5}$	_9		-9	-9
$X.77 \\ X.78 \\ X.79$	9	9 . . 9 9 .	5 5 5	$     \begin{array}{r}       -9 \\       11 \\       -9     \end{array} $	$-5 \\ -5 \\ -5$	$^{-9}_{-11}$	3		-3	$-1 \\ -1$	:	$-525 \\ 315 \\ -525$	$^{-36}_{-207}$	$-45 \\ 75$	$^{45}_{-45}$	$-18 \\ -72$	$^{-9}_{-12}$	$-12 \\ -9 \\ -12$		$-25 \\ 7$	$^{18}_{9}_{-18}$	$^{18}_{9}_{-18}$
$X.80 \\ X.81 \\ X.82$	3 12	$\begin{array}{ccc} -6 & 3 \\ -6 & -6 \end{array}$	-28 $-8$	12		:	-4 :	$-\frac{4}{8}$	:	:	$-\frac{2}{-2}$	$     \begin{array}{r}       414 \\       560 \\       -56     \end{array} $	-156 $156$	$-66 \\ 48$	-66	$2\dot{4}$	$^{-18}_{20}_{4}$	20 -2	-20 -	$\begin{array}{c} 6 \\ 16 \\ 24 \end{array}$	6	6
X.83 X.84 X.85	3	-6 3 : :	$-\frac{1}{4}$	1.4	$\overset{\dot{4}}{-4}$	:		$-\frac{1}{4}$	:	$\begin{array}{c} \dot{4} \\ -4 \end{array}$	$-2 \\ 1 \\ 1 \\ c$	-504	-162	$-24 \\ -24$	$-81 \\ 81$		$-18 \\ -18$	-18	$^{20}_{18}$ $^{18}$	16 24 24	-6 :	-6 :
X.86 X.87		: :	-18 5	$^{-14}_{-3}$	5	$-\dot{3}$	$^{10}_{-3}$	$-\frac{6}{3}$	$-\dot{3}$	$-\dot{3}$	-6 ·	$-504 \\ -189 \\ 225$	162	$-21 \\ -15$	:		54	$-27 \\ -18$		-33 33	:	:

2 3	5 6	5 6	6 5	$\frac{4}{6}$	$^{4}_{6}$	$_{4}^{7}$	5 5	5 5	5 5	3 6	$_{4}^{6}$	$_{4}^{6}$	$_{4}^{6}$	$_{4}^{6}$	7 3	$\frac{2}{6}$	5 4	5 4	$\frac{5}{4}$	5 4	$_{4}^{5}$	3 5
7 13	:	:	<u>:</u>	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	<u>:</u>
$\frac{2P}{3P}$	$\frac{612}{3d}$	$\frac{6_{13}}{3e}$	$\frac{614}{3b}$ $2d$	$\frac{615}{3g}$	$\frac{616}{3f}$ 2b	$\frac{617}{3e} \\ 2f$	$\frac{6_{18}}{3e}$	$\frac{6_{19}}{3c}$	$\frac{620}{3d}$	$\frac{621}{3l}$ $2a$	$\frac{622}{3e}$ $2d$	$\frac{623}{3b}$ 2e	$\frac{624}{3b}$ 2f	$\frac{625}{3g}$	$\frac{626}{3e}$	$\frac{627}{3m}$	$\frac{628}{3c}$	2d	$\frac{630}{3d}$	$\frac{631}{3d}$ 2f	$\frac{632}{3e} \\ 2f$	$\frac{033}{3l}$ $2b$
5P 7P 13P	$\begin{array}{c} 6_{12} \\ 6_{12} \\ 6_{12} \end{array}$	$6_{13} \\ 6_{13} \\ 6_{13}$	$6_{14} \\ 6_{14} \\ 6_{14}$	$6_{15} \\ 6_{15} \\ 6_{15}$	$^{6_{16}}_{6_{16}}$	$6_{17} \\ 6_{17} \\ 6_{17}$	$     \begin{array}{c}       6_{18} \\       6_{18} \\       6_{18}     \end{array} $	$6_{19} \\ 6_{19} \\ 6_{19}$	$\begin{array}{c} 6_{20} \\ 6_{20} \\ 6_{20} \end{array}$	$6_{21} \\ 6_{21} \\ 6_{21}$	$6_{22} \\ 6_{22} \\ 6_{22}$	$\begin{array}{c} 6_{23} \\ 6_{23} \\ 6_{23} \end{array}$	$624 \\ 624 \\ 624$	$2f$ $6_{25}$ $6_{25}$ $6_{25}$	$6_{26}$ $6_{26}$ $6_{26}$	$6_{27}$	$6_{28}$	$6_{29} \\ 6_{29} \\ 6_{29}$	$6_{30}$	$6_{31} \\ 6_{31} \\ 6_{31}$	$6_{32}$	633 633 633
X.1 X.2 X.3	1 1 2	1	1 1 2	1 1 -1	1 1 -1	1 1 2	1 -1	1 1	-1	-1 -1	1 1 2	-1	1 1 2	1 1 -1	1 -1	-1	1 1 -1	1 1 -1	1 1 2	1 1 2	1 1 2	1 1 2
X.4 X.5	$-\frac{7}{7}$ $-3$	$\begin{array}{c} 2 \\ 2 \\ 2 \\ -3 \end{array}$	2 5 5	$-\frac{1}{2}$	$-7 \\ -7 \\ -3$	$-\frac{5}{2}$ $-\frac{5}{2}$	$-\frac{2}{3}$ $-3$	$-1 \\ 5 \\ 5 \\ 2 \\ 2$	-7 $7$ $3$ $-3$	$\begin{array}{c} 6 \\ -6 \\ -1 \end{array}$	$\begin{array}{c} 2 \\ 2 \\ 2 \\ -1 \end{array}$	-5 -5	$-\frac{1}{2}$	$-\frac{1}{2}$ $-\frac{1}{2}$ $-\frac{1}{5}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{2}$	2	$-\frac{1}{1}$	1	$-\frac{1}{2}$	2 2
X.7 X.8	$-3 \\ 1$	$-3 \\ -8$	$\begin{array}{c} 2 \\ 2 \\ -4 \\ \end{array}$	-3 -8 -8	$-3 \\ 1$	-5 $-2$ $-2$	$-\frac{3}{-8}$	-4	$-\frac{3}{1}$	$-\frac{1}{3}$	$-1 \\ -1 \\ 2 \\ 2$	$-\frac{2}{2}$ $-4$ $4$	$-\frac{2}{4}$	-5 $-2$ $-2$	$-5 \\ -2 \\ 2$	$\frac{1}{6}$	$-\frac{2}{4}$	$^{-1}_{2}$	5 -1	1	1 4	-3 1
X.10 X.11	$^{1}_{-14}$	$-8 \\ 4 \\ 2 \\ 2$	$-4 \\ 10 \\ 6$		1 7 11	-4		$-4 \\ -5 \\ 6$	11	3 6	4	6	4 2 2 2	2 8	8	-6 -3	$-1 \\ 2 \\ 2 \\ 2$	-2	-1 -2 3	$-\frac{1}{2}$	$-\frac{4}{2}$	4 2
X.12 X.13 X.14	11 7 7	2 7 7	6 4 4	$-2 \\ 2 \\ 7 \\ 7 \\ 7$	11 7 7	$\begin{array}{c} 8 \\ 8 \\ 3 \\ 3 \\ -10 \end{array}$	$-\frac{2}{7} \\ -7$	6 4 4	$-11 \\ 7 \\ -7$	$-6 \\ -2 \\ -2$	7 7	$-6 \\ 4 \\ -4$		8 3 5	$-8 \\ -3 \\ -3$	$\begin{array}{c} 3 \\ 2 \\ -2 \end{array}$		7 7	3 1 1	$-1 \\ 3 \\ 3 \\ 2$	3 3	$-\frac{2}{-2}$
$X.15 \\ X.16 \\ X.17$	-6 1 1	10	$-3 \\ -3$	10 10	3 1 1	8	$-10 \\ 10$	$-2 \\ -3 \\ -3$	$-\overset{\cdot}{\underset{1}{1}}$	$\begin{array}{c} \cdot \\ 3 \\ -3 \end{array}$	-2 :	-3	-4 5 5	8	$-\frac{\dot{8}}{8}$	${\stackrel{\cdot}{\stackrel{\cdot}{\scriptstyle 6}}}$	2 5 5	:	10 3 3	5 5	2 2 2	-6 $1$ $1$
$X.18 \\ X.19 \\ X.20$	2 2 2	$-16$ $\frac{2}{2}$ $\frac{2}{2}$	$     \begin{array}{r}       -8 \\       -7 \\       -7     \end{array} $	8 2 2	$-\frac{1}{2}$	$     \begin{array}{r}     -4 \\     -2 \\     -2 \\     -2     \end{array} $	$\begin{array}{c} \cdot \\ -2 \\ -2 \\ -2 \end{array}$	$-\frac{4}{7}$	$-\frac{\overset{\cdot}{2}}{\overset{\cdot}{2}}$	$-\frac{8}{8}$	4 2 2	$-\frac{7}{7} \\ -\frac{7}{7} \\ 7$	8 1 1	$-\frac{2}{-2}$	$-\frac{1}{2}$ $-\frac{1}{2}$ $-\frac{1}{2}$	$-\overset{\cdot}{\underset{1}{1}}$	-4 1 1		$-2 \\ 2 \\ 2 \\ 2$	$     \begin{array}{r}       2 \\       -2 \\       -2 \\       -2     \end{array} $	$-\frac{8}{-2}$	2 2 2 2
$X.21 \\ X.22 \\ X.23$	$^{2}_{-10}$	$-10^{2}$	$-7 \\ -7 \\ 2$	$-10^{\frac{2}{2}}$	$^{2}_{-10}$	$-2 \\ -10$	-10	$-7 \\ -7 \\ 2$	$^{-2}_{-10}$	$-\frac{8}{3}$	2 2 2 2 2	2	$\begin{array}{c} 1 \\ 1 \\ 2 \end{array}$	$^{-2}_{-2}_{-10}$	-10	$-1 \\ 1 \\ 3$	$\frac{1}{2}$	$\frac{2}{2}$	$\frac{2}{2}$	$-\frac{2}{2}$	$-\frac{1}{2}$	$^{2}_{-1}$
$X.24 \\ X.25 \\ X.26$	$-10 \\ 22 \\ 14$	$-10 \\ 4 \\ 14$	$\frac{2}{12}$	$     \begin{array}{r}     -10 \\     -2 \\     -7 \\     -10     \end{array} $	$^{-10}_{-11}$	$^{-10}_{16}_{6}$	10	$^{2}_{-6}$	10	-3 :	$\frac{2}{14}$	-2 :	$\frac{2}{4}$	$^{-10}_{-8}$	10	-3 :	$-\frac{2}{2}$	$-\frac{2}{7}$	2 6 2	$-{2 \atop 6}$	2 4 6	$-1 \\ -4 \\ -4$
$X.27 \\ X.28 \\ X.29$	$\begin{array}{c} 2 \\ 4 \\ 4 \\ -20 \end{array}$	20 4 4	$-6 \\ -14 \\ -14$	-2 -2	$-1 \\ -2 \\ -2$	$     \begin{array}{r}       16 \\       -4 \\       -4     \end{array} $	:	3 7 7		:	$\overset{\cdot}{\overset{4}{4}}$	:	$\frac{10}{2}$	$-8 \\ 2 \\ 2$	:	:	$-5 \\ -1 \\ -1$	$-\frac{1}{2}$	$\frac{6}{4}$	$     \begin{array}{r}       10 \\       -4 \\       -4     \end{array} $	$     \begin{array}{r}       4 \\       -4 \\       -4     \end{array} $	$\frac{2}{4}$
$X.30 \\ X.31 \\ X.32$	$\frac{1}{1}$	$^{-20}_{-8}$	$     \begin{array}{r}       4 \\       -1 \\       -1     \end{array} $	$     \begin{array}{r}       10 \\       -8 \\       -8     \end{array} $	10 1 1	$^{-20}_{-4}$	$-\stackrel{\cdot}{\stackrel{\circ}{8}}$	$-2 \\ -1 \\ -1$	-1	$-\overset{\cdot}{\overset{\cdot}{3}}$	4 8 8	$-\overset{\cdot}{\underset{1}{1}}$	$     \begin{array}{r}       4 \\       -1 \\       -1     \end{array} $	$     \begin{array}{r}       10 \\       -4 \\       -4     \end{array} $	$-\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{4}}}$	$-\overset{\cdot}{\overset{\cdot}{3}}$	$-2 \\ -1 \\ -1$	$-\frac{2}{8}$	$-1 \\ -1$	4 5 5	$     \begin{array}{r}       4 \\       -4 \\       -4     \end{array} $	$-2 \\ 1 \\ 1$
$X.33 \\ X.34 \\ X.35$	-3 -3	-8 -3 -3	6 6 3	$-8 \\ -3 \\ -3 \\ -3$	$-3 \\ -3 \\ -3$	$\begin{array}{c} 1 \\ 1 \\ 7 \end{array}$	8 -3 -3 -3 -3 -3	6 6 3	$-3 \\ -3 \\ -3$	-9 9 1	8 8 3 3 3 3 3 3 3	-6 6 3	$10 \\ 10 \\ -5$	$\begin{array}{c} 1 \\ 1 \\ 7 \end{array}$	$-\frac{1}{7}$	-9 9	$10 \\ 10 \\ -5$	-3 -3 3	$-3 \\ -3 \\ 3$	1 1	1 1	-3 -3 -3
$X.36 \\ X.37 \\ X.38$	$     \begin{array}{r}       -3 \\       -3 \\       -3 \\       2     \end{array} $	$-3 \\ -3 \\ -3$	$\begin{array}{c} 3 \\ 3 \\ 3 \\ -2 \end{array}$	$-3 \\ -3 \\ -3$	$-3 \\ -3 \\ -3$	7 7 7	$-\frac{3}{3}$	3 3 3	$-3 \\ -3 \\ -3 \\ 3$	$-1 \\ 1 \\ -1$	3 3	$-3 \\ -3 \\ -3$	$-5 \\ -5 \\ -5$	7 7 7	$     \begin{array}{r}       7 \\       -7 \\       7 \\       -7     \end{array} $	$-1 \\ -1 \\ -1$	-5 $-5$ $-5$	- 3	3 3	1 1	1 1 1	$-3 \\ -3 \\ -3$
X.39 X.40 X.41	$^{-14}_{-14}$	-16 $-16$ $4$	$     \begin{array}{r}       -2 \\       -2 \\       -2     \end{array} $	-3 8 4 4	$-1 \\ -14 \\ -14$	$-8 \\ -2 \\ -2$	$-\frac{1}{4}$	$-\frac{1}{2} \\ -2$	$-14^{\dot{1}}_{14}$	$-12^{\dot{1}2}$	$     \begin{array}{r}       16 \\       -2 \\       -2     \end{array} $	$-\frac{1}{2}$	$-2 \\ -2 \\ -2$	$-\frac{4}{2}$	$-\frac{1}{2}$	$-\frac{3}{3}$	$-2 \\ -2$	$-8 \\ -2 \\ -2$	$-2 \\ -2 \\ -2$	$     \begin{array}{r}       10 \\       -2 \\       -2     \end{array} $	-8 4 4	-3 2 4 4
$X.42 \\ X.43 \\ X.44$	$-8 \\ -8 \\ -8$	1 11 1	5	1 11 1	$-8 \\ -8 \\ -8$	$-1 \\ -3 \\ -1$	$-11 \\ -11 \\ -1$	5	$^{-8}_{-2}$ $^{8}_{2}$	$-3 \\ -6 \\ 3$	$-2 \\ 3 \\ 5 \\ 3 \\ 5$	$-\frac{.}{5}$	$-4 \\ -3 \\ -4$	$-1 \\ -3 \\ -1$	$-\frac{2}{3}$	$-3 \\ -6 \\ 3 \\ 6$	$-4 \\ -3 \\ -4$	5 3	$-\frac{6}{6}$	$     \begin{array}{r}     -4 \\     -6 \\     -4   \end{array} $	5 5	1 2 1
X.45 X.46 X.47	$\begin{array}{c} 2 \\ -6 \\ -6 \end{array}$	$\begin{array}{c} 11 \\ -6 \\ -6 \end{array}$	$     \begin{array}{r}       5 \\       -4 \\       -4     \end{array} $	$\begin{array}{c} 11 \\ -6 \\ -6 \end{array}$	$\begin{array}{c} 2 \\ -6 \\ -6 \end{array}$	$-3 \\ 4 \\ 4 \\ 4 \\ 4$	11 -6 6	-4	$     \begin{array}{r}       2 \\       -6 \\       6 \\       -6 \\       6    \end{array} $	$\begin{array}{c} 6 \\ 3 \\ -3 \end{array}$	8	$^{-4}_{4}$	$-3 \\ 4 \\ 4 \\ 4$	$-3 \\ 4 \\ 4 \\ 4$	$-3 \\ 4 \\ -4 \\ 4$	-3	$-3 \\ 4 \\ 4 \\ 4$	8	-4 2 2	-6 $-2$ $-2$	$-\frac{3}{2}$	3 3
X.48 X.49 X.50	$-6 \\ -6 \\ -6$	$-6 \\ -6 \\ -6$	$-4 \\ -4 \\ 12$	$-6 \\ -6 \\ 3$	-6 -6 3	4	-6 6	$-4 \\ -4 \\ -6$		$-3 \\ -3 \\ -3 \\ 9$	8 8 -6	-4 4	$\frac{4}{4}$ 20	$\begin{array}{c} 4 \\ 4 \\ -1 \end{array}$	-4	$-\frac{3}{3}$	$^{4}_{-10}$	8 3	$\frac{2}{2} - 6$	$-2 \\ -2 \\ 2$	$-2 \\ -2 \\ 2$	$\frac{3}{3}$ $-6$
X.51 X.52 X.53	15 15 -6	$-12 \\ -12 \\ -6$	9 9 6	$ \begin{array}{r}     3 \\     -12 \\     -12 \\     3 \\     3 \end{array} $	15 15 3 3	2 2 2 14	$^{-12}_{12}$	9 -3 -3	$-15 \\ -15 \\ \cdot$	-9	-6 -6	-9	-10	$-\frac{2}{7}$	$-\frac{2}{\cdot}$	:	5 5 5 5 5 5 5	$-6 \\ -6 \\ -3$	-3 -3 6	$-1 \\ -1 \\ 2 \\ 2$	$-4 \\ -4 \\ 2 \\ 2$	$-3 \\ -6 \\ 6$
X.54 X.55 X.56	$     \begin{array}{r}     -6 \\     -4 \\     -4 \\     -9     \end{array} $	-6 $-4$ $-4$ $18$	$     \begin{array}{r}       6 \\       -11 \\       -11 \\       3     \end{array} $	$-4 \\ -4 \\ 18$	-4 $-4$ $-9$	14 8 8	$-\frac{1}{4}$ 18	$-11 \\ -11$	$-\frac{1}{4}$ $-9$	$-\frac{\dot{7}}{7}$	$^{6}_{-8}$	$-11_{11}$	$-10 \\ 5 \\ 5 \\ 3 \\ 3$	$^{-7}_{\ \ 8}$	$-\frac{8}{8}$	$-\frac{\dot{7}}{7}$	5 5 3		6 4 4	$-4 \\ -4$	$-\frac{2}{4}$ $-4$ $-6$	$-6 \\ 5 \\ 5$
X.58 X.59 X.60	-9 11 11	18 2 2	3 6 6	18 2 2	-9 11 11	:	$-18 \\ -2 \\ 2$	3 6 6	$-\frac{3}{9}$ $-\frac{11}{11}$	-6 6	:	$     \begin{array}{r}       3 \\       -3 \\       -6 \\       6     \end{array} $	$\begin{array}{c} 3 \\ -6 \\ -6 \end{array}$	:	:	-6 6	$\begin{array}{c} 3 \\ -6 \\ -6 \end{array}$		-3 3	3 3 3	-6 -6 -6	2 2
X.61 X.62 X.63	-28	8	$-\frac{3}{3}$	$-\frac{7}{4}$	14	-4 6	:	3 3	:		-4 6	3 -3	$-\frac{3}{3}$	2 6 6	6 -6		2 3 3	2 6 6	$-\frac{3}{4}$	$-\frac{3}{4}$	8	8
X.64 X.65 X.66	13 4 13	$-5 \\ 22 \\ -5$	10	$-5 \\ -11 \\ -5$	13 -2 13	$-7 \\ -6 \\ -7$	5 -5	_5	-13 13	-12 $12$	$-3 \\ 10 \\ -3$		$     \begin{array}{r}       -4 \\       -6 \\       -4     \end{array} $	$-\frac{5}{3}$	$-\dot{7}$	-3 3 3	$-\frac{3}{3}$	$-\frac{3}{-5}$	$-\frac{3}{8}$	$-12^{5}_{5}_{3}$	$-1 \\ 6 \\ -1$	4 4 4
$X.67 \\ X.68 \\ X.69$	11	-7	10 10	$-7 \\ -1$	11 8 11	$-3 \\ -2$	$-5 \\ -7 \\ \dot{7}$	10 10	11 -11 -18 18		1	10 -10	$^{6}_{-8}$	$-3 \\ 1 \\ -3 \\ 9$	-3 3	3 -3	6 4	$-\frac{1}{3}$	1 12	$-8 \\ 3$	-3	$\frac{2}{2}$
X.69 X.70 X.71 X.72 X.73	$^{18}_{18}$ $^{-12}$	$ \begin{array}{r}     2 \\     -7 \\     -9 \\     -9 \\     -12 \end{array} $	$^{3}_{-8}$	$     \begin{array}{r}       -7 \\       -9 \\       -9 \\       6   \end{array} $	18 18 6	$-3 \\ 9 \\ 9 \\ 8$	-9 -9	3 3 4	$^{-18}_{18}$	:	$-3 \\ -3 \\ 16$	-10 -3 3	O	$   \begin{array}{c}     9 \\     9 \\     -4 \\     -4   \end{array} $	-9 9		-4	$     \begin{array}{r}       1 \\       -3 \\       -3 \\       -8     \end{array} $	- 4	$ \begin{array}{r} -6 \\ -6 \\ -4 \\ -4 \\ -2 \end{array} $	$-3 \\ 3 \\ -4$	6
$X.74 \\ X.75$	-8	$-24 \\ -8$	-22	$^{6}_{12}_{4}$	-154	$\frac{8}{4}$ 16	:	$^{4}_{-9}$		:	$-16 \\ -12 \\ -16$	:	10 10	$-2 \\ -8$		:	$-4 \\ -5 \\ -5$	-8 6	$-\frac{4}{6}$		$-4 \\ -8 \\ -8$	$_{-6}^{6}$
$X.76 \\ X.77 \\ X.78$	-21	18 6 18	$^{12}_{-9}$	6 18	$^{-9}_{-21}$	$-\frac{2}{8}$	$^{18}_{6}$ $^{-18}$	-9	$^{-9}_{-21}$	9	-6 $-6$	$-9 \\ 12 \\ 9$	$-1 \\ -1 \\ -1$	$-\frac{2}{8}$	$     \begin{array}{r}       2 \\       -8 \\       -2     \end{array} $	9 -9	$-1 \\ 4 \\ -1 \\ 4$	-6 $-6$	-3 -3 -3 -6	$     \begin{array}{r}       -1 \\       -5 \\       -1 \\       -5     \end{array} $	$-\frac{2}{2}$ $-2$	-3 $-3$
X.78 X.79 X.80 X.81	2	36 20	$^{6}_{12}$	$^{-18}_{20}$	$-21 \\ 9 \\ 2$	-8 $-8$	-6 $-20$	$\frac{12}{-3}$	21 -2 2	$-\frac{1}{6}$	:	$ \begin{array}{r} -9 \\ 12 \\ 9 \\ -12 \\ -12 \\ 12 \\ 6 \end{array} $	4 6 4	-8 $-8$	8 8	9 -9 -3 -3	$-\frac{4}{3}$		$-3 \\ -6 \\ 6$	6	$     \begin{array}{r}     -2 \\     -12 \\     4 \\     -12     \end{array} $	-3 2 4
X.82 X.83 X.84	2 9	$\begin{array}{r} 4 \\ 20 \\ -18 \end{array}$	$^{12}_{-6}$	$^{20}_{-18}$	-11 2 9	$-\dot{8}$	$ \begin{array}{r} -18 \\ -6 \\ -20 \\ 20 \\ 18 \\ -18 \end{array} $	$^{-6}_{12}$	-9	-6 :		12 6	$^{-12}_{\ \ 6}$	-8 -8 -8	2 -8 -2 8 8 -8	3	6 4 6	•	6	$-\frac{6}{2}$	$\frac{4}{6}$	4 2
$X.85 \\ X.86 \\ X.87$		-18 9	6	$-18$ $\dot{9}$	9 -18	$^{12}_{9}$	-18 9	$-6 \\ -3 \\ -6$	9 -18	:	12	$-6$ $-\dot{6}$	6 6 6	$-\frac{\dot{6}}{9}$	9	:	$-{}^{6}_{6}$	$-6 \\ -3$	-3 -3	$ \begin{array}{c} -2 \\ -3 \\ -3 \end{array} $	-3	:

2 3 5 7	6 3	6 3	4 4	4 4	4 4	4 4	44	5	2 5 5 3	5	5 3	3		34		3 1 4		4 3	2 4	4	2 4	3 3	3	1 4	2 3
13 2P 3P 5P 7P 13P	$6_{34}$	$\begin{array}{c} 635 \\ 3e \\ 2e \\ 635 \\ 635 \\ 635 \end{array}$	636	$\begin{array}{c} 6_{37} \\ 3f \\ 2d \\ 6_{37} \\ 6_{37} \\ 6_{37} \end{array}$	038	039	040	041	$\begin{array}{c} 2g \\ 6_{42} \\ 6_{42} \end{array}$	$6_{43} \\ 6_{43}$	$\begin{array}{c} 2e \\ 6_{44} \\ 6_{44} \end{array}$	$\frac{6_{45}}{6_{45}}$	$^{646}_{646}$	047	$^{6_{48}}_{6_{48}}$	$\frac{2c}{649}$	$^{650}_{50}$	$^{2g}_{651}_{651}$	$^{2f}_{652}_{652}$	$\begin{array}{c} 2f \\ 6_{53} \\ 6_{53} \end{array}$	$^{2f}_{654}_{654}$	055	$^{656}_{56}$	$\frac{657}{657}$	$^{2g}_{658} \\ ^{658}_{658}$
3P 5PP 7PP 13PP 1X 2 2 X 1X 2 4 X 15 X 15 X 16 X 17 X 18 X 19 X 10 X 11 X 12 X 13 X 13 X 14 X 15 X 16 X 17 X 18 X 19 X 10 X 11 X 11 X 12 X 13 X 14 X 15 X 16 X 17 X 18 X 18 X 18 X 18 X 18 X 18 X 18 X 18	2963446344 -11 -11 -11 -11 -11 -11 -11 -11 -11 -	$\begin{array}{c} 2e \\ 685 \\ 635 \\ 1 \\ 1 \\ -1 \\ 2 \\ 2 \\ -11 \\ -2 \\ 2 \\ -2 \\ 2 \\ -2 \\ 2 \\ -2 \\ 2 \\ -2 \\ 2 \\ $	$\begin{array}{c} 2f\\ 636\\ 636\\ 636\\ 1\\ 1\\ 1\\ 2\\ 2\\ 2\\ 1\\ 1\\ 1\\ 1\\ 1\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\ 1\\ 1\\ 1\\ -2\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\ 1\\ 1\\ 1\\ -1\\ 4\\ 4\\ -5\\ 5\\ 4\\ 4\\ 4\\ 1\\ 1\\ 1\\ 1\\ 1\\ 3\\ 3\\ 1\\ 3\\ 3\\ 1\\ 3\\ 3\\ 1\\ 3\\ 1\\ 3\\ 3\\ 1\\ 3\\ 3\\ 1\\ 3\\ 3\\ 1\\ 3\\ 3\\ 3\\ 3\\ 3\\ 3\\ 3\\ 4\\ 4\\ 4\\ 4\\ 4\\ 4\\ 4\\ 4\\ 4\\ 4\\ 4\\ 4\\ 4\\$	$\begin{smallmatrix} 637 \\ 637 \\ 1 \\ -1 \\ -1 \\ 55 \\ -1 \\ 1 \\ 1 \\ 331 \\ 122 \\ 222 \\ 222 \\ 231 \\ -322 \\ -221 \\ -1 \\ -333 \\ 122 \\ -221 \\ -1 \\ -333 \\ 331 \\ -226 \\ -4 \\ -64 \\ -4 \\ -4 \\ -4 \\ -4 \\ -4 \\ $	$\begin{array}{c} 2f\\ 6388\\ 6388\\ 1\\ 1\\ 2\\ 2\\ 1\\ 1\\ 1\\ 1\\ 1\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\$	$\begin{smallmatrix} 6_{39} \\ 6_{39} \\ 1 \\ -1 \\ -2 \\ 1 \\ 1 \\ 1 \\ 4 \\ 4 \\ 2 \\ 2 \\ 2 \\ 3 \\ 3 \\ 1 \\ -1 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ $	$\begin{array}{c} 6400 \\ 64$	28641166411	$\begin{array}{c} 299 \\ 642 \\ 642 \\ 642 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ $	$egin{array}{c} 299 & 643 & 66443 & 66443 & 66443 & 66443 & 66443 & 66443 & 66443 & 66443 &$	$\begin{array}{c} 2e \\ 644 \\ 644 \\ 644 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ 1 \\ -1 \\ 1 \\ $	$egin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{c} 2f \\ 646 \\ 646 \\ 646 \\ 11 \\ 11 \\ 11 \\ 12 \\ 22 \\ 22 \\ 22 \\ 2$	$egin{array}{c} 6447 & 6447 $	2f86688 $66488$ $-262$ $-26$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{c} 2g\\ 6500\\ 6500\\ -1\\ -1\\ -1\\ -1\\ -1\\ -1\\ -1\\ -1\\ -1\\ -1$	$\begin{array}{c} 2g\\ 2g\\ 651\\ 651\\ 1\\ -1$	$\begin{array}{c} 2f\\ 652\\ 652\\ 1\\ 1\\ 2\\ 2\\ 1\\ 1\\ 1\\ 1\\ 1\\ 1\\ 1\\ 1\\ 1\\ 1\\ 1\\ 1\\ 1\\$	$\begin{array}{c} 2f\\ 653\\ 653\\ 653\\ 653\\ 1\\ 1\\ 1\\ 1\\ 1\\ 1\\ 1\\ 1\\ 1\\ 1\\ 1\\ 1\\ 1\\$	$\begin{array}{c} 2f\\ 654\\ 654\\ 1\\ 1\\ 1\\ -2\\ -2\\ 1\\ 1\\ 1\\ 1\\ 1\\ 1\\ 1\\ 1\\ -2\\ 2\\ 2\\ 2\\ 2\\ 2\\ 1\\ -1\\ -1\\ -1\\ -1\\ -1\\ -1\\ -1\\ -1\\ -1\\$	6655 6655 6555 61 -11 -11 -11 -11 -12 -22 -11 -11 -11 -1	6566 6566565656565656565656565656565656	$\begin{array}{c} 2f \\ 657 \\ 657 \\ \hline \\ 11 \\ -11 \\ 11 \\ 12 \\ -21 \\ -11 \\ 11 \\ $	$\begin{array}{c} 2g \\ 658 \\ 658 \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \\ 1 \\ -1 \\ 1 \\ $
X.46 X.47 X.48 X.50 X.51 X.52 X.53 X.54 X.55 X.56 X.57 X.58 X.59 X.60 X.61 X.62 X.63	5 -5 -5 -5 -5 -3 -3 -3 -3 -3 -3 -3	-8 -8 -6 6	$ \begin{array}{c} -2 \\ -2 \\ -2 \\ -2 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1$	2 2 2 2 3 -3 -3 -3 -4 4 -3 -3 3 2	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$     \begin{array}{r}     -2 \\     -2 \\     -2 \\     -1 \\     -4 \\     -4   \end{array} $	-22 -22 -21 -11 -11 -14 -44 -33 33 33 22	33 33 33 33 33 4 4	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 $	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	11 11 11 11 11 11 22 -11 -11 -11 -11	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	$\begin{bmatrix} 1 & 3 & -3 & 1 & -3 & 1 & -3 & 1 & -3 & 1 & -3 & 1 & -3 & 1 & -3 & 1 & -5 & 1 & $	-2 2 -2 2 -1 1 -1 1	-2 $-2$ $-2$	1 1 1 2 2 2 2 2 2 -1 -1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	-1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1	-1 -1 -1 -1 -1 -1 -1 -1 -1 -1	11 11 11 11 12 22 -11 -11 -11 -11	$-1$ $\vdots$ $-2$ $\vdots$
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X.1 X.2 X.3 X.4 X.5 X.6 X.7 X.8 X.9 X.10	1 1 2 1 1 	1 1 2 -1 -1 -1 -1	1 1 2 -1 -1 1 1	-1 -1 -1 -1 -1 -1	-1 -1 -1 -1 -1 -1 1	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	1 1 2 4 4	$ \begin{array}{c} 1 \\ 1 \\ -1 \\ 3 \\ -2 \\ -2 \\ 3 \\ -3 \end{array} $	$ \begin{array}{c} 1 \\ 1 \\ -1 \\ \cdot \\ 4 \\ 4 \\ \cdot \\ \cdot \\ 3 \end{array} $	1 1 2	1 1 2	1 -1 -1	1 -1 -1	$ \begin{array}{c} 1\\1\\2\\-1\\-1\\1\\1\\1\\.\\-2\\-1\end{array} $	$ \begin{array}{c} 1 \\ -1 \\ 3 \\ -3 \\ -1 \\ 1 \end{array} $	1 2 1 1 -1 -1 -1 2	$-1 \\ 1$	-1 $1$ $-1$ $1$	$ \begin{array}{c} 1 \\ -1 \\ -6 \\ -6 \\ -1 \\ -1 \\ -5 \\ -5 \\ 6 \\ 4 \end{array} $	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c} 1 \\ 1 \\ -1 \\ 2 \\ 3 \\ 3 \\ 1 \\ -2 \end{array} $	1 1 2 -1 -1 -1	1 1 2 2 2 2 3 3 3 2 2 4	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c} 1 \\ 1 \\ -1 \\ -1 \\ -1 \end{array} $	1 1 -1 2 2 3 3 2 2 -2	$ \begin{array}{c} 1\\ 1\\ 2\\ -1\\ -1\\ -2\\ -2\\ -2 \end{array} $
X.12 X.13 X.14 X.15 X.16 X.17 X.18 X.20 X.21 X.22	-1 -1 -1 1 1	-2 -1 -1 -2	-2 1 1 2	-1 1	-1 -1 -1	5 5 -4	$ \begin{array}{c} 3 \\ -1 \\ -1 \\ 8 \\ 3 \\ 3 \\ -1 \\ -1 \end{array} $	5 5 2 -3 -1 -1 -1	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$-\frac{1}{2}$ $-\frac{1}{2}$ $-\frac{1}{2}$	$ \begin{array}{c} -1 \\ -1 \\ 2 \\ \vdots \\ -1 \\ -1 \\ 2 \end{array} $	$ \begin{array}{c} -1 \\ -1 \\ -1 \\ -1 \\ \end{array} $ $ \begin{array}{c} 2 \\ -1 \\ -1 \\ 2 \end{array} $	$ \begin{array}{c} -1 \\ -1 \\ -1 \\ -1 \end{array} $ $ \begin{array}{c} -1 \\ 2 \\ 2 \\ -1 \end{array} $	-1 2 2 2 2 	$-\frac{3}{2}$	1 -2 -2	1 2 -2		4 10 10 10 5 5 5	2 2 2 -3 3 3 -5 4 4 4 4	2 2 -3 -1 -1 -1 4 4 4 4	2 2 2 3 3 4 1 1	$ \begin{array}{c} -1 \\ -1 \\ 6 \\ \\ -2 \\ -2 \\ -2 \\ -2 \\ -2 \end{array} $	4 2 2 1 1 1 1 1 	2 2 3 3 -2 1 1 1	-1 -1 -3 -2 -2 -2 -2 -2	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
X.22 X.23 X.24 X.25 X.26 X.27 X.28 X.29 X.30 X.31 X.32	-2 2 2	1 1 -2 2	-i -1	-1 1	-1 -1 -1	$ \begin{array}{c} -1 \\ 3 \\ 3 \\ 10 \\ -2 \\ -2 \\ 6 \\ 3 \\ 3 \end{array} $	-1 3 6 -2 6 -2 -2 -2 -2	$\begin{array}{c} 3 \\ 3 \\ -5 \\ \vdots \\ 1 \\ 1 \end{array}$	$ \begin{array}{c} -1 \\ 3 \\ -3 \\ 1 \\ -3 \\ 1 \\ -3 \\ \vdots \end{array} $	-2 -2	-2 -2 -2 -2 -2	-2 1 -2 1	i i	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	3 -3	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	-1 1	-1 1	$ \begin{array}{c} -9 \\ -9 \\ -4 \\ -10 \\ -5 \\ \vdots \\ 9 \\ 14 \\ 14 \end{array} $	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{r}     -3 \\     -3 \\     -2 \\     -4 \\     -4 \\     -3 \\     -2 \\     -2 \\     -2 \end{array} $	1 4 4 6 2 2 8 -3 -3	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{c} 3 \\ 3 \\ -4 \\ -2 \\ -1 \\ \\ \vdots \\ -3 \\ -2 \\ -2 \end{array}$	$ \begin{array}{c} 1 \\ 4 \\ 4 \end{array} $ $ \begin{array}{c} -2 \\ -3 \\ -1 \\ -1 \\ -4 \\ -3 \\ -3 \end{array} $	-2 -2 -2 -1	$\begin{array}{c} \cdot \\ -4 \\ 2 \\ 4 \\ \cdot \\ \cdot \\ 2 \\ 2 \end{array}$
X.33 X.34 X.35 X.36 X.37 X.38 X.40 X.41 X.42 X.43 X.44		1	1	-1 · · · · · · · · · · · · ·	-1 1	. 4 4 4 4 6 -3 -3 . 3	-2 -2 -2 -2 -2 -3	. 4 4 4 4 -3 -3 -3 . 3		1 -2 -2 -1 1	$\begin{array}{c} \cdot \\ -2 \\ 1 \\ 1 \\ -2 \\ \cdot \\ $	$\begin{array}{c} \cdot \\ -2 \\ 1 \\ 1 \\ -2 \\ \cdot \\ $	$\begin{array}{c} \cdot \\ \cdot \\ -2 \\ -2 \\ 1 \\ \cdot \\ \cdot$	-1 -1	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	-1 -1	-1 -1 		$ \begin{array}{r} -1 \\ -10 \\ -10 \\ -10 \\ -10 \\ -14 \end{array} $	7722222644455151	$ \begin{array}{c} -1 \\ -1 \\ 2 \\ 2 \\ 2 \\ 4 \\ 4 \\ -3 \\ 1 \\ -3 \\ 1 \end{array} $	$     \begin{array}{r}       -2 \\       -2 \\       -1 \\       -1 \\       -1 \\       -6 \\       -2 \\       -2 \\       -4 \\       1     \end{array} $	$\begin{array}{c} 1 \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -2 \\ -2$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{r} -2 \\ -2 \\ -1 \\ -1 \\ -1 \\ -3 \\ -2 \\ -4 \\ 1 \end{array} $	$\begin{array}{c} 1 \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \\ \end{array}$ $\begin{array}{c} -2 \\ -2 \\ -1 \\ 1 \\ \end{array}$	$\begin{array}{c} -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ 4 \\ \vdots \\ -1 \\ -1 \\ -1 \\ -1 \end{array}$
X.47 X.48 X.49 X.50 X.51 X.52 X.53 X.54 X.55 X.56 X.57	-1 -1 -1 -1 -1 -1 -1	2	2			-3 -3 -3 -3 -3 -5 -5	-4 -4 -1 -1	-3 -3 -3 -3 -4 -4 -5 5			-4 2 -1 -1	· · · · · · · · · · · · · · · · · · ·	-1 -1 -1 -1	-2 -1 -1 -1	3 -3 -1 -1	-2 -1 -1 -1	-1 1 	-1 -1 -1 -1	1 6 6 10 10 	$\begin{array}{c} \cdot \\ \cdot \\ -7 \\ 2 \\ 2 \\ -2 \\ -2 \\ 4 \\ 4 \\ 6 \\ 6 \end{array}$	1 2 2 -2 -2 -4 4 -2 -2	$\begin{array}{c} \cdot \\ \cdot \\ -4 \\ -1 \\ -1 \\ -2 \\ -2 \\ 1 \\ 1 \\ -3 \\ -3 \end{array}$	· · · · · · · · · · · · · · · · · · ·	1 -2 -2 2 2 2	$\begin{array}{c} \cdot \\ \cdot \\ \cdot \\ -1 \\ -1 \\ 1 \\ 1 \\ 1 \\ -3 \\ -3 \end{array}$	-1 2 2 1 1 4 4	-2 -2 -2 -2 -2 -2 -2
X.59 X.60 X.61 X.62 X.63 X.64 X.65 X.66 X.67 X.68 X.70 X.71		$\begin{array}{c} \vdots \\ \vdots \\ 1 \\ 1 \\ -2 \\ \vdots \\ 2 \\ -1 \\ -1 \end{array}$	-1 -1 -2 -2 -1 -1	-i -i -i -i -i -1	-1 -1 -1 -1	-6 -6 -3 -3	3 -3 -3 -6		3 -3 -3 -3					-1 -1 4 1 1	-1 -3 3	1 -1 -1 -1	:		$     \begin{array}{r}       -4 \\       -4 \\       -9 \\       -9 \\       10 \\       -5 \\       10 \\       -10 \\       -5 \\       -10 \\       -5 \\       -5 \\     \end{array} $	$ \begin{array}{r} -4 \\ -7 \\ -7 \\ -2 \\ -1 \\ -2 \\ 6 \\ -5 \\ 6 \\ 3 \\ 3 \end{array} $	$ \begin{array}{c} -4 \\ 5 \\ 5 \\ -2 \\ -1 \\ -2 \\ 6 \\ 3 \\ 6 \\ -5 \\ -5 \end{array} $	$ \begin{array}{c}     -4 \\     -1 \\     -1 \\     -2 \\     -2 \\     -2 \\     -8 \\     -3 \\     -3 \\   \end{array} $	-3	$ \begin{array}{c} -4 \\ -4 \\ 3 \\ 3 \\ 2 \\ 3 \\ -2 \\ 3 \\ 3 \end{array} $	$\begin{array}{c} \cdot \\ \cdot \\ -1 \\ -1 \\ -2 \\ -1 \\ -2 \\ \end{array}$	2 2 2 1 -1 1 -3 1 -3 -3 -3 -3	$\begin{array}{c} 2 \\ 2 \\ \vdots \\ -2 \\ 1 \\ -1 \\ -2 \\ -1 \\ 1 \end{array}$
X.72 X.73 X.74 X.75 X.76 X.77 X.80 X.81 X.82 X.83 X.84 X.85 X.86 X.87	-2		-2				-2 -3 -3 -3	-5	i	-2 -2	$-\frac{1}{2}$			1	· · · · · · · · · · · · · · · · · · ·	· · · · · · · · · · · · · · · · · · ·	2 -2 -1 1	-1	5 5 5 14 4 -4 -4 9	$ \begin{array}{c}     -2 \\     -4 \\     11 \\     -9 \\     11 \\     -9 \\     -6 \\     \cdot $	$ \begin{array}{c}                                     $	$ \begin{array}{c}                                     $	4 8 2	$\begin{array}{c} 1 \\ 1 \\ 1 \\ -2 \\ 4 \end{array}$	-1 -1 -i	· · · · · ·	2

 $Character\ table\ of\ mE\ (continued)$ 

2 3 5	4 3	5 2	5 2	5 2	3 3	3 3	3 3	3 3	4 2	$\frac{4}{2}$	4 2	4 2	2 3	3	5 1	3 2	3 2	3 2	4 1	1 1
13	1210	1211	1210	1210	1214	1215	1210	1217	1210	1210	1200	1201	1200	1200	1204	1205	1200	1207	1200	1 13a
$\frac{2P}{3P}$	$6_{30} \\ 4a$	$\frac{624}{4d}$	$624 \\ 4e$	$6_{17} \\ 4d$	$\frac{629}{4a}$	$637 \\ 4a$	$\frac{636}{4b}$	$\frac{638}{4b}$	$\frac{6_{28}}{4e}$	$\frac{6_{22}}{4c}$	$\frac{630}{4c}$	$\frac{630}{4f}$	$\frac{6_{45}}{4b}$	$\frac{646}{4b}$	$\begin{array}{c} 6_{24} \\ 4g \end{array}$	$\frac{636}{4d}$	637 4 f	$6_{38} \\ 4d$	$6_{30} \\ 4h$	$\frac{13b}{13a}$
$\frac{5P}{7P}$	$12_{10} \\ 12_{10} \\ 12_{10}$	$\frac{12_{11}}{12_{11}}$	$\frac{12_{12}}{12_{12}}$	$\frac{12_{13}}{12_{13}}$	$\frac{12_{14}}{12_{14}}$	$\frac{12_{15}}{12_{15}}$	$\frac{12_{16}}{12_{16}}$	$\frac{12_{17}}{12_{17}}$	$\frac{12_{18}}{12_{18}}$	$\frac{12_{19}}{12_{19}}$	$\frac{12_{20}}{12_{20}}$	$12_{21} \\ 12_{21}$	$\frac{12_{22}}{12_{22}}$	$\frac{12_{23}}{12_{23}}$	$12_{24} \\ 12_{24}$	$\frac{12_{25}}{12_{25}}$	$\frac{12_{26}}{12_{26}}$	$\frac{12_{27}}{12_{27}}$	$\frac{12_{28}}{12_{28}}$	$\frac{13b}{13b}$
$\frac{13P}{X.1}$	$12_{10}^{10}$	$\frac{12_{11}^{11}}{1}$	12 <sub>12</sub>	12 <sub>13</sub>	$\frac{12_{14}^{14}}{1}$	$\frac{12_{15}^{10}}{1}$	$12_{16}^{10}$	1217	12 <sub>18</sub>	$\frac{12_{19}^{13}}{1}$	$\frac{12_{20}^{20}}{1}$	$\frac{12_{21}^{21}}{1}$	$\frac{12_{22}^{22}}{1}$			$\frac{12_{25}^{26}}{1}$	$12\frac{26}{26}$	1227	$12\frac{28}{28}$	$\frac{1a}{1}$
$X.2 \\ X.3$	$\frac{1}{2}$	-1	$\frac{1}{2}$	-1	$-1 \\ -1$	$-1 \\ -1$	$\frac{1}{2}$	$\frac{1}{2}$	$-1 \\ -1$	-1	-1	$\frac{1}{2}$	$-1 \\ -1$	$-1 \\ -1$		-1	$-1 \\ -1$		-1	$\frac{1}{2}$
$X.4 \\ X.5$	$-3 \\ -3 \\ -3$	$-1 \\ 1$	$-1 \\ -1$	$-\frac{2}{2}$	:	$-3 \\ -3$	$-1 \\ -1$	$-1 \\ -1$	$-1 \\ -1$		$-3 \\ 3$	$-1 \\ -1$	$-1 \\ -1$	$-1 \\ -1$	$^{-1}_{1}$	$-1 \\ 1$	$-1 \\ -1$	$-\frac{1}{1}$	$-1 \\ 1$	:
X.6 $X.7$	$-1 \\ -1$			$-3 \\ 3 \\ 2$	$-1 \\ -1$	$-1 \\ -1$				$-\frac{1}{1}$	$-\frac{1}{1}$	$-1 \\ -1$					$-1 \\ -1$		$-\frac{1}{1}$	
$X.8 \\ X.9 \\ X.10$	$\begin{array}{c} 1 \\ 1 \\ -6 \end{array}$	$-\frac{\dot{2}}{2}$	$     \begin{array}{r}       -2 \\       -2 \\       -2     \end{array} $	$-\frac{2}{2}$	$-2 \\ -2$	1 1 3	$-1 \\ -1 \\ -2$	$-1 \\ -1 \\ -2$	$-\frac{1}{2}$	$-\frac{2}{2}$	$-1 \\ -1$	$-1 \\ -1 \\ -2$	$-1 \\ -1 \\ 1$	$-1 \\ -1 \\ 1$	$-\frac{2}{2}$	-1 1	-1		$-\frac{1}{1}$	1
X.11 X.12	1	:	:		$-\frac{1}{2}$	1				$-\frac{1}{2}$	1 -1	1 1			:	:	1 1		1 -1	$-1 \\ -1$
$X.13 \\ X.14$	1 1	$-\frac{2}{2}$	2 2	$-1 \\ 1$	1	1	$-1 \\ -1$	$-1 \\ -1$	2 2	-1	$-1 \\ -1$	$-1 \\ -1$	$-1 \\ -1$	$-1 \\ -1$	$-\frac{2}{2}$	$-\frac{1}{1}$	$-1 \\ -1$	$-\frac{1}{1}$	$-\frac{1}{1}$	
$X.15 \\ X.16$	$-2 \\ -1$	$-\overset{\cdot}{\overset{\cdot}{3}}$	-i	:	$\begin{array}{c} 1 \\ 2 \\ 2 \end{array}$	$-\frac{1}{1}$	:	:	- i	$-\dot{2}$	i	$-2 \\ 1 \\ 1$	:	:	1	:	1	:	- i	:
X.17 X.18	$-\frac{1}{2}$	3 1	$-1 \\ -4$		2	$-1 \\ -1$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-1 \\ 2 \\ 1$	2	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	i	-1 ;		1			$\dot{2}$
X.20 X.21	:	$-\frac{1}{1}$	1 1 1	$-\frac{2}{2}$	:	:	$-\frac{1}{2}$ $-\frac{1}{2}$ 1	-2 1 1	1	:	:	:	-2 1 1	$-\frac{1}{2}$	$-{1\atop 1}$	$-\frac{1}{2}$		$-\frac{2}{1}$		:
X.22 X.23	:	$-\frac{1}{4}$	1	$-2 \\ -2 \\ -2$	·		1 1	$-\frac{1}{2}$	Î.	:			$-\frac{1}{2}$	1 1	-Î	$-\frac{1}{1}$		2 1		:
$X.24 \\ X.25$	2	-4		2	2	-i	1	1		:	:	2	1	1	:	-1	-i	-1	:	$-\dot{2}$
$X.26 \\ X.27 \\ X.28$	$-\frac{2}{2}$	:	$-\frac{4}{2}$	:	$-1 \\ -2$	$-1 \\ 1$	$-2$ $\dot{2}$	-2 $-4$	-2	:	:	$-\frac{2}{2}$	1 2	1	:	:	$-1^{1}$			:
X.29 X.30	:	:	$\frac{2}{2}$	:	:	:	$-\frac{2}{4}$	-4 2 2	$-1 \\ -1$	:	:	:	$-1 \\ -1$	$-1 \\ 2 \\ -1$	:	:		:		:
X.31 X.32	$-1 \\ -1$	$-\frac{1}{3}$	1 1	:	2 2	-1 -1			i 1	$-\frac{1}{2}$	-i	i 1			1 -1		1 1	:	1 -1	
$X.33 \\ X.34$	$-1 \\ -1$	$-\frac{2}{2}$	$\frac{2}{2}$	$-\frac{1}{1}$	$-1 \\ -1$	$-1 \\ -1$	$\frac{1}{1}$	$\frac{1}{1}$	$\frac{2}{2}$	$-1 \\ -1$	$-1 \\ -1$	$-1 \\ -1$	1 1	$\frac{1}{1}$	$-\frac{2}{2}$	$-\frac{1}{1}$	$-1 \\ -1$		$-1 \\ -1$	:
X.35 X.36	$-1 \\ -1$	-1 1	$-1 \\ -1$	$-1 \\ 1 \\ 1$	$-1 \\ -1$	$-1 \\ -1$	$-\frac{2}{1}$	$-\frac{1}{2}$	$-1 \\ -1$	-1 1	-1 1	1	$-\frac{1}{2}$	$-\frac{2}{1}$	-1 1	2 1	1	-2	$-\frac{1}{1}$	:
X.37 X.38 V.30	$-1 \\ -1 \\ -2$	$-\frac{1}{1}$	$-1 \\ -1 \\ 2$	$^{-1}_{1}$	$-1 \\ -1 \\ -2$	$-1 \\ -1 \\ 1$	$-\frac{1}{2}$	$-\frac{2}{1}$	$-1 \\ -1 \\ -1$	$-\frac{1}{1}$	$-\frac{1}{1}$	1 2	$-\frac{2}{1}$	$-\frac{1}{2}$	$-\frac{1}{1}$	$^{-1}_{-2}$	1 1	$\frac{2}{1}$	$-\frac{1}{1}$	:
X.40 X.41	-2	$-\frac{1}{2}$	$-\frac{2}{-2}$	$-\frac{1}{2}$	-2		i 1	1 1	$-\frac{1}{2}$	:			1 1	i 1	$-\frac{1}{2}$	1 -1	-1	i 1		:
$X.42 \\ X.43$	2 2 2	$-\frac{1}{4}$	i	$-1 \\ -1$	$-1 \\ -1$	2 2 2	$-\frac{1}{1}$	$-\frac{1}{1}$	i	$-\frac{1}{1}$	$-\frac{2}{2}$		$-\frac{1}{1}$	$-\frac{1}{1}$	-i	$-1 \\ -1$		$-1 \\ -1$		
X.44 X.45	2 2	$\frac{4}{1}$	i	1 1	$-1 \\ -1$	$\frac{2}{2}$	$-1 \\ 1$	$-1 \\ 1$	i	-1	$-\frac{2}{2}$	:	$-1 \\ 1$	$-1 \\ 1$	1	1 1	:	1 1	:	:
X.46 X.47 X.48	:	:	:	:	:	:			:	:	:	:	:		:	:				
X.49 X.50	-2	:	4	:	1	i	2	2	-2	:		-2	-1	-1		:	i			
$X.51 \\ X.52$	$     \begin{array}{r}       -2 \\       3 \\       3 \\       -2     \end{array} $	$-\frac{1}{1}$	$-1 \\ -1$	$-\frac{2}{2}$	:	3 3	$-1 \\ -1$	$-1 \\ -1$	$-1 \\ -1$	:	$-3 \\ -3$	1 1	$-1 \\ -1$	$-1 \\ -1$	$-\frac{1}{1}$	$-\frac{1}{1}$	1 1	$-\frac{1}{1}$	$-1 \\ -1$	:
X.53 X.54	$-2 \\ -2$	i	$-2 \\ -2$		1 1	1 1	$-\frac{4}{2}$	-2	1	:	:	$\frac{2}{2}$	$-\frac{1}{2}$	-2	1		$-1 \\ -1$			
X.56 X.57	1	$-1 \\ -3$	1 1 1	$-\frac{4}{4}$	$-\dot{2}$	1	1 1	1	1 1 1	$-\dot{2}$	i	- i	1	1 1	$-\frac{1}{1}$	$-1^{1}$	i	$-1^{1}$	_ i	:
X.58 X.59	$-1 \\ -1$	3	1		$-\frac{1}{2}$	$-1^{-1}$			1	$-\frac{2}{2}$	$-\frac{1}{1}$	$-1 \\ -1$			-1		$-1 \\ -1$		1 1	
X.60 X.61	-1		$-\frac{1}{4}$		2	-1	2	2	2	2	-1	-1	-1	-i			-1		-1	:
X.62 X.63 X.64	1	$-1 \\ 1 \\ 2$	$-1 \\ -1 \\ -2$	$-\frac{2}{-1}$	1	· i	$-1 \\ -1 \\ 1$	$-1 \\ -1 \\ 1$	$-1 \\ -1 \\ -2$	_ i	-1	-i	$-1 \\ -1 \\ 1$	$-1 \\ -1 \\ 1$	$-1 \\ 1 \\ 2$	-1 1 -1	-1	-1 1 -1	i	
X.65 X.66	$\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{2}{2}$ $-2$	i	1 1	$-\frac{1}{2}$	$\frac{1}{2}$	2 1	$-1 \\ -2$	i	i	-i	$-\frac{1}{1}$	$-\frac{1}{1}$	-2 -2	i	-i	i	-i	
$X.67 \\ X.68$	$-\frac{1}{4}$			-3	$-\frac{1}{1}$	$-1 \\ -2$	$-\dot{2}$	$-\dot{2}$		-1	-1	1	i	i			1		1	:
X.69 X.70	$-1 \\ -2$	3	i	3	$-1 \\ 1 \\$	$-1 \\ -2$	:	:	i	$-\frac{1}{1}$	1 2	1		:	-1	:	1	:	-1	1
X.71 X.72 V.72	-2 ·	-3 ·	1	-3 ·	1	-2	:	:	1	1	-2	:	:	:	1	:	:		:	1
X.74 X.75	6	-i	$-\frac{1}{2}$ $-1$	:		-3	$     \begin{array}{r}       -2 \\       2 \\       -1 \\       -1 \\    \end{array} $	$ \begin{array}{c} -\frac{1}{2} \\ -1 \end{array} $	1 -1			2	-1	$-\overset{i}{{1}}$			-i			
$X.76 \\ X.77$	$-1 \\ -1$	-1	-1	$-2 \\ -2$	2 2 2 2 2	$-1 \\ -1 \\ -1$	-1	-1	-1	$\begin{array}{c} \dot{2} \\ 2 \\ -2 \end{array}$	$-1 \\ -1$	i 1	-1	-1	-1	-1	1 1		1 1	:
X.69 X.70 X.71 X.72 X.73 X.74 X.75 X.76 X.77 X.78 X.79 X.80	$ \begin{array}{c} -1 \\ -1 \\ -1 \\ -1 \end{array} $	i	$-i$ $\dot{2}$		2	$     \begin{array}{r}       -1 \\       -1 \\       -1 \\       -1 \\     \end{array} $	-1		-1 -i	-2	1 1	$\frac{1}{1}$	-1	-1	i	1	1 1	1	$-1 \\ -1$	
X.80 X.81 X.82	$-\frac{2}{2}$	:		:	$-\frac{2}{2}$	-1 i	:	:	-1	:	:	-2 $-2$	:	:	:		1 1			
X.81 X.82 X.83 X.84	-i	:		:		-1 -1					i	-1	:				-i	. :	i	
$X.85 \\ X.86$	-1	:	$-\dot{2}$	:	2 2	$-1$ $\dot{2}$	-2 -2	$-\dot{2}$	i	-2 2	$-1$ $\dot{2}$	-1	i	i	:	:	-1		-1	-1 ·
X.87	2			-3	-1	- 2				-1	2							•	•	

 $Character\ table\ of\ mE\ (continued)$ 

2 3 5 7 13	1 1	2 1 1	2 i	2 i	3 2 1	1 2 1	2 1		3 4	3 4	2 4	2 4	2 4	2 4	3 3	3 3	3 3	1 4	2 3	2 3	3 2	1 3	1 3	3 1 1	3 i :
2P	13a	14a	$\frac{14b}{14b}$	$\frac{14c}{14c}$	15a 15a 5a 3a 15a 15a	$\frac{5a}{3d}$	5a 3 f	$^{66}_{18c}$	$^{9a}_{6_2}$ $^{18b}$	$^{66}_{18a}$	9c 66 18g 18d 18d	$^{66}_{18e}$	18f 9b 6 <sub>2</sub> 18f 18f 18f	$^{66}_{18d}$	$^{68}_{18i}$	$^{6_{14}}_{18i}$	$^{6_8}_{18h}$	$^{66}_{18k}$	9b 6 <sub>8</sub> 18l 18l 18l	$     \begin{array}{r}       18m \\       9c \\       6_{14} \\       18m \\       18m \\       18m \\     \end{array} $	$^{9a}_{623} \\ 18n$	$^{9e}_{610}$	$\frac{6_{11}}{18p}$	$10a \\ 20a \\ 4a$	$\frac{20b}{4c}$
X.1 X.2 X.3 X.4 X.5 X.6	1 1 2	1 1 2 1 1	i	1 -1 -1 -1	$ \begin{array}{c} 1 \\ -1 \\ 3 \\ 3 \\ 1 \\ 1 \end{array} $	1 2 1	1 -1 -1	1 2 -1 -1	$ \begin{array}{c} 1 \\ -1 \\ 3 \\ -3 \\ 2 \\ -2 \end{array} $	$\begin{array}{c} 1 \\ 1 \\ 2 \\ -1 \\ -1 \end{array}$	1 -1 -1 -1	1 1 2 2 2	-1 $-1$ $-1$ $-1$ $-1$ $-1$ $-1$ $-1$	1 -1 -1 -1	-1 -1 1	$\begin{array}{c} 1 \\ 2 \\ -1 \end{array}$	-1 -1 1	$\begin{array}{c} 1 \\ 1 \\ -1 \\ 2 \\ 2 \\ \end{array}$	-1 $-1$ $-2$ $-2$	$ \begin{array}{c} 1\\ -1\\ -1\\ -1\\ 2\\ 2 \end{array} $	$-\frac{1}{2}$	-1 -1		$ \begin{array}{c} 1\\ 2\\ -1\\ -1\\ -1\\ -1 \end{array} $	-1 -1 -1 1 -1
X.8 X.9 X.10 X.11 X.12 X.13	1 1 -1 -1	2			-3 3 3 2 2	-i	-1 -1	$\begin{array}{c} 1 \\ 1 \\ -2 \\ 2 \\ 2 \\ 1 \\ 1 \end{array}$	3 -3 -5 -5	$\begin{array}{c} 1 \\ 1 \\ -2 \\ 2 \\ 2 \\ 1 \\ 1 \end{array}$	1 1 1 2 2 1 1	$^{4}_{-1}$	3 -3 -1 1	1 1 2 2 1 1	$ \begin{array}{c} 1 \\ -1 \\ 2 \\ -2 \\ 1 \\ -1 \end{array} $	$     \begin{array}{r}       -1 \\       -1 \\       -2 \\       \hline       1     \end{array} $		$ \begin{array}{r} -2 \\ -2 \\ -2 \\ -1 \\ -1 \\ 1 \end{array} $	$\begin{array}{c} 2 \\ -1 \\ 1 \\ 1 \end{array}$	-1 -1 1	-1 1	-i	-1	-2 -1 -1	-1 1
X.14 X.15 X.16 X.17 X.18 X.19 X.20	2	-1 -1 -1 -1	1 −1	-1 -1 -1 1	-1	-1 2	-1 -1	$ \begin{array}{r} -2 \\ -2 \\ 2 \\ -1 \\ -1 \end{array} $	-i -1	$ \begin{array}{r} -2 \\ -2 \\ 2 \\ -1 \\ -1 \end{array} $	$ \begin{array}{r} -2 \\ -2 \\ -1 \\ -1 \\ -1 \end{array} $	$\begin{array}{c} 1 \\ 1 \\ -4 \\ -1 \\ -1 \end{array}$	$-\frac{3}{3}$	$ \begin{array}{r} -2 \\ -2 \\ -1 \\ -1 \\ -1 \end{array} $	$-\frac{1}{2}$	$ \begin{array}{c} 4 \\ -2 \\ -1 \\ -1 \end{array} $	$-\frac{1}{2}$ $-\frac{1}{1}$	$\begin{array}{c} 1 \\ 1 \\ 2 \\ -1 \\ -1 \end{array}$	$-1 \\ 1 \\ -1 \\ 1 \\ 1$	-2	-1 1	-1 -2	· · · · · · · · · · · · · · · · · · ·	-2	
X.21 $X.22$ $X.23$ $X.24$ $X.25$ $X.26$	-2 -2	-1 -1	-1 -1	-1 1	3 3 -3 -2	-2	1	$ \begin{array}{r} -1 \\ -1 \\ -1 \\ 4 \\ 2 \\ -4 \end{array} $	$     \begin{array}{c}       -1 \\       1 \\       3 \\       -3 \\       \vdots \\       \vdots     \end{array} $	$ \begin{array}{r} -1 \\ -1 \\ -1 \\ 4 \\ 2 \\ -4 \end{array} $	$ \begin{array}{r} -1 \\ -1 \\ -1 \\ -1 \\ -2 \\ -1 \\ 2 \end{array} $	$\frac{2}{2}$	-1 $3$ $-3$ .	$-1 \\ -2 \\ -1 \\ 2$	-1 -1 -1 1	$-1 \\ -1$	-1 $1$ $-1$ $1$ $1$ $1$	-1 $-1$ $-1$ $-1$ $-1$ $-1$	$-1 \\ -1 \\ -1 \\ 1 \\ \vdots \\ \vdots \\ \cdots$	$     \begin{array}{r}       -1 \\       -1 \\       -1 \\       -1 \\       -1 \\     \end{array} $	-1 -1 -1 1	1	-1 -2	1 1 -2	1 -1
X.28 X.29 X.30 X.31 X.32 X.33		-2 -2			$ \begin{array}{c}     -3 \\     1 \\     1 \\     -1 \\     -1 \end{array} $	1 1 -1	1 1 -1 -1	-2 -2 -2 1 1	3 -3	-2 -2 -2 1 1	1 1 1 1 1			1 1 1 1 1	1 -1		1 -1	$\begin{array}{c} 1 \\ 1 \\ -2 \\ -2 \\ \end{array}$	-2 -2 :	1 1 -1 -1	-i			$\begin{array}{c} \cdot \\ 2 \\ -1 \\ -1 \\ -1 \\ -1 \end{array}$	-1 -1 1 1 -1
X.35 X.36 X.37 X.38 X.39 X.40 X.41					-1 2 2				$     \begin{array}{r}       4 \\       -4 \\       -4 \\       -3 \\       3     \end{array} $	2 1	-1 1 1	-4 -2 -2	$     \begin{array}{r}       -2 \\       2 \\       -2 \\       2     \end{array} $	-1 1 1		-2 1 1		-2 -2 -2	-2 2	1 1		$ \begin{array}{c} 1 \\ 2 \\ -2 \\ -1 \end{array} $	$     \begin{array}{c}       -2 \\       -1 \\       1 \\       2     \end{array} $	$-\frac{1}{2}$	
X.42 X.43 X.44 X.45 X.46 X.47		1	-1	1 -1				$ \begin{array}{c} -2 \\ -1 \\ -2 \\ -1 \\ A \\ \bar{A} \end{array} $	-3 -3 -3 3	$ \begin{array}{c} -2 \\ -1 \\ -2 \\ -1 \\ \bar{A} \\ A \end{array} $	$ \begin{array}{c} -2 \\ -1 \\ -2 \\ -1 \\ A \\ \bar{A} \end{array} $	1	3 -3	$ \begin{array}{c} -2 \\ -1 \\ -2 \\ -1 \\ \bar{A} \\ A \end{array} $	$ \begin{array}{c} -2 \\ 1 \\ 2 \\ -1 \\ A \\ A \end{array} $	-i -i -1	$-2$ $1$ $2$ $-1$ $\bar{A}$ $\bar{A}$	1 2 1 2	$-\frac{1}{2}$	-i -i -1 -1	i -i -1				
X.48 X.49 X.50 X.51 X.52 X.53 X.54		1 1	-1 1	-1 -1	1 3 3	$-\dot{2}$	i	A A	-3 3	A Ā	A A	•	•	A Ā	Ā Ā		A A			-1 -1	-1 1			-2 1 1	-1
X.55 X.56 X.57 X.58 X.59 X.60		-1 -1	-i -i :	-i -i :	1 1 -1 -1	1 1 -1 -1	1 1 -1 -1	-1 -1	5 -5	-1 -1 -1	-1 -1 -1	-1 -1 -1 -1	$-1 \\ 1 \\ -3 \\ 3$		-1 1 -2 -2		$-\frac{1}{1}$ $-\frac{1}{2}$		-i 1 -1	1	-1 -1	-1 1	-1 1	1 1 1 1 1	
X.61 X.62 X.63 X.64 X.65 X.66 X.67					-2 -3 -3	-2	1	$ \begin{array}{c}     -2 \\     -2 \\     -2 \\     -1 \end{array} $	-3	2 -2 -2 -2 -1	$ \begin{array}{c} -1 \\ -2 \\ 1 \\ -2 \\ -1 \end{array} $	-4 1 4 1 2		$ \begin{array}{c} -1 \\ -2 \\ 1 \\ -2 \\ -1 \end{array} $	-2 -1	2		$\begin{array}{c} 2 \\ \cdot \\ \cdot \\ -2 \\ 1 \\ 2 \end{array}$	-i -1 1 2	-1	1			1 1	-1 -1
X.68 X.69 X.70 X.71 X.72 X.73	1	2						-4 -1	3	-1 -4 -1	$-1$ $2$ $-1$ $\dot{A}$ $\dot{A}$	:	•	-1 -1 -1	i	-2 $-2$		$-\frac{1}{2}$	-2 : : :	i 1	-i				
X.73 X.74 X.75 X.76 X.77 X.78 X.79 X.80 X.81		-2			-3		-1	-2 : :		-2	1			1				1		-i				2	
X.81 X.82 X.83 X.84 X.85 X.86 X.87	-1 -1				$     \begin{array}{r}       -2 \\       1 \\       -2 \\       1 \\       1 \\       3 \\     \end{array} $	$-\frac{1}{2}$ $\frac{1}{1}$ $\frac{1}{1}$	1 1 1 1	4 2		2 4 2 ·	$-\frac{2}{2}$	$     \begin{array}{r}       -1 \\       -2 \\       -1 \\       \cdot \\       \cdot \\       \cdot \\    \end{array} $	3 -3	-2 -2 2	-2 2	:	-2 2	-1 1 -1	1 -i					2 1 1 2	-1 -1 1

 $Character\ table\ of\ mE\ (continued)$ 

2 3 5 7 13	$\frac{1}{1}$	$\frac{4}{1}$	$_{1}^{4}$	1	1	3 1 1	2 1 1	1 1 1	i	i	$_{1}^{1}$	$\begin{smallmatrix}2\\1\\1\end{smallmatrix}$
7 13	1		:	i	i				i	i	1	
2P	$\frac{21a}{21a}$	$\frac{24a}{123}$	$\frac{24b}{122}$	26a	$\frac{26b}{13b}$	$\frac{30a}{15a}$	$\frac{30b}{15a}$	15b	$\frac{39a}{39b}$	$\frac{39b}{39a}$	21a	$\frac{60a}{30a}$
$\frac{3P}{5P}$	7a $21a$	24a	240	26b	$\frac{26b}{26a}$	$^{10a}_{63}$	10c 61	64	39b	$\frac{13b}{39a}$	$\frac{14a}{42a}$	$\frac{20a}{12_1}$
$\frac{7P}{13P}$	91a	24a	$\frac{24b}{24b}$	2a	2a	$\frac{30a}{30a}$	30b	30c	$\frac{39b}{3a}$	3a	42a	
X.1 X.2 X.3 X 4	$\begin{array}{c} 1 \\ 1 \\ -1 \end{array}$	$\begin{array}{c} 1 \\ 1 \\ -1 \end{array}$	$\begin{array}{c} 1 \\ 1 \\ -1 \end{array}$	$-1^{1}$	$-1 \\ -1$	$\begin{array}{c} 1 \\ 1 \\ -1 \end{array}$	$\begin{array}{c} 1 \\ 1 \\ -1 \end{array}$	$-1^{1}$	$\begin{array}{c} 1 \\ 1 \\ -1 \end{array}$	$-1 \\ -1$	1	1 1 -1
X.4 X.5	1 1	-1	-1			$-1 \\ -1 \\ -1$	1	:	-1	-1	$-1 \\ 1 \\ 1$	$-1 \\ -1 \\ -1$
X.6 $X.7$		$-1 \\ -1$	$-1 \\ -1$	:		1	$-1 \\ -1$	$-\frac{1}{1}$	:			$-1 \\ -1$
X.8	:	$-1 \\ -1$	$\frac{1}{1}$	$-1 \\ -1$	$-1 \\ -1$	:	:	:	$\frac{1}{1}$	1 1	:	i i
X.10 X.11 X.12 X.13	-1	:	:	-i	-i	$-\frac{1}{1}$	$-1 \\ 1 \\ 1$	:	-i	-1	-1	-1
$X.12 \\ X.13 \\ X.14$	:		:	1	1	$-1 \\ 2 \\ 2 \\ -1$	1	$-\overset{\cdot}{\overset{\cdot}{1}}$	-1 ·	-1		-1 ·
$X.15 \\ X.16$	-i	$-1^{i}$	1 1	:		$-\overline{1}$	1		:		-1	i
$X.17 \\ X.18 \\ Y.10$	-1 i	$-\frac{1}{1}$	$-1 \\ -1$	:	:	:	:	:	-i	-i	-1	:
$X.19 \\ X.20 \\ X.21$	1 1		:		:	:		:	:		$-1 \\ -1 \\ -1$	
X.22 X.23 X.24	1	1	-1	i		-i	-1		:		-1	i
X 25		1	$-1 \\ -1 \\ .$	:	:	$-1 \\ 1$	$-1 \\ -1$	:	i	i	:	$\frac{1}{1}$
X.26 X.27 X.28	1 -1	i	-i	:	:	-2		:	:		1 1	:
X.29 X.30	-1	-i	i			i	1		:		1	-i
X.29 X.30 X.31 X.32 X.33		:	:	:	:	1 1	$-1 \\ -1$	$-\frac{1}{1}$	:		:	$^{-1}_{-1}$
X.33 X.34	:	1 1	1 1	:		$-1 \\ -1$	$-1 \\ -1$	$-1^{1}$	:		:	$-1 \\ -1$
X.34 X.35 X.36 X.37 X.38 X.39		:	:	:		:			:		:	:
X.38 X.39						-i	1					i
$X.40 \\ X.41$	:			:	:	$\frac{2}{2}$		$^{-1}_{1}$	:	:	:	:
$X.42 \\ X.43 \\ X.44$		$-\frac{1}{1}$	$-\frac{1}{1}$	:		:			:		:	:
$X.45 \\ X.46$	-1	-1	-1			:		i			1	:
X.47 X.48	$-1 \\ -1$		:			:			:	:	1	
X.49 X.50 X.51	-1	$-\dot{1}$	$-\dot{1}$	:	:	-1	-1	:	:	:	1	1 1
X.52 X.53		:	:	:		-1	-1	:	:		:	1
X.52 X.53 X.54 X.55 X.56 X.57 X.58 X.59		:	:	:	:	:		:	:	:	:	:
X.50 X.57 X.58	$-1 \\ -1$		:			i 1	1 1	$-1 \\ -1$	:		$-1 \\ -1$	1 1
X.59 X.60			i			$-1 \\ -1$	1 1	$-1 \\ -1$				1 1
$X.61 \\ X.62 \\ X.63$	:	1	-i	:	:	-2	-1		:	:	:	i
X.64 X.65	:	1 i	-1 i			1	-1		:		:	1
$X.66 \\ X.67$												
X.68 X.69	:	-1 ;	-1 ;			:	:	:	i		:	:
X.70 X.71 X.72	i	-1	-1 -1	-1 1	-1 1		:		1	1	-1	:
$X.73 \\ X.74$	1	-1 -1				1	1			1	-1	-i
X.75 X.76		-1	-i	:	:	:	:	:	:	:	:	
X.78 X.79		1	$-{1\atop 1}$			-1		:		:		:
X.80 X.81	1	-1 :	1			$-\frac{1}{2}$	-1	-i	:	:	1	-i -i
X.82 X.83			:	i	i	$-\frac{1}{2}$	-1 i	_ i	-1	- i	:	-1 i
X.69 X.70 X.71 X.72 X.73 X.74 X.75 X.76 X.77 X.80 X.81 X.82 X.83 X.84 X.85 X.86 X.87		-i	i	-1	-1	1	1	-1 1	-1	-1		1 1 -1
X.87	i	-1	-1	:	:	-1 ·				:	i	

2 3 5 7	10 10 1 1 1	10 9 1 1	$^{10}_{\begin{subarray}{c}7\\1\\1\end{subarray}}$	10 6 1 1	10 5 1	10 4 1	10 4	10 3	9 10 1 1	6 10 :	5 10 :	5 8 1	7 8	4 8 1	6 8	4 8 :	4 8 :
2P 3P 5P 7P	$ \begin{array}{c c} 1a \\ 1a \\ 1a \\ 1a \\ 1a \\ 1a \end{array} $	$ \begin{array}{r} 1 \\ 2a \\ 1a \\ 2a \\ 2a \\ 2a \end{array} $	$ \begin{array}{r} 2b \\ 1a \\ 2b \\ 2b \\ 2b \\ 2b \end{array} $	$ \begin{array}{r} 2c \\ 1a \\ 2c \\ 2c \\ 2c \\ 2c \end{array} $	$ \begin{array}{c} 2d \\ 1a \\ 2d \\ 2d \\ 2d \\ 2d \end{array} $	$\begin{array}{c} 2e \\ 1a \\ 2e \\ 2e \\ 2e \\ 2e \end{array}$	$\begin{array}{c} 2f \\ 1a \\ 2f \\ 2f \\ 2f \\ 2f \end{array}$	$\begin{array}{c} 2g \\ 1a \\ 2g \\ 2g \\ 2g \\ 2g \end{array}$	$ \begin{array}{r} 3a \\ 3a \\ 1a \\ 3a \\ 3a \\ 3a \end{array} $	3b 3b 1a 3b 3b 3b	3c $3c$ $1a$ $3c$ $3c$	3d 3d 1a 3d 3d 3d	3e 3e 1a 3e 3e 3e	$ \begin{array}{c} 3f \\ 3f \\ 1a \\ 3f \\ 3f \end{array} $	3g 3g 1a 3g 3g 3g	3h 3h 1a 3h 3h	$ \begin{array}{c} 3i\\ 3i\\ 1a\\ 3i\\ 3i\\ 3i \end{array} $
13P X.88 X.89 X.90 X.91 X.92 X.93 X.94 X.95 X.96 X.97	5670	$\begin{array}{r} 2a \\ -5265 \\ 5460 \\ 5460 \\ -5460 \\ -5460 \\ -5460 \\ -5460 \\ \end{array}$	$\begin{array}{r} 2b \\ 225 \\ -140 \\ 420 \\ -140 \\ 420 \\ 980 \\ -700 \\ -140 \\ -700 \\ 630 \\ \end{array}$	$ \begin{array}{r} 2c \\ -225 \\ -140 \\ 420 \\ -420 \\ \hline 700 \\ 140 \\ -700 \\ \end{array} $	$-140 \\ 100 \\ 150$	140 100	$ \begin{array}{r} 2f \\ 33 \\ 20 \\ 20 \\ -12 \\ 20 \\ 52 \\ -12 \\ 20 \end{array} $	$ \begin{array}{c} 2g \\ -33 \\ 20 \\ 20 \\ -20 \\ -12 \\ -12 \\ -12 \\ -12 \\ -12 \end{array} $	$\begin{array}{r} 3a \\ 5265 \\ 5460 \\ 5460 \\ -2730 \\ 5460 \\ -2730 \\ 5460 \\ 5460 \\ -2835 \end{array}$	$ \begin{array}{r} 3b \\ 162 \\ -129 \\ 33 \\ 276 \\ 33 \\ -48 \\ 114 \\ -129 \\ 114 \\ -162 \end{array} $	$ \begin{array}{r} 3c \\ 162 \\ -129 \\ 33 \\ -138 \\ 33 \\ 24 \\ 114 \\ -129 \\ 114 \\ 81 \end{array} $	3d -30 -30 -30 150 60 60 60	3e 81 60 96 42 96 -30 6 60 61 62	$ \begin{array}{r} 3f \\ 60 \\ -30 \\ 15 \\ -30 \\ -75 \\ 60 \\ 60 \\ 60 \\ . \end{array} $	3 <i>q</i> 81 60 96 -21 96 15 6 60 6 -81	$ \begin{array}{c} 3h \\                                   $	$ \begin{array}{c} 3i \\ 6 \\ -21 \\ -48 \\ -21 \\ 6 \\ -21 \\ 6 \\ -21 \end{array} $
X.98 X.100 X.101 X.102 X.103 X.104 X.105 X.106 X.107 X.108 X.107	5824 5824 5824 5824 6552 6552 7020 7020 7020 7280 7280	-5824 5824 -5824 -6552 6552 -7020 -7020 7020 7020 -7280 7280	896 896 -896 -896 -504 -504 540 540 540 -560 -560	-896 896 -896 504 -504 -540 540 540 560 -560	64 64 64 -24 -24 60 60 60 80 80	$     \begin{array}{r}       -64 \\       64 \\       64 \\       -64 \\       24 \\       -60 \\       -60 \\       60 \\       60 \\       -80 \\       80 \\    \end{array} $		8 -8 -12 -12 12 12 16 -16	5824 5824 5824 5824 6552 7020 7020 7020 7020 7280 7280	$     \begin{array}{r}       -8 \\       -8 \\       -8 \\       234 \\       234 \\       -27 \\       -27 \\       -27 \\       -10 \\       -10 \\    \end{array} $	$     \begin{array}{r}       -8 \\       -8 \\       -8 \\       234 \\       234 \\       -27 \\       -27 \\       -27 \\       -10 \\       -10 \\    \end{array} $	154 154 154 154 45 45  -10 -10	$     \begin{array}{r}     -8 \\     -8 \\     -8 \\     -8 \\     72 \\     72 \\     -54 \\     -54 \\     -54 \\     -64 \\     -64 \\     -64 \\   \end{array} $	154 154 154 154 45 45 	-54 $-54$ $-64$	$     \begin{array}{r}       -8 \\       -8 \\       -8 \\       -8 \\       18 \\       18 \\       54 \\       -27 \\       -27 \\       -27 \\       -10 \\       -10 \\     \end{array} $	
X.1109 X.1111 X.1112 X.113 X.114 X.115 X.116 X.117 X.118 X.119 X.120	7280 7280 7280 7280 7371 7371 8190 8190 8190 8190	7280 7280 7280 7280 7371 -7371 -8190 8190 -8190	$ \begin{array}{r} -360 \\ -560 \\ -560 \\ -560 \\ -560 \\ 819 \\ 819 \\ -210 \\ -210 \\ 630 \\ -210 \\ -210 \end{array} $	-560 -560 -560 -560 -819 -819 -210 -210 -210	80 80 80 80 51 51 30 30 -90 30	-80 80 -80 80 51 -51 -30 30	$ \begin{array}{r} -16 \\ -16 \\ -16 \\ -16 \\ -21 \\ -21 \\ -18 \\ -18 \\ -50 \end{array} $	$     \begin{array}{r}       -16 \\       16 \\       -16 \\       16 \\       -16 \\       -21 \\       21 \\       18 \\       -18 \\       -18 \\       18 \\   \end{array} $	7280 7280 7280 7280 7371 7371 8190 8190 -4095 8190 8190	-10 -10 -10 -10 -10 81 81 171 171 414 171	-10 $-10$ $-10$ $-10$ $-10$ $81$ $81$ $171$ $171$ $-207$ $171$ $171$	-10 $-10$ $-10$ $-10$ $-10$ $81$ $81$ $-45$ $-45$ $90$ $-45$ $-45$	$     \begin{array}{r}       -64 \\       -64 \\       152 \\       152 \\       81 \\       81 \\       -45 \\       -45 \\       -45 \\       -45 \\     \end{array} $	-10 $-10$ $-10$ $-10$ $-10$ $81$ $81$ $-45$ $-45$ $-45$ $-45$	$     \begin{array}{r}     -64 \\     -64 \\     152 \\     152 \\     81 \\     81 \\     -45 \\     -45 \\   \end{array} $	$\begin{array}{c} 44 \\ 44 \\ -10 \\ -10 \\ \end{array}$	$-10 \\ -10$
X.121 X.122 X.123 X.124 X.125 X.126 X.127 X.128 X.129 X.130	8190 8736 9072 10530 10920 10920 11648 11648 11648	11648	$ \begin{array}{r} -210 \\ -1050 \\ 1120 \\ -1008 \\ 450 \\ 840 \\ -1400 \\ -280 \\ -1792 \\ 1792 \\ . $	210	150 $96$ $-48$ $-30$ $200$ $200$ $-280$ $128$ $128$ $128$	128	14 32 48 66 40 -24 40		$\begin{array}{c} -4095 \\ -4368 \\ -4536 \\ -5265 \\ -5460 \\ -5460 \\ -5460 \\ -5824 \\ -5824 \\ 11648 \end{array}$	$     \begin{array}{r}       -72 \\       312 \\       324 \\       324 \\       66 \\       228 \\       -258 \\       -16 \\       -16 \\       -16 \\    \end{array} $	$ \begin{array}{r}   36 \\   -156 \\   -162 \\   -162 \\   -33 \\   -114 \\   129 \\   8 \\   -16 \end{array} $	$   \begin{array}{r}     90 \\     132 \\     162 \\     \hline     -60 \\     120 \\     120 \\     308 \\     308 \\     -124 \\   \end{array} $	144 -48	$     \begin{array}{r}     -45 \\     -66 \\     -81   \end{array} $ $     \begin{array}{r}     30 \\     -60 \\     -60 \\     -154 \\     -154   \end{array} $	$ \begin{array}{r} -72 \\ 24 \\ -81 \\ -96 \\ -60 \\ -60 \\ 8 \\ -16 \end{array} $	$ \begin{array}{r} 36 \\ -12 \\ -42 \\ -42 \\ -16 \\ -16 \\ -16 \\ -16 \\ -16 \\ -16 \end{array} $	36 -12 -42 -42 12 -16
X.133 X.134 X.135 X.136 X.137 X.138 X.139 X.140	14040 $14040$ $14560$ $14560$ $14560$ $14742$ $14742$	-11648   -14742 14742	-1008 $1080$ $1080$ $-1120$ $-1120$ $-1120$ $630$ $630$ $1638$ $-420$	-630 630	$   \begin{array}{c}     128 \\     -48 \\     120 \\     160 \\     160 \\     160 \\     102 \\     102 \\     102 \\     60 \\   \end{array} $	-128 -102 102	$ \begin{array}{r} -16 \\ 24 \\ 24 \\ -32 \\ -32 \\ -32 \\ 6 \\ 6 \\ -42 \\ -36 \end{array} $	-6 6	$\begin{array}{c} 11648 \\ -6552 \\ -7020 \\ -7020 \\ -7280 \\ -7280 \\ -7280 \\ 14742 \\ 14742 \\ -7371 \\ -8190 \end{array}$	$     \begin{array}{r}       -16 \\       468 \\       -54 \\       -20 \\       -20 \\       -20 \\       162 \\       162 \\       342 \\    \end{array} $	$     \begin{array}{r}     -16 \\     -234 \\     27 \\     27 \\     10 \\     10 \\     10 \\     162 \\     162 \\     -81 \\     -171 \\   \end{array} $	-20 -	$ \begin{array}{r} 144 \\ -108 \\ -108 \end{array} $	-45 10 10	$-72 \\ 54 \\ 54$	$^{36}_{-54}$	-16 36 108 -54 88 -20 -20
$X.148 \\ X.149 \\ X.150$	16380 16380 16640 16640 16640 17472 17472 17472	16640	$     \begin{array}{r}     -420 \\     -420 \\     -420 \\     -1280 \\     1280 \\     -1280 \\     1280 \\     896 \\     -896 \\     -896 \\     896   \end{array} $	$\begin{array}{c} 420 \\ -420 \\ \hline \\ -1280 \\ 1280 \\ 1280 \\ -1280 \\ -896 \\ -896 \\ 896 \\ 896 \end{array}$	60 60 60  -64 -64 -64	-60 60	28 28 -36	-28 28	16380 16380 -8190 16640 16640 16640 17472 17472 17472 17472	99 99 342 -208 -208 -208 -24 -24 -24 -24 -24	$     \begin{array}{r}       99 \\       99 \\       -171 \\       -208 \\       -208 \\       -208 \\       -24 \\       -24 \\       -24 \\       -24 \\       -24 \\       -24 \\       -24 \\     \end{array} $	-90 -90 -90 80 80 80 80 30 30 30 -80	$ \begin{array}{r} -36 \\ -36 \\ -90 \\ -64 \\ -64 \\ -64 \\ -24 \\ -24 \\ -24 \\ -24 \\ 64 \end{array} $	-90 -90 45 80 80 80 30 30 30 -80	-24 - 24	$     \begin{array}{r}       18 \\       18 \\       72 \\       \hline       8 \\       8 \\       8 \\       8 \\       -24 \\       \hline       -24 \\       \hline       -24 \\       \hline       -24 \\       \hline       -8 \\     \end{array} $	18 18 -90 8 8 8 8 -24 -24 -24 -24 -24
X.154 X.155 X.156 X.157 X.158 X.159	17920 17920 17920 19683 19683 21840	$\begin{array}{c} 17920 \\ -17920 \\ -17920 \\ 19683 \\ -19683 \\ -21840 \end{array}$	-729 -729 560	-729 729 -560	-81 -81 -80 -80		-48	27 -27 48	17920 17920 17920 19683 19683 21840	-224 - 224	-224 -224 -224 -30	-80 -80 -80	64 64 64	-80 -80 -80 -80 -30	64 64 64	-8 -8 -8	-8 -8 -8
X.161	22113 23296 29484 32760 33280 33280 34944 34944 35840 35840 39366		560 189 189 1260 -840 2560 -2560 1792 -1792 -1458 1120	-189 189	-171	-80 171 -171 	9 9 12 56	-9 9	$\begin{array}{c} 21840 \\ 22113 \\ 22113 \\ -11648 \\ -14742 \\ -16380 \\ -16640 \\ -17472 \\ -17472 \\ -17920 \\ -17920 \\ -17920 \\ -19683 \\ -21840 \end{array}$	243 $243$ $-32$ $324$ $198$ $-416$ $-416$ $-48$ $-48$ $-448$	243 $243$ $16$ $-162$ $-99$ $208$ $24$ $24$ $24$ $224$ $224$	-162 $-180$ $160$ $160$ $60$	-72	124 81 90 -80 -80 -30 -30 80 80	81 36 64 64 24		36 16 16 -48 -48 -16
X.174	44226		378		-342		18	•	-22113	486	-243						

2 3 5 7	3 8	3 8	3 7	2 7 :	2 7 :	1 7	7 3 1	8 3	7 2 1	8 2	8 2	7 2	8 1	7 1	$\begin{smallmatrix} 4\\2\\1\\. \end{smallmatrix}$	9 7 1 1	6 9	9 5 1	5 7 1	7 7	6 7 :
2P 3P 5P 7P 13P	3j 3j 1a 3j 3j 3j	3k 3k 1a 3k 3k 3k 3k	3l 3l 1a 3l 3l 3l	3m 3m 1a 3m 3m 3m	$ \begin{array}{c} 3n \\ 3n \\ 1a \\ 3n \\ 3n$	30 30 1a 30 30 30	$\begin{array}{c} 4a \\ 2d \\ 4a \\ 4a \\ 4a \\ 4a \end{array}$	$\begin{array}{c} 4b \\ 2f \\ 4b \\ 4b \\ 4b \\ 4b \end{array}$	$\begin{array}{c} 4c \\ 2d \\ 4c \\ 4c \\ 4c \\ 4c \\ 4c \end{array}$	$\begin{array}{c} 4d \\ 2f \\ 4d \\ 4d \\ 4d \\ 4d \\ 4d \end{array}$	4e	4f	$\begin{array}{c} 4g \\ 4g \\ 4g \end{array}$	4h	5a 5a 5a 1a 5a 5a	$ \begin{array}{r} 6_1 \\ 3_a \\ 2_b \\ 6_1 \\ 6_1 \\ 6_1 \end{array} $	$\begin{array}{r} 6_2 \\ 3b \\ 2a \\ 6_2 \\ 6_2 \\ 6_2 \end{array}$	6 <sub>3</sub> 3a 2d 6 <sub>3</sub> 6 <sub>3</sub>	$\begin{array}{r} 6_4 \\ 3d \\ 2a \\ 6_4 \\ 6_4 \\ 6_4 \end{array}$	65 3e 2a 65 65 65	66 3b 2b 66 66 66
X.88 X.89 X.90 X.91 X.92 X.93	6 -21 24 -21 -3 -21	$ \begin{array}{r}                                     $	15 -3 -12 -3 24 6	$ \begin{array}{r}     -3 \\     -3 \\     6 \\     -3 \\     6 \end{array} $	15 -3 6 -3 -12 6	-3 -3 -3 -3	5 -20 20	-3 12 4 12 4 -4 -4	-5	3 12 4 -4 -4	-3 -4 4 12 4 -4 -4	-3 -4 -4	3 -4 4 -4	3		$ \begin{array}{r} 225 \\ -140 \\ 420 \\ 70 \\ 420 \\ -490 \end{array} $	$ \begin{array}{r} -162 \\ -129 \\ 33 \\ -33 \\ -114 \end{array} $	$     \begin{array}{r}       -15 \\       -140 \\       100 \\       -10 \\       100 \\       -90 \\       100 \\    \end{array} $	60 -30 30 -60	-81 60 96 -96 -6	$ \begin{array}{r}     -38 \\     -5 \\     -39 \\     76 \\     -39 \\     8 \\     2 \end{array} $
X.94 $X.95$ $X.96$ $X.97$ $X.98$ $X.99$	-21 -21 -8 -8 -8	-21 -21 -8 -8 -8	15 6 -8 -8	$ \begin{array}{r}     6 \\     -3 \\     6 \\     -8 \\     -8 \\     -8 \end{array} $	15 6 -8 -8	$ \begin{array}{c}     6 \\     -3 \\     6 \\     -8 \\     -8 \\     -8 \end{array} $	-10 16 16 16 -16	12 -4 6	-16 16 -16	-12 -4	-4 -4 -4 -10	6	4 -4		-1 -1 -1	-700 -140 -700 -315 896 896 -896	129 114 8 -8 -8	-140 $-140$ $100$ $-75$ $-64$ $-64$	-60 $-60$ $-154$ $154$ $154$	-60 6	$ \begin{array}{r} -5 \\ 2 \\ -18 \\ 32 \\ 32 \\ -32 \end{array} $
X.101 $X.102$ $X.103$ $X.104$ $X.105$ $X.106$	$     \begin{array}{r}       -8 \\       18 \\       18 \\       54 \\       -27 \\       -27 \\     \end{array} $	$     \begin{array}{r}       -8 \\       18 \\       18 \\       -27 \\       54 \\       54     \end{array} $	-8 : :	-8 9 9	-8 : :	-8 9 9	-16 4 4	4 4 4	16 -4 4	-4 -4 4	4 4 4	4 4	-4 -4 4		$     \begin{array}{r}       -1 \\       -3 \\       -3 \\       \vdots \\     \end{array} $	$   \begin{array}{r}     -896 \\     -504 \\     -504 \\     540 \\     540 \\     540   \end{array} $	$     \begin{array}{r}       8 \\       -234 \\       234 \\       27 \\       27 \\       -27 \\     \end{array} $	$     \begin{array}{r}       64 \\       -24 \\       -24 \\       60 \\       60 \\       60    \end{array} $	-154 -45 45	$     \begin{array}{r}       8 \\       -72 \\       72 \\       54 \\       54 \\       -54 \\     \end{array} $	$ \begin{array}{r} -32 \\ -18 \\ -18 \\ -27 \\ -27 \\ -27 \end{array} $
X.107 X.108 X.109 X.110 X.111 X.112 X.113	$     \begin{array}{r}       54 \\       -10 \\       -10 \\       44 \\       -10 \\       -10     \end{array} $	-27 $44$ $44$ $-10$ $-10$ $-10$ $-10$	8 8 8 8 -10 -10	$ \begin{array}{r} -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -10 \\ -10 \end{array} $	8 8 8 8 -10 -10	$ \begin{array}{r} -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -10 \\ -10 \end{array} $		4		4	4		4			540 $-560$ $-560$ $-560$ $-560$ $-560$ $-560$	$     \begin{array}{r}       -27 \\       10 \\       -10 \\       10 \\       -10 \\       10 \\       -10 \\     \end{array} $	80 80 80 80 80 80 80	$ \begin{array}{c}     10 \\     -10 \\     10 \\     -10 \\     10 \\     -10 \\     -10 \end{array} $	$     \begin{array}{r}     -54 \\     64 \\     -64 \\     64 \\     -64 \\     -152 \\     152     \end{array} $	-27 34 34 34 34 34 34
X.114 $X.115$ $X.116$ $X.117$ $X.118$ $X.119$ $X.120$	-45 $-45$ $9$ $36$ $36$	36 36 36 9 -45 -45	-9 -9 -9 -9	18	-9 -9 -9 -9		$     \begin{array}{r}       -1 \\       -1 \\       10 \\       10 \\       10 \\       10 \\       10 \\     \end{array} $	$   \begin{array}{c}     -9 \\     -9 \\     2 \\     2 \\     2 \\     2   \end{array} $	$ \begin{array}{c} -1 \\ 1 \\ -10 \\ 10 \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ .$	$   \begin{array}{c}     -9 \\     9 \\     -2 \\     2 \\     \hline     -2 \\     -2   \end{array} $	$ \begin{array}{c} -1 \\ -1 \\ 2 \\ 2 \\ -2 \\ 2 \\ 2 \end{array} $	$     \begin{array}{r}       -1 \\       -1 \\       2 \\       2 \\       2 \\       2     \end{array} $	$ \begin{array}{c} -1 \\ 1 \\ -2 \\ 2 \\ -2 \\ -2 \end{array} $	$ \begin{array}{c} -1 \\ 1 \\ -2 \\ 2 \\ -2 \\ -2 \end{array} $	1	$\begin{array}{c} 819 \\ 819 \\ -210 \\ -210 \\ -315 \\ -210 \\ -210 \end{array}$	$     \begin{array}{r}       81 \\       -81 \\       -171 \\       171 \\       \hline       . \\       171 \\       -171     \end{array} $	51 30 30 45 30 45	$     \begin{array}{r}       81 \\       -81 \\       45 \\       -45 \\       &                             $	$     \begin{array}{r}       81 \\       -81 \\       45 \\       -45 \\       \end{array} $	$   \begin{array}{c}     9 \\     9 \\     -21 \\     -21 \\     -18 \\     -21 \\     -21 \\     -21   \end{array} $
X.121 $X.122$ $X.123$ $X.124$ $X.125$ $X.126$	$ \begin{array}{c} -18 \\ 6 \\ \vdots \\ 21 \\ 21 \end{array} $	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c} 18 \\ -12 \\ \vdots \\ -6 \\ 12 \end{array} $	6 -6 12	-9 6	-3 -3 -6	10 -8 10	-18 -6 -8 -8	-10	-2	6 -6 8 -8	2 -8 -6	-2-	- 2	-4 2	525 $ -560 $ $ 504 $ $ -225 $ $ -420 $ $ 700$	-171	$     \begin{array}{r}       -75 \\       -48 \\       24 \\       15 \\       -100 \\       -100     \end{array} $		45	$     \begin{array}{r}       -24 \\       40 \\       -36 \\       -36 \\       -78 \\       4     \end{array} $
X.127 X.128 X.129 X.130 X.131 X.132 X.133	-16 -	$ \begin{array}{r} -6 \\ 8 \\ 8 \\ -16 \\ -16 \\ -18 \\ -54 \end{array} $	30 -16 -16 20 20	$ \begin{array}{r} -6 \\ -16 \\ -16 \\ 2 \\ 2 \\ 18 \end{array} $	-15 8 8 20 20	3 8 8 2 -9	-32 32	24			-8	8			-2 -2 -2 -2 -6	140 896 -896	-16 16	$     \begin{array}{r}       140 \\       -64 \\       -64 \\       128 \\       128 \\       -60 \\     \end{array} $	-124 124	-16 16	$     \begin{array}{r}       -10 \\       -64 \\       64 \\       &                             $
X.134 $X.135$ $X.136$ $X.137$ $X.138$ $X.139$	$ \begin{array}{r} -54 \\ 10 \\ -44 \\ 10 \end{array} $	27 -44 10 10	16 16 -20	-2 -2 -20	-8 -8 10	1 1 10	-10 -10	-6 -6			8 -6 -6	-2 -2			· · · · · · · · · · · · · · · · · · ·	-540 560 560 560 630 630	-162 162	$     \begin{array}{r}     -60 \\     -80 \\     -80 \\     -80 \\     102 \\     102   \end{array} $	81 -81	81 -81	$     \begin{array}{r}       -54 \\       68 \\       68 \\       68 \\       -18 \\       -18     \end{array} $
X.140 $X.141$ $X.142$ $X.143$ $X.144$ $X.145$ $X.146$	45 18 18 -36 8	18 18 45 8	$ \begin{array}{r} -18 \\ -9 \\ -9 \\ -18 \\ -10 \\ -10 \end{array} $	-9 -9 -8 8	9 -9 -9 9 -10 -10	-9 -9 -8 8	-2 20 20	-18 4 12 12 4	•	-12 12	$ \begin{array}{c} -2 \\ 4 \\ -4 \\ -4 \\ 4 \end{array} $	-2 4 4	4 -4			$     \begin{array}{r}       -819 \\       210 \\       -420 \\       -420 \\       210 \\       -1280 \\       1280 \\   \end{array} $	-99 99 -208 -208	-51 -30 60 60 -30	90 -90 -80 80	$\begin{array}{r} 36 \\ -36 \\ -64 \\ -64 \end{array}$	$     \begin{array}{r}       18 \\       -42 \\       39 \\       39 \\       -42 \\       16 \\       -16 \\    \end{array} $
X.147 $X.148$ $X.149$ $X.150$ $X.151$ $X.152$	$   \begin{array}{r}     8 \\     8 \\     -24 \\     -24 \\     -24 \\     -24   \end{array} $	$   \begin{array}{r}     8 \\     8 \\     -24 \\     -24 \\     -24 \\     -24   \end{array} $	$ \begin{array}{r} -10 \\ -10 \\ 12 \\ 12 \\ 12 \\ 12 \end{array} $	$   \begin{array}{r}     8 \\     8 \\     -6 \\     -6 \\     -6 \\     -6   \end{array} $	$ \begin{array}{r} -10 \\ -10 \\ 12 \\ 12 \\ 12 \\ 12 \\ 12 \end{array} $	8 -6 -6 -6	-16 16 16 -16		16 16 16 -16 -16						-3 -3 -3 -3	-1280 1280 896 -896 -896 896	208 208 24 -24 -24 -24 -24	-64 -64 -64	$     \begin{array}{r}     -80 \\     -80 \\     -30 \\     30 \\     -30 \\     30   \end{array} $	64 $64$ $24$ $-24$ $24$ $-24$	$ \begin{array}{r} 16 \\ -16 \\ 32 \\ -32 \\ -32 \\ 32 \end{array} $
X.153 X.154 X.155 X.156 X.157 X.158 X.159		-8 -8 -8 -8	-8 -8 -8 -8	10 10 10 10 	-8 -8 -8 -8		9 9	-9 -9		-9 9	3 3	-3 -3	3 - -3	-3 -3	3 3	-729 -729 560	-224 $224$ $224$		:	64 64 -64 -64 -24	-34
X.160 X.161 X.162 X.163 X.164 X.165	24  16 -18	24  16 -18 -8	40 -18	$-18^{\circ}$	-20 9	$-\frac{1}{9}$	-9 -9 -20	$ \begin{array}{r}  -3 \\  -3 \\  -3 \\  -12 \\  24 \end{array} $	9 -9		1 1 -12 -8	$\begin{array}{c} \vdots \\ 3 \\ 3 \\ -4 \\ \vdots \end{array}$	-i -	-3 3	3 3 -4 4	560 189 189 -630 420	-30 -243 243	$     \begin{array}{r}     -80 \\     -171 \\     -171 \\     -128 \\     -102 \\     -60   \end{array} $	-30 : : :		$ \begin{array}{r} -34 \\ 27 \\ 27 \\ -36 \\ 78 \\ 22 \\ \end{array} $
X.166 $X.167$ $X.168$ $X.169$ $X.170$ $X.171$ $X.172$	$     \begin{array}{r}       -8 \\       24 \\       24 \\       8 \\       8     \end{array} $	-8 24 24 8 8	-20 24 24 -16 -16		$ \begin{array}{c} 10 \\ -12 \\ -12 \\ 8 \\ 8 \end{array} $	$ \begin{array}{r}     6 \\     6 \\     -10 \\     -10 \end{array} $		-18			6	-6				1280 -896 896		64 64		•	64 -64
$X.173 \\ X.174$			12	-24	-6 ·	12	$-18^{\circ}$	$-\dot{6}$	:	:	$\dot{2}$	6	:	:	$\dot{6}$	$-560 \\ -189$	:	80 171	:	:	$^{-68}_{54}$

2 3 5 7	5 7	6 6	9 4	4 7	$\begin{array}{c} 4 \\ 7 \\ \vdots \end{array}$	5 6	5 6	6 5	4 6	4 6	7 4	5 5	5 5	5 5	3 6	6 4	6 4 :	6 4 :	6 4	7 3	2 6	5 4 ·
2P 3P 5P	$\begin{array}{c} 6_7 \\ 3_c \\ 2_b \\ 6_7 \end{array}$	$\frac{68}{3b}$ $\frac{2c}{68}$	69 3a 2f 69	$\frac{6_{10}}{3h}$ $\frac{3h}{2a}$ $6_{10}$	$\begin{array}{c} 6_{11} \\ 6_{11} \\ 2a \\ 6_{11} \end{array}$	$\begin{array}{c} \frac{1}{6_{12}} \\ \frac{3d}{2b} \\ 6_{12} \end{array}$	$\begin{array}{c} 6_{13} \\ 3e \\ 2b \\ 6_{13} \end{array}$	$\frac{6_{14}}{3b}$ $\frac{3b}{2d}$ $6_{14}$	$\begin{array}{r} 6_{15} \\ 3g \\ 2b \\ 6_{15} \end{array}$	$\frac{6_{16}}{3f}$ $\frac{3f}{2b}$ $6_{16}$	$\frac{6_{17}}{3e}$ $\frac{3e}{2f}$ $6_{17}$	$\begin{array}{c} 6_{18} \\ 3e \\ 2c \\ 6_{18} \end{array}$	$\begin{array}{c} 6_{19} \\ 3c \\ 2d \\ 6_{19} \end{array}$	$\begin{array}{c} 6_{20} \\ 3d \\ 2c \\ 6_{20} \end{array}$	$\begin{array}{c} \frac{621}{3l} \\ 2a \\ 621 \end{array}$	$\begin{array}{c} 6_{22} \\ 3e \\ 2d \\ 6_{22} \end{array}$	$\begin{array}{r} 6_{23} \\ 3_{0} \\ 2_{0} \\ 6_{23} \\ \end{array}$	$\frac{624}{3b}$ $2f$ $624$			$6_{27} 6$	$\frac{.}{28}$ $3c$ $2f$ $28$
7P 13P X.88 X.89 X.90	$\begin{array}{r} 6_{7}^{\prime} \\ 6_{7}^{\prime} \\ -18 \\ -5 \\ -39 \end{array}$	$     \begin{array}{r}       68 \\       \hline       68 \\       \hline       18 \\       -5 \\       -39     \end{array} $	69 69 33 20 20	$\begin{array}{c} 6_{10} \\ 6_{10} \\ \vdots \\ -21 \end{array}$	$\begin{array}{c} 6_{11} \\ 6_{11} \\ \vdots \\ -21 \end{array}$	$6_{12}$	$     \begin{array}{r}       6_{13} \\       6_{13} \\       9 \\       4 \\       6     \end{array} $	$     \begin{array}{r}       6_{14} \\       6_{14} \\       \hline       -6 \\       -5 \\       1     \end{array} $	$6_{15} \\ 6_{15} \\ 9 \\ 4 \\ 6$	$     \begin{array}{r}       6_{16} \\       6_{16} \\       \hline       -18 \\       4 \\       6     \end{array} $	$     \begin{array}{r}       6_{17} \\       6_{17} \\       9 \\       -4 \\       8     \end{array} $	$     \begin{array}{r}       6_{18} \\       6_{18} \\       \hline       -9 \\       4 \\       6     \end{array} $	$     \begin{array}{r}       6_{19} \\       6_{19} \\       \hline       -6 \\       -5 \\       1     \end{array} $	$620 \\ 620 \\ 18 \\ 4 \\ 6$	$6_{21} \\ 6_{21} \\ \vdots \\ 15 \\ -3$	$     \begin{array}{r}       6_{22} \\       6_{22} \\       \hline       -3 \\       4 \\       4     \end{array} $	$6_{23} \\ 6_{23} \\ 6 \\ -5 \\ 1$	624 $624$ $6$ $-1$ $-7$	$     \begin{array}{r}       6_{25} \\       6_{25} \\       9 \\       -4 \\       8     \end{array} $	$     \begin{array}{r}       6_{26} \\       6_{26} \\       \hline       -9 \\       -4 \\       8     \end{array} $	$\begin{array}{c} 6_{27}^{-7} & 6 \\ 6_{27} & 6 \\ \hline -3 & -3 \\ -3 & -3 \end{array}$	28 28 6 -1 -7
X.91 X.92 X.93 X.94 X.95	$     \begin{array}{r}       -38 \\       -39 \\       -4 \\       2 \\       -5     \end{array} $	5	$ \begin{array}{r} 6 \\ 20 \\ -26 \\ -12 \\ 20 \end{array} $	2i 2i -6	-6	6	$-14 \\ 6 \\ -10 \\ -16 \\ 4$	-5	$^{-16}_{\ \ 4}$	$^{6}_{-13}_{-16}$	$     \begin{array}{r}     -6 \\     8 \\     -14 \\     6 \\     -4   \end{array} $	-6 $16$ $-4$	$-10 \\ 1 \\ 10 \\ -5$	-6 $16$ $-4$	$\begin{array}{c} \dot{3} \\ -6 \\ -15 \end{array}$	$ \begin{array}{r} 2 \\ 4 \\ -6 \\ -2 \\ 4 \end{array} $	$-1 \\ -10 \\ 5$	$     \begin{array}{r}       12 \\       -7 \\       -8 \\       -6 \\       -1     \end{array} $	$\begin{array}{c} 3 \\ 8 \\ 7 \\ 6 \\ -4 \end{array}$	$-\frac{1}{8}$ $-\frac{1}{6}$	3 - -6 - 3 -	$     \begin{array}{r}       -6 \\       -7 \\       4 \\       -6 \\       -1     \end{array} $
X.96 X.97 X.98 X.99 X.100	2 9 32 - 32 -32 -	-32 -32 -32	-12 -3 :	-21	-21		-16 $-18$ $-4$ $-4$ $4$	10 6 -8 -8 -8	-16 9 $-4$ $-4$ 4	$-16 \\ -18 \\ -4 \\ -4 \\ 4$	6 18	$-16$ $\begin{array}{c} & 1 \\ 4 \\ -4 \\ 4 \end{array}$	10 -3 -8 -8 -8	-16 -4 -4 4	6 -8 -8	$     \begin{array}{r}       -2 \\       -6 \\       4 \\       4 \\       4   \end{array} $	10	-6 6	6 -9	6	6 - 8 -8 -8	-6 -3
X.101 X.102 X.103 X.104 X.105	$-27 \\ -27$	$     \begin{array}{r}       32 \\       18 \\       -18 \\       27 \\       27     \end{array} $	$-8 \\ 12 \\ 12$	$^{8}_{18}$ $^{18}$ $^{-54}$ $^{27}$	$^{8}_{18}$ $^{18}_{27}$ $^{-54}$		$^{4}_{-18}$ $^{-18}$	$     \begin{array}{r}     -8 \\     -6 \\     -6 \\     -3 \\     -3   \end{array} $	$^{4}_{-18}$ $^{-18}$ $^{:}$	4 9 9	$     \begin{array}{r}       -8 \\       -8 \\       -6 \\       -6     \end{array} $	$^{-4}_{18}$ $^{-18}$	$     \begin{array}{r}     -8 \\     -6 \\     -3 \\     -3 \\     -3 \\     -3     \end{array} $	-4 -9 9	8	-6 -6	$^{8}_{6}$ $^{6}_{3}$ $^{3}$	$ \begin{array}{r} -2 \\ -2 \\ -3 \\ -3 \end{array} $	$ \begin{array}{r} -8 \\ -8 \\ -6 \\ -6 \end{array} $	$-\frac{8}{6}$		-2 -2 -3 -3
X.106 X.107 X.108 X.109 X.110	-27 - 34 - 34 - 34 -	$^{34}_{-34}$	$ \begin{array}{r} 12 \\ -16 \\ -16 \\ -16 \end{array} $	$-10 \\ -44$	$     \begin{array}{r}       54 \\       -27 \\       -44 \\       44 \\       10     \end{array} $	$ \begin{array}{c}     -2 \\     -2 \\     -2 \\     -2 \end{array} $	$-2 \\ -2$	$     \begin{array}{r}       -3 \\       -3 \\       -10 \\       -10 \\       -10     \end{array} $	$ \begin{array}{c}     -2 \\     -2 \\     -2 \\     -2 \end{array} $	$ \begin{array}{c}     -2 \\     -2 \\     -2 \\     -2 \end{array} $	-6 -6 8 8	$-\frac{2}{2}$	$-10 \\ -10 \\ -10$	-2 -2 2	-8 -8 -8	$     \begin{array}{r}     -6 \\     -6 \\     -4 \\     -4 \\     -4   \end{array} $	$     \begin{array}{r}       -3 \\       -3 \\       10 \\       -10 \\       10     \end{array} $	$     \begin{array}{r}       -3 \\       -3 \\       2 \\       2 \\       2     \end{array} $	-6 -6 8 8	$     \begin{array}{r}       -6 \\       -8 \\       8 \\       -8     \end{array} $	-1 -1 1	-3 -3 2 2
X.111 $X.112$ $X.113$ $X.114$ $X.115$	34 9 9	-34 34 9 -9	$-21 \\ -21$	-10 -10		$     \begin{array}{r}       -2 \\       -2 \\       -2 \\       9 \\       9     \end{array} $	$     \begin{array}{r}       -2 \\       -2 \\       9 \\       9   \end{array} $	$     \begin{array}{r}       -10 \\       -10 \\       -10 \\       -3 \\       -3 \\    \end{array} $	$     \begin{array}{r}       -2 \\       -2 \\       -2 \\       9 \\       9   \end{array} $	$-\frac{2}{9}$	$     \begin{array}{r}       8 \\       -16 \\       -16 \\       9 \\       9   \end{array} $	$     \begin{array}{r}       2 \\       -2 \\       9 \\       -9     \end{array} $	$     \begin{array}{r}       -10 \\       -10 \\       -10 \\       -3 \\       -3 \\    \end{array} $	$     \begin{array}{r}       -2 \\       2 \\       -2 \\       9 \\       -9     \end{array} $	8 -10 -10	$     \begin{array}{r}       -4 \\       -4 \\       -3 \\       -3 \\     \end{array} $	$-10 \\ 10 \\ -10 \\ -3 \\ 3$	$\begin{array}{c} 2 \\ 2 \\ -3 \\ -3 \end{array}$	-16 $9$ $9$	$^{8}_{-16}$ $^{8}_{-9}$	$ \begin{array}{c} -1 \\ 10 \\ -10 \end{array} $	2 2 -3 -3
X.116 $X.117$ $X.118$ $X.119$ $X.120$	$     \begin{array}{r}       -21 \\       -21 \\       9     \end{array} $ $     \begin{array}{r}       -21 \\       -21 \\     \end{array} $	-21 -21	$     \begin{array}{r}       -18 \\       -18 \\       25 \\       -18 \\       -18     \end{array} $	$     \begin{array}{r}       45 \\       -45 \\       \hline       36 \\       -36 \\    \end{array} $	$-36 \\ 36 \\ -45 \\ 45$	$ \begin{array}{r} -3 \\ -3 \\ -18 \\ -3 \\ -3 \end{array} $	-3 -3 36 -3 -3	3	$     \begin{array}{r}       -3 \\       -3 \\       -18 \\       -3 \\       -3     \end{array} $	-3 -3 -3 -3	3 4 3 3	$-\frac{3}{3}$	3 9 3 3	$-3 \\ -3 \\ -3 \\ 3$	-9 -9 9	$     \begin{array}{r}       3 \\       3 \\       -12 \\       3 \\       3     \end{array}   $	$-3 \\ 3 \\ -3 \\ -3$	$\begin{array}{c} 3 \\ 3 \\ -2 \\ 3 \\ 3 \end{array}$	3 -2 3 3	-3 3 -3		3 1 3
X.121 $X.122$ $X.123$ $X.124$ $X.125$	$     \begin{array}{r}       12 \\       -20 \\       18 \\       18 \\       39 \\    \end{array} $	•	-7 $-16$ $-24$ $-33$ $-20$	•		$     \begin{array}{r}       -42 \\       4 \\       18 \\       -36 \\       12     \end{array} $	18 12	$ \begin{array}{r} 24 \\ 24 \\ -12 \\ -12 \\ 2 \end{array} $	$     \begin{array}{r}     -6 \\     -20 \\     18 \\     -9 \\     -6   \end{array} $	$     \begin{array}{r}       21 \\       -2 \\       -9 \\       18 \\       -6     \end{array} $	-16 18 16	•	$     \begin{array}{r}       -12 \\       -12 \\       \hline       6 \\       -1 \\     \end{array} $	•	•	-6 8		$\begin{array}{r} 8 \\ 8 \\ 12 \\ 12 \\ -14 \end{array}$	8 -9 -8			$     \begin{array}{r}       -4 \\       -4 \\       -6 \\       -6 \\       7     \end{array} $
X.126 $X.127$ $X.128$ $X.129$ $X.130$	$     \begin{array}{r}       -2 \\       5 \\       32 \\       -32 \\       & .     \end{array} $		$-{}^{12}_{-20}$		-16	$-32 \\ 8 \\ -8 \\ .$	$-\frac{8}{8}$	$ \begin{array}{r} 20 \\ -10 \\ -16 \\ -16 \\ -16 \end{array} $	$     \begin{array}{r}       16 \\       -4 \\       -4 \\       4     \end{array} $	$     \begin{array}{r}       16 \\       -4 \\       -4 \\       4     \end{array} $	12 -8		$-10 \\ 8 \\ 8 \\ -16$		20	-4 8 8 8	-16	-12 -2	-6 4		· · · · 2	6
X.131 X.132 X.133 X.134 X.135	18 27 27 -34	:	8 -12 -12 16	16	16	18 -4 -4	-36 -4	$^{-6}_{-20}$	18		-16 $-12$ $-12$ $16$		-16 6 3 10		-20 : :	8 -12 -12 -8 -8	16	$     \begin{array}{r}       -4 \\       -6 \\       -6 \\       4   \end{array} $	6 6 -8		-2	2 3 3 -2 -2
X.136 X.137 X.138 X.139 X.140	$ \begin{array}{r} -34 \\ -34 \\ -18 \\ -18 \\ -9 \\ 21 \end{array} $	18 -18	16 16 6 6 21 18			$^{-4}_{\ \ 9}_{\ \ 18}$		$     \begin{array}{r}     -20 \\     -20 \\     -6 \\     -6 \\     -6 \\     6     \end{array} $	2 9 9 -9 3	2 9 9 -9 3	$ \begin{array}{r} 16 \\ -32 \\ -9 \\ -9 \\ 18 \end{array} $	-9 9	10 10 -6 -6 3 -3	-9 9		-8 3 -6 6	-6 -6	$\begin{array}{c} 4 \\ 4 \\ 6 \\ 6 \\ -6 \\ 6 \end{array}$	-8 16 -9 -9 -9	-9		-2 -6 6 3
X.141 X.142 X.143 X.144 X.145 X.146	39 - 39 21 16	-39 39 16 -16	28 28 18	-18 18	-18 18	$     \begin{array}{r}     -6 \\     -6 \\     -6 \\     16 \\     -16   \end{array} $	$ \begin{array}{r} -6 \\ -6 \\ -6 \\ 16 \\ -16 \end{array} $	15 15 6	$ \begin{array}{r} -6 \\ -6 \\ 3 \\ 16 \\ -16 \end{array} $	$ \begin{array}{r} -6 \\ -6 \\ 3 \\ 16 \\ -16 \end{array} $	6 4 4 6	6 -6 16 -16	15 15 -3		9 -9 -10 -10	6	-15 15 :	-5 -5 6	$-3 \\ 4 \\ -3 \\ .$	$-\frac{i}{4}$	9 - -9 - 8 8	-5 -5 -3
X.147 X.148 X.149 X.150 X.151		-16 16 -32 -32 -32				16 -16 -4 4 4	$ \begin{array}{r}  16 \\  -16 \\  -4 \\  4 \\  4 \end{array} $		$ \begin{array}{r} -16 \\ -16 \\ -4 \\ 4 \end{array} $	16 -16 -4 4		-16 -16 -16 -4 -4		-16 $-16$ $4$ $-4$	$   \begin{array}{c}     10 \\     10 \\     10 \\     -12 \\     12 \\     -12   \end{array} $	-4 -4 -4	-8 -8				$     \begin{array}{r}       -8 \\       -8 \\       \hline       6 \\       -6 \\       \hline       6     \end{array} $	
X.152 X.153 X.154 X.155 X.156	32	32		$     \begin{array}{r}       -24 \\       -8 \\       -8 \\       8    \end{array} $	$     \begin{array}{r}       24 \\       -8 \\       -8 \\       8    \end{array} $	-4	-4	8	-4	-4		-4	8 8	-4	12 -8 -8 -8	-4	-8				$ \begin{array}{r}     -6 \\     10 \\     10 \\     -10 \\     -10 \end{array} $	:
X.157 X.158 X.159 X.160 X.161	-34 -34 - 27 -	-34		-24 24	-24	2 2	2 2	10 10 -9	2 2			-2 2	10 10 10 -9	-2 2	-6 6	4 4	-10 10 9	6 6 -9			$^{12}_{-12}$	6 6 -9
$X.162 \\ X.163 \\ X.164$	27 18 -39 16	27	9 -6		•		18 -12	$-9 \\ -32$	-9 6	:	-18 -18 8		-9 16 6 -15			16 6	-9 :	-9 $12$ $-10$				-9 -6 5
X.167 X.168 X.169 X.170 X.171	-16					32 -8 8	32 -8 8	16 16	-16 4 -4 :				-8 -8			-8 -8						
$X.172 \\ X.173 \\ X.174$	$^{34}_{-27}$	:	-27 $48$ $-9$			4	4	20 -18	$-\frac{1}{2}$	$-\frac{1}{2}$			-10 9			8	•	$^{12}_{-18}$		:	: -	-6 9

 $Character\ table\ of\ mE\ (continued)$ 

2 5 3 4 5 .	5 5 4 4	5 4	3 6 5 3	6 3	4 4	1 4 1 4	44	44	2 5	5 3	5 3	5 3	3 4	3 4	3 4	3 4	3 4	$\frac{4}{3}$	4 3	2 4	2 4
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\frac{632}{3e}$ $\frac{3e}{2f}$ $\frac{632}{632}$	$\begin{array}{c} \vdots \\ \underline{633} \ 634 \\ 3l \ 3b \\ 2b \ 2g \\ \underline{633} \ 634 \\ \underline{633} \ 634 \\ \underline{633} \ 634 \\ \end{array}$	635 6 3e 2e 635 6 635 6	$\begin{array}{c} \vdots \\ 636 & 637 \\ 3i & 3j \\ 2f & 2a \\ 36 & 637 \\ 36 & 637 \end{array}$	7 638 f 3h d 2f 7 638 7 638	639 3g 2f 639 639	$\frac{640}{3f}$ $2f$ $640$ $640$	$\frac{6_{41}}{3n}$ $\frac{3b}{6_{41}}$	$\frac{6_{42}}{3e}$ $\frac{3e}{6_{42}}$ $\frac{6_{42}}{6_{42}}$	$\frac{6}{643}$ $\frac{3d}{2g}$ $\frac{6}{643}$ $\frac{6}{643}$	$\frac{644}{3d}$ $\frac{3d}{2e}$ $644$ $644$	$\frac{645}{3j}$ $2f$ $645$ $645$ $645$	$\frac{646}{3k}$ $\frac{3k}{2f}$ $\frac{646}{646}$	$\frac{647}{3l}$ $\frac{647}{647}$ $\frac{647}{647}$	648 3l 2f 648 648	$\begin{array}{c} \vdots \\ 6_{49} \\ 3l \\ 2c \\ 6_{49} \\ 6_{49} \end{array}$	$\begin{array}{c} \vdots \\ 650 \\ 3h \\ 2g \\ 650 \\ 650 \end{array}$	$\begin{array}{c} \frac{651}{3i} \\ 2g \\ 651 \\ 651 \end{array}$	$\frac{652}{3m}$ $\frac{2f}{652}$ $\frac{652}{652}$	$\frac{653}{3n}$ $2f$ $653$ $653$ $653$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$     \begin{array}{r}       -3 \\       -4 \\       2 \\       -6 \\       2     \end{array} $	633 634	3 4 4 -4 -4 2 -4	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{r} 639 \\ -3 \\ -4 \\ 2 \\ 3 \\ 2 \\ 1 \\ -4 \end{array} $	$   \begin{array}{r}     640 \\     \hline     6 \\     -4 \\     2 \\     -3 \\     2 \\     -5 \\     -4 \\   \end{array} $	$ \begin{array}{c}     -5 \\     -3 \\     -2 \\     -3 \\     -4 \\     2 \\     -5 \\   \end{array} $	$     \begin{array}{r}       642 \\       \hline       3 \\       -4 \\       2 \\       \hline       -2 \\       \hline       4 \\       \end{array} $	$ \begin{array}{r} 643 \\ -6 \\ -4 \\ 2 \\ -2 \\ \vdots \\ 4 \end{array} $	$ \begin{array}{c} 644 \\ -2 \\ 2 \\ -4 \\ -4 \\ 4 \end{array} $	$ \begin{array}{c} 645 \\ 2 \\ -1 \\ -1 \\ 1 \\ 3 \\ 2 \\ 3 \end{array} $	$ \begin{array}{c} 646 \\                                  $	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c} 649 \\ -5 \\ -3 \\ 3 \\ -2 \\ 5 \\ 2 \end{array} $	$ \begin{array}{c} 650 \\ -1 \\ 1 \\ -3 \\ -2 \\ 3 \end{array} $	$ \begin{array}{c} 651 \\ 2 \\ -1 \\ 1 \\ -3 \\ -2 \\ 3 \end{array} $	$ \begin{array}{c} 652 \\ -1 \\ -1 \\ -6 \\ -1 \\ -2 \\ -1 \end{array} $	$ \begin{array}{c} 653 \\ -1 \\ -1 \\ -1 \\ -2 \\ -1 \end{array} $
X.97 3 X.98 4 X.100 4 X.101 4 X.102 . X.104 -6 X.105 -6 X.107 -6 X.107 -6 X.109 -4 X.109 -4 X.111 -4 X.113 -4 X.114 -3 X.115 -3 X.116 3 X.116 3 X.117 3	-1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -	-2	-4	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		2 · · · · · · · · · · · · · · · · · · ·	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	6 · · · · · · · · · · · · · · · · · · ·	-4 -4 4 4 	· · · · · · · · · · · · · · · · · · ·	-1 1 -2 2 -2 2 -2 2 -2 3 3 3 -3	-2 -2 -2 -2 -3 -3 -3 -2 -2 -2 -2 -2 -2 -3 -3 -3 -3 -3 -3 -3 -3 -3 -3 -3 -3 -3	-2 -2 -2 -3 -3 -3 -6 2 2 -4 -4 2 2	-22 -23 -36 66 -34 -42 22 22 22	-2 -2 -2 -3 -4 -4 -4 -2 -2 -2 -3 -3 -3 -3 -3 -4 -4 -4 -4 -4 -4 -4 -4 -4 -4 -4 -4 -4	-2 -2 -2 -3 -3 -3 -3 -3 -3 -3 -3 -3 -3 -3 -3 -3	$\begin{array}{c} 4 \\ -4 \\ 4 \\ -4 \\ \vdots \\ \vdots \\ 2 \\ -2 \\ 2 \end{array}$	2 -2 -6 3 -3 6 -2 2 4 -4 -2 2	2 -2 3 -6 6 -3 4 -4 -2 2 -2 2		-2 -2 -2 -4 -4 -4 2 2 -3 -3
X.118 6 X.119 3 X.120 3 X.121 . X.122 . X.123 3 X.125 -4 X.126 -2 X.127 -4 X.129 -4 X.130 8 X.131 8 X.131 8 X.131 8 X.132 6 X.134 6 X.134 6 X.134 6 X.135 4	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c} 4 \\ -3 \\ -3 \\ -4 \\ 8 \\ 12 \\ -6 \\ 4 \\ -8 \\ -4 \\ \vdots \end{array} $	-3 3 3 -3 -3 -6 4	3	6 -4 4 -4 	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		-33	-33	3 -3	$\begin{array}{c} 3 \\ 1 \\ \vdots \\ 2 \\ 2 \\ \vdots \\ -3 \\ -2 \\ \vdots \\ 2 \\ 3 \\ -6 \\ -2 \\ \end{array}$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	-3 -3 -2 -4 -2 -2 -2 -4 -4	4 3 3 2 -4 -2 -2 -4	-33		3 -3	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
X.136 4 X.137 4 X.138 3 X.139 3 X.140 3 X.141 2 X.142 . X.144 3 X.144 5 X.145 . X.146 . X.147 . X.148 . X.149 -4 X.150 -4 X.150 -4	4 4 4 4 3 -3 -3 -3 -6 -6 -6 -6 -2 -6 -2 -6 -2 -2 -2 -2 -2 -2 -2 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1	$ \begin{array}{r} 4 \\ -3 \\ -3 \\ -6 \\ -6 \\ -2 \\ -2 \end{array} $	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	3	4 - 2 4 - 3 - 2 - 6 - 2 - 6 - 3 - 3 - 4 - 5 - 6 - 5 - 5 - 5 - 5 - 5 - 5 - 5 - 5 - 5 - 5	2 -8 4 2 4 3 · · · · · · · · · · · · · · · · · · ·	-2 -2 -3 -3 -3 -2 -2 -3 3	-2 -2 -3 -3 -3 -2 -2 -3 3	2 2 3 3 3 -2 -2 -2 -4 4 4			$\begin{array}{c} \cdot \\ -3 \\ 3 \\ \cdot \\ -6 \\ \cdot \\ \cdot \\ \cdot \\ -2 \\ 2 \\ -2 \\ 2 \end{array}$	-2 4 -2 -3 -2 -2 -2	-2 -2 -2 -2 -2 -3 -3	-8 4 4 	-8 4  -6 1 1 6 	-3 3 -2 2 -2 4 4 -4	· · · · · · · · · · · · · · · · · · ·		-2 -2 4	-2 4 -2
X.161 X.162 X.163 -8 X.164 -3 X.165 X.166 X.167 X.168 X.169 4 X.170 X.171 X.171 X.172	2	6	-4	4	-4 -4 -2 -2 -2				-4 · · · · · · · · · · · · ·	-66 6	-6 6 		22	22	22	22	-4 · · · · · · · · · · · · · · · · · · ·			22	-i

2 3 5 7 13	2 4	3 3	3 3	1 4	2 3	2 1 i	5 1	5 1	5	5 ·	3 5	2 5	2 5	1 5	$\overset{1}{\overset{4}{\cdot}}$	1 4 :	4	4	$^{4}_{1}_{1}$	4 1 1	3 1 1	4 i	3 1	6 3 1	7 3 :	7 2
2P :	54 54	$\begin{array}{c} 3l \\ 2g \\ 6_{55} \\ 6_{55} \end{array}$	$^{2g}_{656}$ $^{656}_{656}$	$\begin{array}{c} 3o \\ 2f \\ 657 \\ 657 \end{array}$	$6_{58} \\ 6_{58}$	7a $7a$ $1a$	8a 8a 8a 8a 8a 8a	$\frac{8b}{8b}$	8c 4b 8c 8c 8c 8c 8c	8d $8d$	$\frac{9a}{9a}$	9b 9b 3b 9b 9b 9b	9c 9c 3b 9c 9c 9c	9d	9e 9e	9f 9f	9g 9g	9h 9h	10a	$5a \\ 10b \\ 2a \\ 10b$	$10c \\ 2b \\ 10c$	5a $10d$ $2e$ $10d$	$ \begin{array}{c} 5a \\ 10e \\ 2c \end{array} $	$12_{1}$	$12_2$ $6_9$ $4b$ $12_2$ $12_2$ $12_2$	$ \begin{array}{c}                                     $
X.88 X.89 - X.90 - X.91 X.92 - X.93 - X.94	- i	-1 -1 -1 : : : : : : : : :	$ \begin{array}{c} -1 \\ -1 \\ 1 \\ \vdots \\ 1 \\ \vdots \\ \vdots \\ \vdots \\ 2 \end{array} $	$     \begin{array}{r}       -1 \\       -1 \\       3 \\       -1 \\       1     \end{array} $	-i -1 i	1	-1 	-1 			-3 -6 -6 -6 -3 -3	-3 -6 -3 -3	$ \begin{array}{r}     -3 \\     -6 \\     3 \\     -6 \end{array} $	-3 -3 -3 -3 -3 -3 1 1 1	· · · · · · · · · · · · · · · · · · ·	· · · · · · · · · · · · · · · · · · ·			-1 -1 -1 -1 1 1				-1 -1 -1 -1	5  10 -10  5 16 16 -16	-3 12 4 -6 4 2 -4 12 -4 -3	-3 -4 -4 -6 4 2 -4 -4 -5
$X.109 - X.110 \ X.111 \ X.112 \ X.113 \ X.114 \ X.115 \ X.116 \ X.117$	-4 -4 2 2 2 2 2 2 3	$\begin{array}{c} \cdot \\ \cdot \\ -4 \\ -2 \\ 2 \\ -2 \\ \cdot \\ -3 \\ 3 \end{array}$	$\begin{array}{c} 2 \\ 4 \\ -4 \\ -2 \\ 2 \\ \end{array}$	-1 -1 -1 2 2	$ \begin{array}{c} 1 \\ -1 \\ -2 \\ 2 \end{array} $	:		1 1	-1 -1			$-1 \\ -1 \\ -1$	-1	$     \begin{array}{r}       -1 \\       -1 \\       -1 \\       -1     \end{array} $	$     \begin{array}{r}       2 \\       -1 \\       -1 \\       -1 \end{array} $	-1 -1 -2 2 -1 -1 -1	$-1 \\ 2 \\ -1 \\ -1$	2 2 -1 -1 -1 -1 -1		-11	-1 -1 -1		-i		4 4 4 	4 4 4 
X.120 -	-2 -3 -3 -1 2	-3 3 	3 -3	1 2 -1		2	2 -2 -2 -2	-2			$-6 \\ 2 \\ 2 \\ 2 \\ 2$	-6 -6 -6 -2 2		3 3 -1 -1 2	· · · · · · · · · · · · · · · · · · ·		-1		-4 2	· · · · · · · · · · · · · · · · · · ·	· · · · · · · · · · · · · · · · · · ·	-2		$ \begin{array}{c} -5 \\ 10 \\ 10 \\ -5 \\ 4 \\ -5 \\ \vdots \\ 16 \\ -16 \\ \vdots \end{array} $	-11 2 2 9 3 -4 4 -12	1 2 2 -3 3 -4 4 4
X.131 X.132 X.133 X.134 X.135 X.136 - X.137 - X.138 X.139 X.140 X.141 - X.142	· · · · · · · · · · · · · · · · · · ·	-1		-1 -1 -1 -2 -2	-i	-2 -2 -2	· · · · · · · · · · · · · · · · · · ·	· · · · · · · · · · · · · · · · · · ·			2 		2 2 	2	-1 4	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	-1	-1 :	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	2 	2 		$\begin{array}{c} -4 \\ \vdots \\ -10 \\ -10 \\ 1 \\ -10 \\ \cdot \end{array}$	$ \begin{array}{c}     -4 \\     -4 \\     -4 \\     \hline     -6 \\     -6 \\     9 \\     -2 \\     12 \\     12 $	$ \begin{array}{c}     -4 \\     -4 \\     -4 \\     \hline     -6 \\     -6 \\     -1 \\     -2 \\     -4 \\     -4 \\   \end{array} $
X.143 X.144 X.145 X.146 X.146 X.147 X.148 X.149 X.150 X.151 X.152 X.153 X.153 X.154 X.155	1 3	1	1	1	1	1 1 1 1 1 					$ \begin{array}{c}     -4 \\     -4 \\     -4 \\     -4 \\     3 \\     3 \\     3 \\     4 \\     4 \end{array} $	······································	$\begin{array}{c} \cdot \cdot \\ -4 \\ -4 \\ -4 \\ -3 \\ 3 \\ 3 \\ 4 \\ 4 \end{array}$	2	$-1 \\ -1$	-1	-1	$-1 \\ -1$				-i -1 1 -1 1	. 1	$\begin{array}{c} -10 \\ \cdot \\ \cdot \\ \cdot \\ -16 \\ 16 \\ 16 \\ -16 \\ \cdot \\ \cdot \\ \end{array}$	12 -2	-4 -2
X.166	-i			-i		-1 -1 -1	1 -1 -1	-1 -1 -1 : : 1 1	1 -1	-1 -1 -1 -1 -1	-3 -3 -3 4	-3 -3 -3 -4	-3 -3 -2 -2	-3 -3 -3 -2 -2	1 	1 			$ \begin{array}{c}     \cdot \\     -1 \\     -1 \\     \cdot \\     \cdot \\     -1 \\     -1 \\     -1 \\     -4 \\     4 \end{array} $	-3 -3 -3 -3 -3 -3	1	1	-1 i	9 9 -9 -9 10	-9 -9 -3 -3 -3 -6 -12	; 3 3 1 1 6 4
X.167 X.168 X.169 X.170 X.171 X.172 X.173 X.174						2					6 6 8	$\begin{array}{c} 6 \\ 6 \\ -4 \\ -4 \\ \end{array}$	$ \begin{array}{r} 4 \\ -3 \\ -3 \\ -4 \\ -4 \end{array} $	$-3 \\ -3$	2	2	-1 -1	$-1 \\ -1$	2 2		$\begin{array}{c} \dot{2} \\ -2 \\ \dot{2} \\ -\dot{2} \\ -\dot{2} \end{array}$		•	16 -16 -9 -9		-3 -1

2 5 3 3	5 3	$\frac{6}{2}$	4 3	4 3	$\frac{4}{3}$	4 3	5 2	5 2	5 2	3 3	3 3	3	3 3	$\frac{4}{2}$	$\frac{4}{2}$	$\frac{4}{2}$	$\frac{4}{2}$	2 3	2 3	5 1
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	12 <sub>5</sub>	12 <sub>6</sub>	$\frac{127}{127}$	12 <sub>8</sub>	12 <sub>9</sub>	: 12 <sub>10</sub>	$\frac{12}{12}$	12 <sub>12</sub>	12 <sub>13</sub>	12 <sub>14</sub>	12 <sub>15</sub>	12 <sub>16</sub>	: 12 <sub>17</sub>	: 12 <sub>18</sub>	12 <sub>19</sub>	: 12 <sub>20</sub>	12 <sub>21</sub>	: 12 <sub>22</sub>	12 <sub>23</sub>	$\frac{12}{12}$
$^{6}_{24}$ $^{6}_{4b}$ $^{7}_{12_{4}}$ $^{7}_{12_{4}}$ $^{7}_{12_{4}}$	$^{6_{17}}_{4b}$ $^{12_{5}}_{12_{5}}$ $^{12_{5}}_{12_{5}}$	$ \begin{array}{r} 6_{3} \\ 4f \\ 12_{6} \\ 12_{6} \\ 12_{6} \end{array} $	$^{6}_{28}$ $^{4}_{4b}$ $^{12}_{7}$ $^{12}_{7}$	$^{625}_{4b}$ $^{128}_{128}$ $^{128}$	$ \begin{array}{c} 6_{22} \\ 4a \\ 12_{9} \\ 12_{9} \\ 12_{9} \end{array} $	$^{630}_{4a}$ $^{12}_{12}$ $^{12}_{10}$ $^{12}_{12}$	$^{624}_{4d}$ $^{12}_{11}$ $^{12}_{11}$ $^{12}_{11}$	$^{624}_{4e}$ $^{12}_{12}$ $^{12}_{12}$ $^{12}_{12}$	$^{6_{17}}_{4d}$ $^{12_{13}}_{12_{13}}$ $^{12_{13}}$	$^{629}_{4a}$ $^{12}_{14}$ $^{12}_{14}$ $^{12}_{14}$	$\begin{array}{c} 12_{15} \\ 6_{37} \\ 4a \\ 12_{15} \\ 12_{15} \\ 12_{15} \\ 2 \\ \end{array}$	$^{636}_{4b}$ $^{12}_{16}$ $^{12}_{16}$ $^{12}_{16}$	$^{638}_{4b}$ $^{12}_{17}$ $^{12}_{17}$ $^{12}_{17}$	$^{6}_{4e}^{4e}$ $^{12}_{18}^{12}$ $^{12}_{18}$ $^{12}_{18}$	$^{622}_{4c}$ $^{12}_{19}$ $^{12}_{19}$ $^{12}_{19}$	$^{630}_{4c}$ $^{12}_{20}$ $^{12}_{20}$ $^{12}_{20}$	$^{630}_{4f}$ $^{12}_{21}$ $^{12}_{21}$ $^{12}_{21}$	$^{645}_{4b}$ $^{12}_{22}$ $^{12}_{22}$ $^{12}_{22}$	$^{646}_{4b}$ $^{12}_{23}$ $^{12}_{23}$ $^{12}_{23}$	$^{624}_{4g}$ $^{12}_{24}$ $^{12}_{24}$
8	$ \begin{array}{c} -3 \\ 4 \\ -6 \\ 4 \\ 2 \\ 2 \\ -6 \\ \vdots \end{array} $	-3 -2 -2 -3 -3	3 1 1 2 2 3 2 3	$ \begin{array}{c} -3 \\ 4 \\ 3 \\ 4 \\ -1 \\ 2 \\ 3 \\ \vdots \\ \vdots \\ 3 \end{array} $	-2	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c}     1 \\     -1 \\     -2 \\     -3 \\     2 \end{array} $	1 i	3 4 -4 -2 2	i -i -i	1 -1	1 1 2 -1 -1	i i	-1 1 2 2 -1 2 -1	· · · · · · · · · · · · · · · · · · ·	· · · · · · · · · · · · · · · · · · ·	-2 -2 	i i	1 1 -1 -1 -1 -1	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
1	-2 -2 -2 -2 -2	444	1 1 1 1	-2 -2 -2 -2 -2	-2 -2 -2	1 1 	-1 -1 -1 1 1	1 1 1 1 1	2 2 2 -2 -2 -2	-2 -2 2 2 -2 -2 -2	2 1 1 	1 -2 -2 -1 1	$ \begin{array}{c}     -2 \\     1 \\     1 \\     -2 \end{array} $	1 1 1 1 1	-2 2 -2 2 -2 -2	$ \begin{array}{c} -\bar{2} \\ -1 \\ 1 \\ . \\ . \\ . \\ . \\ . \\ . $	1 1	$ \begin{array}{c}     -2 \\     -2 \\     1 \\     -2 \\     \vdots \\    $	1 -2 -2 -2 1	-1 -1 1 1
3 3 -1 -1 -2 -1 -1	-3 -3 -1 -1 -1 -1 -1	$\begin{array}{c} \cdot \\ -1 \\ -1 \\ 2 \\ 2 \\ -1 \\ 2 \\ -1 \end{array}$	3 3 -1 -1 -1 -1 -1	-3 -3 -1 -1 -1 -2 -1 -1	-1 -1 1 1 4 1 1 4	-1 -1 -1 1 -2 1 -2	3 -3 -1 -1 -1 1	$ \begin{array}{c}                                     $	-3 3 1 -1 -1 1	$ \begin{array}{c}                                     $		2 2 2 -2 -1 -1	-1 -1 -2 2 2	-1 -1 -1 -1 -1 -1 -1	-1 -1 -1 1 -1	-1 -1 -1 1 -1 -1	-1 -1 -1 -1 -1 -1 2 -1 -1	2	2 2 2 1 -1 -1	-1 1 1 -1 -1 -1
2 4 6	-6 8 4	4 3	-1 -2 -3	3 -4 -2	$ \begin{array}{c} 4 \\ -2 \\ \vdots \\ 4 \\ -4 \\ \vdots \\ -4 \end{array} $	$     \begin{array}{c}       -2 \\       4 \\                           $		2 4 -2		$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$     \begin{array}{c}       -2 \\       \cdot \\       -2 \\       2 \\       \cdot \\       \cdot    \end{array} $	-2 -2 -2	-2 -2 -2	$ \begin{array}{c}     -1 \\     -2 \\     1 \end{array} $			-2	-1 1	-1 1	
2 2	-4 -4		-1 -1	2 2				2 2			-1	$-\frac{1}{2}$	-4	-1 -1				$-\frac{1}{2}$	-1	•
$\begin{array}{c} \cdot \\ \cdot \\ -2 \\ 3 \\ -3 \\ -2 \\ \cdot \end{array}$	$\begin{array}{c} 3\\ 3\\ -6\\ -2\\ \cdot\\ -2\\ \cdot\\ \end{array}$	$ \begin{array}{c} -2 \\ -2 \\ 1 \\ -2 \\ \vdots \\ -2 \end{array} $	-3 1 3 3 1	3 3 3 1	$ \begin{array}{c} -1 \\ -1 \\ -2 \\ 2 \\ \vdots \\ 2 \end{array} $	$ \begin{array}{c} -1 \\ -1 \\ -2 \\ 2 \end{array} $	-3 3	$ \begin{array}{c}     -2 \\     -2 \\     -1 \\     -1 \\     -2 \\   \end{array} $	-3 3	$ \begin{array}{c} -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ \end{array} $	$ \begin{array}{c} -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ \end{array} $	4 -2	-2 -4	1 1 -1 -1 1	-1 -1	-1 -1	$\begin{array}{c} 1 \\ 1 \\ -2 \\ -2 \\ \end{array}$		-2 -2	
					-2 -2 -2 -2 :	-2 -2 -2 -2 2				-2 -2 -2 -2 2	-2 -2 -2 2				-2 -2 -2 2 2	-2 -2 -2 2 2				
-3 -3 -3		-3 -3	-3 -3 -3	-3		-2	3 -3 -3			1				1 1		-	2			-1 -1 -1
-6					-4 -4 -:	$\begin{array}{c} \cdot \\ \cdot \\ -4 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{array}$				-2 2	$-\dot{2}$			-1						

2 3 5 7	3 2	3 2	3 2 :	4 1	1	1	2 1 i	2 1	2 i	3 2 1	1 2 1	2 1	3 4	3 4	3 4	2 4	2 4	2 4	2 4	3 3	3 3	3 3	$\begin{array}{c} 1 \\ 4 \\ \vdots \\ \end{array}$	2 3
$ \begin{array}{r} 13\\ \hline 2P\\ 3P\\ 5P\\ 7P\\ 12P\\ 12P\\ 12P\\ 12P\\ 12P\\ 12P\\ 12P\\ 12$	$12_{25}$ $6_{36}$ $4d$ $12_{25}$ $12_{25}$ $12_{25}$	$\begin{array}{r} 12_{26} \\ 6_{37} \\ 4f \\ 12_{26} \\ 12_{26} \end{array}$	$12_{27}$ $6_{38}$ $4d$ $12_{27}$ $12_{27}$	$ \begin{array}{r} 12_{28} \\ 6_{30} \\ 4h \\ 12_{28} \\ 12_{28} \end{array} $	13a 13b 13a 13b 13b	$     \begin{array}{r}       13b \\       13a \\       13b \\       13a \\       13a \\       13a     \end{array} $	$     \begin{array}{r}       14a \\       7a \\       14a \\       14a \\       2b     \end{array} $	14b $7a$ $14b$ $14b$ $14b$	14c 7a 14c 14c 2c	15a 15a 5a 3a 15a 15a	$15b \\ 15b \\ 5a \\ 3d \\ 15b \\$	15c $15c$ $5a$ $3f$ $15c$	$     \begin{array}{r}       18a \\       9a \\       66 \\       18c \\       18a \\     \end{array} $	$     \begin{array}{r}       18b \\       9a \\       6_2 \\       18b \\       18b \\     \end{array} $	18c 9a 66 18a 18c	18d 9c 66 18g 18d	$     \begin{array}{r}       18e \\       9b \\       66 \\       18e \\       18e     \end{array} $	18f 9b 6 <sub>2</sub> 18f 18f	$     \begin{array}{r}       18g \\       9c \\       66 \\       18d \\       18g     \end{array} $	18h 9a 68 18j 18h	18i 9a 6 <sub>14</sub> 18i 18i	$     \begin{array}{r}       18j \\       9a \\       68 \\       18h \\       18j \\     \end{array} $	$^{66}_{18k}$	$     \begin{array}{r}                                     $
X.89 X.90 X.91 X.92 X.93 X.94 X.95 X.97 X.100 X.101 X.102 X.103 X.104 X.105 X.104 X.105	1 -1 1 -1 -1 	1226 -1 -1 -1 	1 -1 1 -1	12 <sub>28</sub>	1a	1a 			-i -1	-1 -1 -1 -1 -1 -3 -3	15b		1	-3 -6	1	18d	1 4 2 -1 1 1 1 1 1 1 1	-3 -3 -3 -3 -1 1 1 -1 -1	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	18h 1 1 -1 -1 -1 -1	18 <i>i</i> -2 -2 -2 -1 1 1 1 1	18jj	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	18t 1 1 -1 -1 -1 -1
X.107 X.108 X.109 X.110 X.111 X.112 X.113 X.114 X.115 X.116 X.117 X.118 X.119 X.120 X.121	1 	-1 -1 -1 -1 -1 -1 -1 -1	-2 	-1 -1 -1 -1 -1 -1			1	-1		· · · · · · · · · · · · · · · · · · ·	1 1 1 	· · · · · · · · · · · · · · · · · · ·	1 1 1 1 1 1 	-1 -1 -1 -1 -1 -1	1 1 1 1 1 1 	1 1 1 1 1 1 	1 1 1 1 1 1 	1 -1 -1 -1 -1 -1 -1	1 1 1 1 1 1 	-1 -1 -1 -1 -1 :	-1 -1 -1 -1 -1 -1 -1	-1 -1 -1 -1 -1 :	1 1 1 1 1 1 1 1	-1 1 -1 1 -1 1 
X.123 X.124 X.125 X.126 X.127 X.128 X.130 X.131 X.132 X.133 X.134		1			-2	-2	2 			-1 1 1 -2 -2 -2 3	2 	-1	-2 2 2 -2 -2	· · · · · · · · · · · · · · · · · · ·	-2 2 2 -2 -2	1 -1 -1 -1 1	-2 2 2 -2 -2	· · · · · · · · · · · · · · · · · · ·			-4 2 2 2 2 2 2 2 2 2		1 -1 -1 -1 1	
X.135 $X.136$ $X.137$ $X.138$ $X.140$ $X.141$ $X.141$ $X.142$ $X.143$ $X.144$		1 1 1 1		-i -i 1						2 2 2 -1	-1 -1 -1 2	-1 -1 -1 -1	2 2 2 		2 2 2 	-1 -1 -1 -1	2 2 2 		-1 -1 -1		-2 -2 -2		-1 -1 -1	
X.145 X.146 X.147 X.148 X.149 X.150 X.151 X.152 X.153 X.154 X.155							-1 -1 -1 -1 -1	1 1 -1 -1 -1	-1	-3 -3 -3 -3 -3			-2 -2 -2 -1 1 1 -1	-4 -4 4 -3 3 -3 4 4 -4	$ \begin{array}{c}     -2 \\     2 \\     -2 \\     2 \\     -1 \\     1 \\     -1 \\     \vdots   \end{array} $	-2 -2 -2 -1 1 1 -1	-2 -2 -2 -1 1 1 -1	2 2 -2 -2 -3 3 -3 -2 -2	$ \begin{array}{c}     -2 \\     2 \\     -2 \\     2 \\     -1 \\     1 \\     -1 \\     \vdots   \end{array} $	-2 2 -2 -2 1 1 -1 -1	$-1 \\ -1$	-2 2 -2 -2 1 1 -1 -1	-2 -2 -2 -1 1 1 -1	$ \begin{array}{c}                                     $
X.156 $X.157$ $X.158$ $X.169$ $X.161$ $X.161$ $X.162$ $X.163$ $X.164$ $X.165$		-i			C 1 1	D 1 1		-i	-i		· · · · · · · · · · · · · · · · · · ·	-1 1	-1 -1 -1	-4 -3 -3	-1 -1 -1 :	-1 -1 -1	-1 -1 -1	2	-1 -1 -1 :	-1 -1 -1	1 1 1 	-1 -1 	-1 -1 -1 :	
X.166 X.167 X.168 X.169 X.170 X.171 X.172 X.173 X.174							$-\frac{1}{2}$			3 3 -3 -3			$ \begin{array}{c} 4 \\ -4 \\ -2 \\ 2 \\ \vdots \\ -2 \\ \end{array} $		4 -4 -2 2	-2 2 1 -1 -1	2		$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		-2 -2 -2		$ \begin{array}{c} -2 \\ 2 \\ -1 \\ -1 \\ \vdots \\ 1 \end{array} $	

Character table of mE (continued)

2 3 5 7 13	2 3	3 2	1 3	1 3	3 1 1	3 i	1 1 1	1		1 : i	1 i	3 1 1	2 1 1	1 1 1	1	i i	1 1 1	1
2P 3P 5P 7P 3P	$     \begin{array}{r}       18m \\       9c \\       6_{14} \\       18m \\       18m \\       18m \\     \end{array} $	9a	9e	9f	20a	10a 20h	21a 7a 21a 3a 21a	$     \begin{array}{r}       8a \\       24a \\       24a \\       24a   \end{array} $	$12_{2} \\ 8b \\ 24b \\ 24b \\ 24b \\ 24b$	26a	26b 13b 26b	10a	15a	-10h	$\frac{39b}{13a}$	39a	140	30a
.88 .89 .90 .91 .92	$ \begin{array}{c} 1 \\ -2 \\ -1 \\ -2 \end{array} $	-2 -2	:				1	-1	-1								1	
.94 .95 .96 .97 .98	1 1 1 1 1	$     \begin{array}{r}       -1 \\       -1 \\       1     \end{array} $	-1 1		. 1	-1 -1 -1		i	i 1			-1 -1 -1	1 1 1	1 -1 -1	-i	-1		1 1 -1
$01 \\ 02 \\ 03 \\ 04 \\ 05 \\ 06$	1	-1			-1 -1 -1 -1	1 1 -1	-1 -1 -1					-1 -1 1 1	-1 1 1 1	1			1 1 1	-1 -1 -1 -1
07 08 09 10 11	$ \begin{array}{r} -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{array} $	$-1 \\ -1 \\ -1 \\ -1 \\ 1$	$\begin{array}{c} 2 \\ 1 \\ -1 \\ 1 \end{array}$	$-1 \\ -2 \\ 2 \\ 1$			-1			•							1	
13 14 15 16 17 18	-1	-1	-1	-1	-1 -1	-1 1		1 1 -1	1 1	•		1 1	-1 -1	-1 -1				-i -1 :
20 21 22 23 24 25					· · ·		-1	1	-1			2 -1	-i		i	i	- i	-i
6 7 8 9 0	$ \begin{array}{r} -1 \\ -1 \\ -1 \\ -1 \\ 2 \\ 2 \end{array} $	-2 -2								•		1 1 -2 -2	1 -1	1 -1				i -1
$\frac{2}{3}$ $\frac{4}{5}$ $\frac{6}{7}$ $\frac{7}{6}$	1 1 1				-2		1					-1	-1				-1 -1	1
$   \begin{array}{c}     8 \\     9 \\     0 \\     1 \\     2 \\     3 \\     4   \end{array} $					-2 -2			-i	-1	•		2 2 -1	1	-1 -1				i
5 6 7 8 9	-1 -1		-1 -1 1 1	-1 -1 1 1	$-\frac{1}{1}$	1	1 1 1 1					1					-1 -1 -1	-i
1 2 3 4 5 5 6	-1 -1	-1		$-1 \\ -1 \\ -1$		-1 -1						1	-1 1		C D D C	D C C D		-1 -1
58 59 50 51 52	1 1	-i			-1 1 1	1 -1 1	-1	1	-1 1 1	-1 -1	-1 -1	$ \begin{array}{c} -1 \\ -1 \\ \vdots \\ -1 \\ -1 \\ 2 \end{array} $	1 -1 -1		1 1	1		-1
63 64 65 66 67 68	-2 1				-2 2		-1 -1			•		$-\frac{2}{2}$ $-\frac{1}{1}$	-1 1				1 -1	
70 71 72 73 74	-1 -1				-2 -2		i	-i	1 -1			i	-i		-D -C -1	- <i>C</i> - <i>D</i> -1	1	

where  $A = -6\zeta(3) - 3$ ,  $B = -12\zeta(3) - 6$ ,  $C = -\zeta(13)^{11} - \zeta(13)^8 - \zeta(13)^7 - \zeta(13)^6 - \zeta(13)^5 - \zeta(13)^2 - 1$ , D = -C - 1, E = 2D, F = 2C.

**B.4.** Character table of  $H(Fi'_{24}) = \langle b, p, r \rangle$ 

2 3 5 7	$\begin{array}{c} 21 \\ 7 \\ 1 \\ 1 \end{array}$	$\begin{array}{c} 21 \\ 7 \\ 1 \\ 1 \end{array}$	19 4 1	20 2 1	14 4 1	14 4 1	17 3	17 3	16 1	13 2	8 7 1 1	12 6	10 7	5 7	8 5	5 6	6 4 :	15 5 1	15 2
2P 3P 5P	$\begin{array}{c} 1a \\ 1a \\ 1a \\ 1a \end{array}$	$ \begin{array}{c} 2a \\ 1a \\ 2a \\ 2a \end{array} $	$\begin{array}{c} 2b \\ 1a \\ 2b \\ 2b \end{array}$	$\begin{array}{c} 2c \\ 1a \\ 2c \\ 2c \end{array}$	$\begin{array}{c} 2d \\ 1a \\ 2d \\ 2d \end{array}$	$\begin{array}{c} 2e \\ 1a \\ 2e \\ 2e \end{array}$	$\begin{array}{c} 2f \\ 1a \\ 2f \\ 2f \end{array}$	$\begin{array}{c} 2g \\ 1a \\ 2g \\ 2g \end{array}$	$ \begin{array}{c} 2h \\ 1a \\ 2h \\ 2h \end{array} $	$\begin{array}{c} 2i \\ 1a \\ 2i \\ 2i \end{array}$	3a $3a$ $1a$ $3a$	$\begin{array}{c} 3b \\ 3b \\ 1a \\ 3b \end{array}$	$\begin{array}{c} 3c \\ 3c \\ 1a \\ 3c \end{array}$	$\begin{array}{c} 3d \\ 3d \\ 1a \\ 3d \end{array}$	3e $1a$ $3e$	$\begin{array}{c} 3f \\ 3f \\ 1a \\ 3f \end{array}$	$\begin{array}{c} 3g \\ 3g \\ 1a \\ 3g \end{array}$	$ \begin{array}{r} 4a \\ 2a \\ 4a \\ 4a \end{array} $	$\begin{array}{c} 4b \\ 2a \\ 4b \\ 4b \end{array}$
$\begin{array}{r} 7P \\ \hline X.1 \\ X.2 \\ X.3 \end{array}$	1a 1 1 21	$\frac{2a}{1}$ 21	$\frac{2b}{1}$ 21	2c 1 1 21	$ \begin{array}{r} 2d \\ 1 \\ -1 \\ 9 \end{array} $	$ \begin{array}{r} 2e \\ 1 \\ -1 \\ 9 \end{array} $	2f 1 1 5	$\frac{2g}{1}$ $\frac{1}{5}$	2h 1 1 5	$\frac{2i}{1} - \frac{1}{1}$	$\frac{3a}{1}$ 21	3b 1 1 3	$\frac{3c}{1}$ $-6$	3d 1 1 -6	3e 1 1 3	3f 1 1 3	3g 1 1 3	$\frac{4a}{1}$ 21	$\begin{array}{r} 4b \\ \hline 1 \\ 1 \end{array}$
$X.4 \\ X.5 \\ X.6$	21 30 35	$\frac{21}{30}$ 35	$\frac{21}{30}$ 35	21 30 35	-9 15	-9 15	$-\frac{5}{3}$	$-\frac{5}{3}$	$-\frac{5}{3}$	$-1 \\ -i$	$^{21}_{-15}_{35}$	$^{3}_{6}$	$^{-6}_{-6}$	$^{-6}_{12}_{8}$	3 8	$     \begin{array}{r}       3 \\       -3 \\       -1     \end{array} $	$-\dot{1}$	21 30 35	$   \begin{array}{c}     5 \\     5 \\     -2 \\     3   \end{array} $
$X.7 \\ X.8 \\ X.9 \\ X.10$	35 35 35 42	35 35 35 42	35 35 35 42	35 35 35 42	$-5 \\ 5 \\ -15$	$-5 \\ 5 \\ -15$	3 3 10	3 3 10	3 3 10	-5 5 1	$\begin{array}{r} 35 \\ 35 \\ 35 \\ -21 \end{array}$	$   \begin{array}{c}     8 \\     8 \\     -1 \\     12   \end{array} $	$   \begin{array}{c}     8 \\     8 \\     -3   \end{array} $	8 8 8 6	$^{-1}_{-1}_{8}$	$   \begin{array}{c}     8 \\     8 \\     -1 \\     -6   \end{array} $	$-1 \\ -1 \\ -1$	35 35 35 42	3 3 10
$X.11 \\ X.12 \\ X.13$	90 90 140	90 90 140 140	90 90 140 140	90 90 140 140	$     \begin{array}{r}       -30 \\       30 \\       -20 \\       20     \end{array} $	$-30 \\ 30 \\ -20 \\ 20$	10 10 12 12	10 10 12 12	10 10 12 12	$     \begin{array}{r}     -6 \\     6 \\     -4 \\     4   \end{array} $	90 90 140 140	$9 \\ -4 \\ 4$	9 9 5	9 9 5	$\begin{array}{c} 9 \\ 9 \\ -4 \end{array}$	$^{9}_{-4}$	5	90 90 140	$10 \\ 10 \\ 12 \\ 12$
$X.14 \\ X.15 \\ X.16 \\ X.17$	140 189 189 210	$     \begin{array}{r}       189 \\       189 \\       210     \end{array} $	189 189 210	189 189 210	-9 9	-9 9	$-3 \\ -3 \\ 18$	$-3 \\ -3 \\ 18$	$-3 \\ -3 \\ 18$	-9 9	$189 \\ 189 \\ -105$	-4 6	$   \begin{array}{r}     5 \\     27 \\     27 \\     -15   \end{array} $	5 27 27 30	-4 :	-4 -3	5	140 189 189 210	$-3 \\ -3 \\ 18$
$X.18 \\ X.19 \\ X.20 \\ X.21$	210 210 210 210	210 $210$ $210$ $210$	210 $210$ $210$ $210$	$210 \\ 210 \\ 210 \\ 210$	30 -30	30 -30	$-14 \\ -14 \\ 2 \\ 18$	$-14 \\ -14 \\ 2 \\ 18$	$-14 \\ 2 \\ 18$	-10 10	$     \begin{array}{r}       210 \\       -105 \\       210 \\       -105     \end{array} $	$\begin{array}{c} 3 \\ 6 \\ 3 \\ 24 \end{array}$	$-15 \\ 21 \\ 12$	$\begin{array}{c} 21 \\ 30 \\ 21 \\ -24 \end{array}$	3	$ \begin{array}{r}     3 \\     -3 \\     3 \\     -12 \end{array} $	3	210 210 210 210	$-14 \\ -18$
$X.22 \\ X.23 \\ X.24 \\ X.25$	280 280 280 280	280 280 280 280	280 280 280 280	280 280 280 280	$     \begin{array}{r}     -40 \\     -40 \\     40 \\     40   \end{array} $	$     \begin{array}{r}     -40 \\     -40 \\     40 \\     40   \end{array} $	-8 -8 -8	-8 -8 -8	-8 -8 -8	$^{8}_{-8}$	280 280 280 280	1 1 1	10 10 10 10	10 10 10 10	10 10 10 10	1 1 1	1 1 1 1	280 280 280 280	-8 -8 -8
$X.26 \\ X.27 \\ X.28$	315 315 315	$     \begin{array}{r}       315 \\       315 \\       315     \end{array} $	$\frac{315}{315}$	$     \begin{array}{r}       315 \\       315 \\       315     \end{array} $	$^{-15}_{75}$	$^{-15}_{75}$ $^{15}$	$\frac{11}{11}$	$^{11}_{11}_{11}$	$\frac{11}{11}$	$^{9}_{-9}$	$     \begin{array}{r}       315 \\       315 \\       315     \end{array} $	$     \begin{array}{r}       18 \\       -9 \\       18     \end{array} $	$     \begin{array}{r}       -9 \\       -9 \\       -9     \end{array} $	$-9 \\ -9 \\ -9$	$^{-9}_{18}$ $^{-9}$	$^{18}_{-9}$ $^{18}$	:	$     \begin{array}{r}       315 \\       315 \\       315     \end{array} $	11 11 11
$X.29 \\ X.30 \\ X.31 \\ X.32$	315 378 378 420	315 378 378 420	$     \begin{array}{r}       315 \\       58 \\       58 \\       420     \end{array} $	$     \begin{array}{r}       315 \\       -6 \\       -6 \\       420     \end{array} $	-75 $-36$ $-36$ $60$	-75 $-36$ $-36$ $60$	11 42 42 4	11 42 42 4	11 10 10 4	$     \begin{array}{r}       -3 \\       12 \\       -12 \\       -4     \end{array} $	315 420	-9 54 54 6		-9 -39	18 9 9 6	-9 6	-3	$     \begin{array}{r}       315 \\       -6 \\       -6 \\       420     \end{array} $	$     \begin{array}{r}       11 \\       -6 \\       -6 \\       4     \end{array} $
X.33 X.34 X.35 X.36	420 420 560 560	$420 \\ 420 \\ 560 \\ 560$	$420 \\ 420 \\ 560 \\ 560$	$420 \\ 420 \\ 560 \\ 560$	-60 :	-60	$\begin{array}{r} 4 \\ 4 \\ -16 \\ -16 \end{array}$	$\begin{array}{r} 4 \\ 4 \\ -16 \\ -16 \end{array}$	$\begin{array}{r} 4 \\ 4 \\ -16 \\ -16 \end{array}$	4	$     \begin{array}{r}       420 \\       -210 \\       560 \\       560     \end{array} $	$\begin{array}{c} 6 \\ 30 \\ 20 \\ 2 \end{array}$	$     \begin{array}{r}       -39 \\       -3 \\       \hline       20 \\       -34     \end{array} $	-39 $6$ $20$ $-34$	6 2 2	$-15 \\ 20 \\ 2$	-3 2 2	420 420 560 560	$\begin{array}{r} 4 \\ 4 \\ -16 \\ -16 \end{array}$
X.37 X.38 X.39 X.40	560 630 672	560 630 672 720	560 630 672 720	560 630 672 720	:	:	$-16 \\ -10 \\ 32 \\ 16$	$-16 \\ -10 \\ 32 \\ 16$	$ \begin{array}{r} -16 \\ -10 \\ 32 \\ 16 \end{array} $	:	560 -315 -336 -360	$\begin{array}{c} 2 \\ 18 \\ 12 \\ -18 \end{array}$	$-34 \\ 36 \\ 6$	$ \begin{array}{r} -34 \\ -72 \\ -12 \\ -36 \end{array} $	2	$     \begin{array}{r}       2 \\       -9 \\       -6 \\       9     \end{array} $	2	560 630 672 720	$-16 \\ -10 \\ 32 \\ 16$
$X.41 \\ X.42 \\ X.43$	720 720 729 729	720 729 729	720 $720$ $729$ $729$	720 729 729	-81 81	-81 81	16 9 9	16 9 9	16 9 9	-9 9	-360 - 729 - 729	-18 :	18	-36 :	:	9		720 729 729	16 9 9
X.44 X.45 X.46 X.47	768 768 840 896	-768 $768$ $840$ $896$	768 840 896	768 840 896	64	64	-64 8	64 8	8		$     \begin{array}{r}       -6 \\       -384 \\       -420 \\       \hline       896     \end{array} $	$   \begin{array}{r}     96 \\     24 \\     -12 \\     -4   \end{array} $	$     \begin{array}{r}       -24 \\       -24 \\       -33 \\       32     \end{array} $	$     \begin{array}{r}     -6 \\     48 \\     66 \\     32     \end{array} $	-4	$     \begin{array}{r}       6 \\       -12 \\       6 \\       -4     \end{array} $	$-\frac{1}{4}$	768 840 896	8
$X.48 \\ X.49 \\ X.50 \\ X.51$	1280	896 1260 1280 1280	896 1260 1280	896 1260 1280	-64	-64	12 64	$12^{\dot{2}}$ $-64^{\dot{4}}$	12		$^{896}_{-630}$ $^{1280}_{20}$	$     \begin{array}{r}     -4 \\     -18 \\     -16 \\     32     \end{array} $	$ \begin{array}{r} 32 \\ -9 \\ -16 \\ 56 \end{array} $	$     \begin{array}{r}       32 \\       18 \\       -16 \\       -7     \end{array} $	-4 $-16$ $-8$	$-4 \\ 9 \\ -16 \\ -4$	-4 2 8	896 1260 1280	12
$X.52 \\ X.53 \\ X.54 \\ X.55$	1280 - 1458 1512	1280 $1458$ $1512$ $1701$	$1458 \\ 1512 \\ -27$	$1458 \\ 1512 \\ 37$	: 27	: 27	$     \begin{array}{r}       64 \\       18 \\       -24 \\       117     \end{array} $	$     \begin{array}{r}     -64 \\     18 \\     -24 \\     117     \end{array} $	$^{18}_{-24}$	19	$   \begin{array}{r}     20 \\     -729 \\     -756   \end{array} $	32 81	$\frac{56}{-27}$	$-7$ $5\dot{4}$	8	-4 :	8	$1458 \\ 1512 \\ -27$	$^{18}_{-24}$ $^{-27}$
X.56 X.57 X.58 X.59	$\frac{1701}{1890}$	$\begin{array}{c} 1701 \\ 1890 \\ 2016 \end{array}$	$^{-27}_{1890} \\ _{-32}$	$^{37}_{1890} \\ -32$	-27	-27 :	$^{117}_{-30}$ $^{96}$	$^{117}_{-30}$ $^{96}$	$^{21}_{-30}$ $^{-32}$	$-19 \\ 32$	-945	81 120	27 36	-54	6	3	9 6	$^{-27}_{1890}_{32}$	$^{-27}_{-30}$
$X.60 \\ X.61 \\ X.62$	$\begin{array}{r} 2268 \\ 2268 \\ 3584 - \end{array}$	2016 2268 2268 3584	$-32 \\ 348 \\ 348$	-32 $-36$ $-36$	$-144 \\ 144 \\ -192$	$-144 \\ 144 \\ 192$	$\begin{array}{r} 96 \\ 60 \\ 60 \\ -128 \end{array}$	96 60 60 128	$ \begin{array}{r} -32 \\ -4 \\ -4 \end{array} $	-32	56	120 32	$   \begin{array}{r}     36 \\     -81 \\     -81 \\     -16   \end{array} $	· · 2	6 27 27 44	3 -4	6 8	$^{32}_{-36}$ $^{-36}$	$^{32}_{-36}$ $^{-36}$
X.63	3584 - 3584 - 3584 -	3584 3584 3584 3780	580	-60	$^{128}_{-128}$ $^{129}$	128	-128 $-128$ $-128$ $-36$	128 128 128 36	-28	:	56 56 56	$176 \\ 176 \\ 32 \\ 216$	$-16 \\ -16 \\ -16 \\ 27$	2 2 2	$\begin{array}{c} 8 \\ 8 \\ 44 \\ -18 \end{array}$	$     \begin{array}{r}       -22 \\       -22 \\       -4     \end{array} $	8 8 8	-60	-60
$X.67 \\ X.68$	4480 - 4480 - 4480 -	4480 $4480$ $4480$	:	:	$-160 \\ 160$	-160 $160$ $-160$	-32 $-32$ $-32$	32 32 32	:	:	70 70	$112 \\ 112 \\ -32$	$^{-128}_{-128}$	16 16 -11	28 28 28	$-14 \\ -14 \\ 4$	-8 -8 -8	:	:
$X.72 \\ X.73$	4480 - 4480 - 5670	$\frac{4480}{5670}$	870	-90	$-160 \\ -160 \\ -160 \\ 180$	-160 $160$ $160$ $180$	-32 $-32$ $-32$ $150$	32 32 32 150		12	70 70	-32 $-32$ $-32$ $162$	$^{88}_{-81}$	-11 -11 -11	28 28 28	4 4 4	-8 -8 -8	-90	6
$X.74 \\ X.75 \\ X.76 \\ X.77$	5670	5670 5670 5670 7560	870 870 870 1160	$     \begin{array}{r}       -90 \\       -90 \\       -90 \\       -120     \end{array} $	$-180 \\ -180 \\ -180$	$^{180}_{-180}$	$     \begin{array}{r}     -42 \\     -42 \\     150 \\     -120     \end{array} $	$     \begin{array}{r}     -42 \\     -42 \\     150 \\     -120     \end{array} $	-10	-36 -36 -12		162	$     \begin{array}{r}       162 \\       162 \\       -81 \\       -189     \end{array} $		27 27 18	:		$-90 \\ -90 \\ -90 \\ -120$	$\begin{array}{c} 6 \\ 6 \\ 6 \\ 72 \end{array}$
	7560 7560 7680 – 7680 –		1160 1160	$-120 \\ -120 \\ -120 \\ .$	-360 360	-360 360	$     \begin{array}{r}       168 \\       168 \\       -128 \\       -128    \end{array} $	168 168 128 128	40 40	-24 24		-54 -54 -96	54 54 -96 -96	12	$\begin{array}{r} 45 \\ 45 \\ -24 \\ -24 \end{array}$	12 12		-120 $-120$ $-120$	-24
$X.82 \\ X.83 \\ X.84$	8064 8064 8505	8064 8064 8505	-128 $-135$	$-128 \\ 185$	135	135	153	153	5 <u>7</u>	64 -64 23	:	$156 \\ 156 \\ 162$	144 144	12	24 24 24	12 12 12	6 -9	128 128 -135	9
X.85 X.86 X.87 X.88	8505 8505 8505 8960 –	8505 8505 8505 8960	-135 -135 -135	100	$-135 \\ 135 \\ -135 \\ .$	100	153 $153$ $153$ $-64$	153 153 153 64	57	-23 23 -23		$     \begin{array}{r}       162 \\       -81 \\       -81 \\       224     \end{array} $	176	-22	-16	-28	18 18	-135 -135 -135	9 9 9

Character table of  $H(\mathrm{Fi}_{24}')$  (continued)

$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	11 12 10 1 . 1 	10 10 9 9 1 1 1 1 1 	9 10 10 9 6 1 1 1	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	7 2h 2h 2h 5a 7 4r 4s 4t 5a 7 4r 4s 4t 1a	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
X.1 1 1 1 1 $X.2$ -1 -1 -1 1 $X.3$ 9 9 1 5 $X.4$ -9 -9 -1 5	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	3 3 3 3 3 3 3 3	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$egin{array}{cccccccccccccccccccccccccccccccccccc$	-2  10  -2  10	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$		$egin{array}{cccccccccccccccccccccccccccccccccccc$
$egin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	189 . 27 27
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	10 10 2  8 8 . 8 8 .	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$		$\begin{array}{cccccccccccccccccccccccccccccccccccc$
X.29 - 75 - 75 - 3 11 X.30	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$1-2 \cdot -2 \cdot 3$	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$		4	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	. 32 . 32 . 16 . 16 . 16 . 16 -3 9-3 9	-9 - 9 3	3 3 1 3	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
$egin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		-3 -3 1 -3 		$\begin{array}{cccccccccccccccccccccccccccccccccccc$
X.48 - 64 - 64	$\begin{array}{cccccccccccccccccccccccccccccccccccc$		4	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	. 81
$egin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	8 . 8 . 88 44 . 4 . 4	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$egin{array}{cccccccccccccccccccccccccccccccccccc$	12 -12 12 4 -8 . 8 .	16	16	4 4 	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
X.67 X.68 X.69 X.70 X.71	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	8	8 . 8 8 8 . 8		$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
X.72	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
X.80 X.81 X.81	. 824	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	88	8 3 3	$\begin{array}{c} \cdot & 108 - 189 \\ \cdot & -54 & 54 \\ \cdot & -54 & 54 \\ \cdot & -120 & 96 & 96 - 12 \\ -120 & 96 & 96 - 12 \\ \end{array}$
X.8264 $X.83$ 64 $X.84-25$ . 15 . 237		$\begin{array}{cccccccccccccccccccccccccccccccccccc$		6 6	. 156 144
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$9 - 15 \ 9 \ 9$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	9 -3 -3 -3	$\vec{1} - \vec{7} = \vec{5}$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$

2 3 5	8 5	9 4	10 3	5 6	$^{11}_{\ 2}$	8 3	8 3	8 3	8 3	8 3	8 3	8 3	8 3	8 3	8 3	6 4	7 3	5 4	5 4	5 4	5 4	7 2	5 3
2P 3P 5P 7P	6 <sub>5</sub> 3e 2a 6 <sub>5</sub> 6 <sub>5</sub>	6 <sub>6</sub> 2 <sub>b</sub> 6 <sub>6</sub> 6 <sub>6</sub>	67 3b 2b 67 67	68 3f 2a 68 68	69 3b 2c 69	$6_{10}$ $3b$ $2f$ $6_{10}$ $6_{10}$	$6_{11}$ $3a$ $2g$ $6_{11}$ $6_{11}$	$6_{12}$ $3b$ $2e$ $6_{12}$ $6_{12}$	6 <sub>13</sub> 3b 2d 6 <sub>13</sub> 6 <sub>13</sub>	$ \begin{array}{r}     6_{14} \\     3a \\     2f \\     6_{14} \\     6_{14}   \end{array} $	$6_{15}$	$6_{16}$ $3c$ $2g$ $6_{16}$ $6_{16}$	$\frac{6_{17}}{3e}$ $\frac{3e}{2g}$ $\frac{6_{17}}{6_{17}}$	$6_{18}$ $3e$ $2f$ $6_{18}$ $6_{18}$	$6_{19}$ $3b$ $2g$ $6_{19}$ $6_{19}$	$6_{20}$ $3g$ $2a$ $6_{20}$ $6_{20}$	$6_{21}$ $3e$ $2b$ $6_{21}$ $6_{21}$	$\begin{array}{r} 6_{22} \\ 3d \\ 2d \\ 6_{23} \\ 6_{22} \end{array}$	622	$\begin{array}{c} 6_{24} \\ 3d \\ 2e \\ 6_{24} \\ \end{array}$	$\begin{array}{c} 3d \\ 2e \\ 624 \\ 625 \end{array}$	$\frac{2i}{626}$ 6	$\frac{.}{27}$ $3e$ $2d$ $27$
X.1 X.2 X.3 X.4 X.5 X.6 X.7 X.8 X.9 X.11 X.12 X.13 X.14 X.15 X.17 X.24 X.21 X.22 X.21 X.23 X.24 X.25 X.27 X.23 X.23 X.23 X.23 X.24 X.25 X.26 X.27 X.27 X.27 X.27 X.27 X.27 X.27 X.27	65 11 33 33 8 8 1-1 -1 8 9 9 -4 -4 -4	$\begin{array}{c} 6_{6} \\ \hline 1\\ -6_{6} \\ -6_{8} \\ 8\\ 8\\ 8\\ 8\\ -3\\ 9\\ 9\\ 5\\ 5\\ 5\\ 27\\ -25\\ -15\\ -21\\ 10\\ 10\\ 10\\ -9\\ -9\\ -5\\ -39\\ -39\\ -39\\ -39\\ -39\\ -39\\ -39\\ -39$	$\begin{array}{c} 6\dot{7} \\ \hline 1 \\ 3 \\ 3 \\ 6 \\ -1 \\ 8 \\ 8 \\ -1 \\ 1 \\ 1 \\ 1 \\ 2 \\ 4 \\ -4 \\ -4 \\ \cdot \\ $	$\begin{array}{c} 6_8 \\ \hline 1 \\ 3 \\ 3 \\ -3 \\ -1 \\ 8 \\ 8 \\ -1 \\ -6 \\ -1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ $	$\begin{array}{c} 69 \\ -1 \\ 33 \\ 36 \\ -11 \\ 88 \\ -11 \\ 12 \\ 99 \\ -44 \\ -4 \\ -4 \\ -4 \\ -4 \\ 11 \\ 11 \\ 1$	$\begin{array}{c} 6_{10} \\ \hline 1 \\ -1 \\ -1 \\ -1 \\ -2 \\ 3 \\ 3 \\ 4 \\ 4_{11} \\ 1 \\ \vdots \\ 6_{11} \\ -2_{11} \\ -1_{11} \\ 1_{12} \\ -1_{11} \\ 6_{11} \\ 6_{12} \\ -1_{12} \\ -1_{13} \\ -1_{14} \\ -1_{15} \\$	$\begin{array}{c} 6_{11} \\ 1 \\ 1 \\ 5 \\ 5 \\ 5 \\ 1 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3$	$\begin{array}{c} 6_{12} \\ -1 \\ -3 \\ 3 \\ 3 \\ -2 \\ 2 \\ 2 \\ -3 \\ 3 \\ -22 \\ 2 \\ 2 \\ \\ \end{array}$	$\begin{array}{c} 6_{13} \\ -1 \\ -1 \\ -3 \\ 3 \\ 3 \\ -2_{2} \\ -3 \\ 3 \\ -2_{2} \\ 2 \\ \vdots \\ 3 \\ -1 \\ -1 \\ 1 \\ 1 \\ 1 \\ -1 \\ 6 \\ -6 \\ 6 \\ 6 \\ 6 \\ 6 \\ \end{array}$	$ \begin{array}{c} 6144 \\ 11 \\ 15 \\ 55 \\ 51 \\ 33 \\ 33 \\ 33 \\ -50 \\ 100 \\ 100 \\ 122 \\ 27 \\ 72 \\ 29 \\ -8 \\ -8 \\ -8 \\ -8 \\ -8 \\ -8 \\ -8 \\ -8$	$\begin{array}{c} 615 \\ 1 \\ 2 \\ 2 \\ -2 \\ \end{array}$ $\begin{array}{c} -2 \\ \cdot \\ \cdot \\ \cdot \\ 11 \\ -3 \\ -3 \\ 3 \\ 3 \\ -3 \\ 5 \\ \cdot \\ -2 \\ -2 \\ -2 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1$	$\begin{array}{c} 616 \\ 1 \\ 1 \\ 2 \\ 2 \\ -2 \\ \vdots \\ 1 \\ 1 \\ 1 \\ 1 \\ -3 \\ 3 \\ 3 \\ 3 \\ -3 \\ 3 \\ 3 \\ 3 \\ -3 \\ 5 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \\ 1 \\ 1 \\ 1 \\ 1 $	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{c} 6_{188} \\ \hline 1 \\ 1 \\ -1 \\ -1 \\ 4 \\ \vdots \\ 3_{3} \\ 3_{3} \\ \vdots \\ 4_{4} \\ 1_{1} \\ \vdots \\ -1_{4} \\ 4_{-1} \\ \vdots \\ -2_{-2} \\ -2_{-2} \\ -2_{-2} \\ -1_{2} \\ 2_{-2} \\ 2$	$\begin{array}{c} 6_{199} \\ \hline 1 \\ 1 \\ -1 \\ -2 \\ 3 \\ 3 \\ 4 \\ 4 \\ 1 \\ 1 \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	620 1 1 3 3 3 3 3 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1	$\begin{array}{c} 621 \\ \hline 1 \\ 1 \\ 3 \\ 3 \\ 3 \\ . \\ . \\ . \\ . \\ . \\ . \\ .$	$\begin{array}{c} 622\\ \hline 1\\ -1\\ \\ -$	$\begin{array}{c} 623 \\ \hline 1 \\ \hline -1 \\ \\ -6 \\ 4 \\ -4 \\ -6 \\ \hline \\ -3 \\ \hline 3 \\ \hline 7 \\ -7 \\ -9 \\ 9 \\ \\ \hline \\ 3 \\ \\ -3 \\ \\ \hline \\ -A \\ \overline{A} \\ A \\ A \\ -A \\ \overline{A} \\ 3 \\ \\ \hline \\ 3 \\ \\ 3 \\ \\ \hline \\ -3 \\ \\ \end{array}$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
X 35 X 36 X 337 X 339 X 441 X 442 X 443 X 444 X 45 X 45 X 55 X 55 X 56 X 55 X 56 X 57 X 58 X 56 X 57 X 58 X 57 X 58 X 58 X 58 X 58 X 58 X 58 X 58 X 58	-8 -8 -8 -8 -8 -8 -8 -8 -8 -8 -8 -8 -8 -	18 -24 -33 32 32		9	20 22 188 121 -188 -188 -188 -188 -198	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	2 2 2	$ \begin{array}{c} 16 \\ -6 \\ 12 \\ -12 \\ 6 \\ -6 \\ 6 \\ \vdots \\ 18 \end{array} $		$\begin{array}{c} -166 \\ -166 \\ -166 \\ -8 \\ -8 \\ -8 \\ -8 \\ -8 \\ -15 \\ -8 \\ -8 \\ -8 \\ -8 \\ -8 \\ -8 \\ -8 \\ -$	$\begin{array}{c} -22 \\ -2 \\ -4 \\ -1 \\ -3 \\ -3 \\ -3 \\ -3 \\ -3 \\ -8 \\ -8 \\ -8$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	2 2 2 2 4 4 4 4 4 4 4 4 1 8 8 8 8 8 8 8 8 8 8 8	2 2 2 2 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	2 2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3	2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	$\begin{array}{c} D \\ \bar{D} \\ \bar{D} \\ \vdots \\ \vdots \\ -88 \\ \cdot \\$		$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	 6	

Character table of  $H(\mathrm{Fi}_{24}')$  (continued)

2 3 5 7	5 3	5 3	5 3	5 3	5 3	5 3	5 3	7	5 2	3 2	1 1	2	9 1	8	8	8	8	8	8	6 1	6	6	6	5 4	3 4	$\overset{1}{\overset{4}{\cdot}}$	2 2 3 3	2 3
2P 3P 5P 7P	$^{3d}_{2g}_{6_{28}}$	$^{2g}_{629}$	$^{2f}_{630}$	$^{2f}_{631}$	$^{2e}_{632}$	$^{2g}_{633}$	$^{2f}_{634}$	$\frac{3e}{2h} \\ 6_{35}$	$\begin{array}{c} 3g \\ 2c \\ 636 \end{array}$	$\begin{array}{c} 3g \\ 2i \\ 637 \end{array}$	7a $7a$	4a 8a 8a	4b 8b 8b	8c 8c	4j $8d$ $8d$	8e 8e	4f 8f 8f	8g 8q	8h $8h$	8i	8j	8k	8l 8l 8l	3c	$\frac{3c}{9b}$	3c :	9d 9e 9e 9d 3d 3d 9e 9d 9d 9e	$\frac{1}{d}$
X.1 X.2 X.3 X.4	1 1 2 2	$\begin{array}{c} 1 \\ 1 \\ -1 \\ -1 \end{array}$	$\begin{array}{c} 1 \\ 1 \\ -1 \\ -1 \end{array}$	$\frac{1}{2}$	$-1^{-1}$	$\begin{array}{c} 1 \\ 1 \\ -1 \\ -1 \end{array}$	$     \begin{array}{c}       1 \\       1 \\       -1 \\       -1    \end{array} $	$\begin{array}{c} 1 \\ 1 \\ -1 \\ -1 \end{array}$	3	$-{1\atop 1}$	1	1	1 1 1	$-1 \\ -1 \\ 1 \\ -1$	$\begin{array}{c} 1 \\ -1 \\ -3 \\ 3 \end{array}$	$-{1 \atop 1}$	1 1 1 1	1 1 1 1	$-{1 \atop 1}$	$     \begin{array}{c}       1 \\       1 \\       -1 \\       -1     \end{array} $	$-1^{-1}$	$     \begin{array}{c}       1 \\       1 \\       -1 \\       -1    \end{array} $	$-1^{1}$	1		1		1
$X.5 \\ X.6 \\ X.7 \\ X.8 \\ X.9$	4	3 3	-2 3 3	4	-1	1 3	1 3	3	$-1 \\ -1 \\ -1 \\ -1$	$-1 \\ -1 \\ -1 \\ 1$		-1 -5 5	3 3	$     \begin{array}{r}       3 \\       -1 \\       1 \\       -3     \end{array} $	$-1 \\ -3 \\ -3 \\ 1$		$-1 \\ -1 \\ -1$	$-1 \\ -1 \\ -1$	$-\frac{1}{3}$	-1	$-\frac{1}{1}$	$ \begin{array}{c} 2 \\ -1 \\ -1 \\ -1 \\ -1 \end{array} $	$\substack{-1\\1\\1}$	$\begin{array}{c} 2\\2\\-1\end{array}$		2 -	$\begin{array}{c} \dot{2} & \dot{2} \\ -1 & -1 \\ -1 & -1 \\ 2 & \dot{2} \end{array}$	
X.10 X.11 X.12 X.13 X.14	$     \begin{array}{r}       -2 \\       1 \\       -3 \\       -3     \end{array} $	1 1	$-2 \\ 1 \\ 1 \\ \cdot \\ \cdot$	-2 $1$ $1$ $-3$ $-3$	$-\frac{1}{3}$ $-\frac{1}{2}$	1 1		1					$\frac{5-2}{4}$	$-\frac{2}{2}$	$-\frac{1}{2}$ $-4$ $4$	2	2	2 2	$-\frac{2}{2}$	-2		-2		-i	-3 -1 -1		· -1 -1	1
$X.15 \\ X.16 \\ X.17 \\ X.18 \\ X.19$	6 5	:	-1 -2	$\begin{array}{c} 3 \\ 3 \\ 6 \\ -2 \end{array}$		:	-3 -1 1	-1	3			-10	5 2	-3 $-2$	$-1 \\ 1 \\ 2$	-3	$\frac{1}{2}$	$-\frac{1}{2}$	-3 $-2$	$\begin{array}{c} 1 \\ 1 \\ 2 \\ -2 \end{array}$	-1 1		-1 1	-3 -3	3 3		:	
$X.20 \\ X.21 \\ X.22 \\ X.23$	$   \begin{array}{r}     5 \\     -2 \\     -2   \end{array} $	-1 $-2$ $-2$	-1 $-2$ $-2$	$   \begin{array}{r}     5 \\     -2 \\     -2   \end{array} $	2 2		-1 1 1	-1 $-2$ $-2$	3 1	1 -1 -1		8	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	2	-2	2	$-\frac{1}{2}$	-2	2	2		2		3 1 1	1 1	-3 1 1	B Ē Ē E	3
X.24 X.25 X.26 X.27 X.28	$     \begin{array}{r}       -2 \\       -2 \\       -1 \\       -1 \\       -1     \end{array} $	$-2 \\ -1$	$-\frac{1}{2}$	$     \begin{array}{r}       -2 \\       -2 \\       -1 \\       -1 \\       -1     \end{array} $	$     \begin{array}{r}       -2 \\       -2 \\       -3 \\       \hline       3     \end{array} $	$\begin{array}{c} 1 \\ 1 \\ 2 \\ -1 \\ 2 \end{array}$	2	$-2 \\ -1 \\ 2$	1	1 1			$\begin{array}{ccc} 3 & . \\ -1 & . \\ 3 & -1 \end{array}$	3	5 -1 -5	3		-1	1 3 -1	1 1 1	-i -i 1	1 1 1	-1 -1 1 1	1 1	1		B E B E	
X.29 X.30 X.31 X.32 X.33	-1 · i		$\begin{array}{c} 2 \\ 3 \\ 3 \\ -2 \end{array}$	-1	-3 -3	-1 $-2$ $-2$	-1 $-2$	$\begin{array}{c} 2 \\ 1 \\ 1 \\ -2 \end{array}$	-3	-1 1		-3 -4		$-3 \\ -4 \\ 4 \\ .$	1	-3 :	-1	$-1 \\ -2$	-3		$-1 \\ -2 \\ 2 \\ .$		$-\frac{1}{2}$ $-\frac{2}{2}$	9 9				
X.34 X.35 X.36 X.37	$     \begin{array}{r}       -2 \\       -4 \\       2 \\       2     \end{array} $	$-\frac{2}{2}$	$-\frac{2}{2}$ $\frac{2}{2}$	$-2 \\ -4 \\ 2 \\ 2$		$\begin{array}{c} 1 \\ -4 \\ 2 \\ 2 \end{array}$	$\begin{array}{c} 1 \\ -4 \\ 2 \\ 2 \end{array}$	4 2 2 2	2 2 2 2				-4				-4	-4						-1	-1	$ \begin{array}{c}  -3 \\  -1 \\  -1 \\  -1 \end{array} $	$\begin{array}{c} \cdot \\ \cdot \\ -1 & -1 \\ -1 & -1 \end{array}$	2 1
X.38 X.39 X.40 X.41 X.42	-4	$-\frac{2}{2}$	$ \begin{array}{c} 2 \\ -2 \\ -2 \\ -2 \end{array} $	$     \begin{array}{r}       8 \\       -4 \\       4 \\       4    \end{array} $		$-1 \\ 2 \\ 1 \\ 1 \\ .$	$-1 \\ 2 \\ 1 \\ 1 \\ .$	-444			$-1 \\ -1 \\ -1 \\ 1$			3	3	3	1			-2 -1		-2 -1	1	-3 :	:	3		
X.43 X.44 X.45 X.46 X.47		4 2	$-\frac{i}{2}$	2 2		$-\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}$	2		-4		$ \begin{array}{c} 1 \\ -2 \\ -2 \end{array} $		0 -3 · · · · · 8 · · ·	-3	-3	-3	1	1	-3	-1 :	-1	-1 :	-1 :	3		-3 3 -3 -1 -	: -i -1	1
$X.48 \\ X.49 \\ X.50 \\ X.51 \\ X.52$	-6 $-1$ $-1$	-4 -4	4 4	-6	2	-3 $-4$ $-4$	-3 4 4		-4 2		-1 -1 -1		4			:	-4 :	-4 :			:	:		$-1$ $\begin{array}{c} 2 \\ -4 \\ -4 \end{array}$	2 2	$ \begin{array}{c} -1 - \\ 2 \\ -1 - \\ -1 - \end{array} $	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	1 2 1
X.53 X.54 X.55 X.56 X.57	6			6	:				1 1	-1	2	- i			-1 1	-i -1		1		-2 3 3		$ \begin{array}{c} -2 \\ -1 \\ -1 \\ 2 \end{array} $	-1		:			
$X.58 \\ X.59 \\ X.60 \\ X.61$		3	3	-6	-3 3	3 3	3 3	$-1 \\ -1$	-2	$-\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}$			. 4	4 -4		-4 4			4 -4					6 6	3 3			
$X.62 \\ X.63 \\ X.64 \\ X.65 \\ X.66$	2 2 2 2	-4	$     \begin{array}{r}       -2 \\       4 \\       4 \\       -2 \\     \end{array} $	$     \begin{array}{r}       -2 \\       -2 \\       -2 \\       -2 \\     \end{array} $	$     \begin{array}{r}       6 \\       -8 \\       8 \\       -6 \\     \end{array} $	$\begin{array}{c} 2 \\ 2 \\ -4 \\ \end{array}$	$ \begin{array}{c} 4 \\ -2 \\ -2 \\ 4 \end{array} $						4				-4	· . 4							$ \begin{array}{c} 2 \\ -4 \\ -4 \\ 2 \end{array} $	$-1 \\ 2 - \\ 2 - \\ -1 \\ \cdot$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2 1 1 2
X.67 X.68 X.69 X.70		2	$     \begin{array}{r}       -2 \\       -2 \\       -2 \\       -2     \end{array} $	4 4 1 1	2		$     \begin{array}{r}       -2 \\       -2 \\       4 \\       4   \end{array} $								•	:			•						$     \begin{array}{r}       -2 \\       -2 \\       -2 \\       -2     \end{array} $		1 1 1 B E E E E E E E E E E E E E E E E	3
X.71 X.72 X.73 X.74 X.75 X.76 X.77	-1 -1	$-\frac{2}{6}$	$     \begin{array}{r}       -2 \\       -6 \\       -3 \\       -3     \end{array} $	1	$     \begin{array}{r}       -2 \\       -2 \\       6 \\       -3 \\       3     \end{array} $	-4 :	4	- <u>i</u>						-4 4			-2	$-\frac{1}{2}$	4		2 2 -2		$-\frac{1}{2}$	4	-2 -2	1	B E B E	
X.76 X.77 X.78 X.79 X.80		$ \begin{array}{r} -3 \\ -6 \\ -6 \\ 3 \\ -4 \end{array} $	$     \begin{array}{r}       -6 \\       -6 \\       3 \\       3 \\       4     \end{array} $	4	-6 -3 3		4	2 1 1					. 8	8		4	. 2	:	-4 :		-2			$-9 \\ -9 \\ -9 \\ -9 \\ .$	:	:		
X.81 X.82 X.83 X.84	-4	-4 :	4	4		-4 :	4		$ \begin{array}{c} -2 \\ -2 \\ -1 \end{array} $	$-\frac{1}{2}$	1	— 5	 	3	-1	-i	i	-3 -3	i			i	-1	6 6	3 3		:	
X.85 X.86 X.87 X.88		-8	8	2	:	4	-4		2	$-\frac{2}{2}$	:	-5	$\frac{5}{5} - \frac{3}{3}$	-3	$-\frac{1}{1}$	$-\frac{1}{1}$	. 1	$-3 \\ -3$	$-\frac{1}{1}$	-3 -3 -3	$-\frac{1}{1}$	1 1 1	$-{1\atop 1}$	$-\dot{4}$	2	: -1	· · ·	2

2 3 5 7	6 1 1	5 1	5 1	5 i	4 i	4 1	9 5	7 5	9 3	6 4	4 5	8 2	8 2	7 2	5 3	5 3	6 2	6 2	6 2	6 2	6 2	7 1
5P	10a 5a 10a 2a 10a	$\frac{10c}{2c}$	$\frac{10b}{2c}$	10d $2b$	$     \begin{array}{r}       10e \\       5a \\       10e \\       2d \\       10e     \end{array} $	2e	$\frac{4a}{121}$	$     \begin{array}{r}       12_2 \\       6_3 \\       4a \\       12_2 \\       12_2     \end{array} $	$     \begin{array}{r}       12_{3} \\       6_{2} \\       4a \\       12_{3} \\       12_{3}     \end{array} $	$ \begin{array}{r} 12_4 \\ 6_5 \\ 4_a \\ 12_4 \\ 12_4 \end{array} $	$     \begin{array}{r}       12_{5} \\       6_{8} \\       4a \\       12_{5} \\       12_{5}     \end{array} $	$ \begin{array}{r} 12_{6} \\ 6_{9} \\ 4_{6} \\ 12_{6} \\ 12_{6} \end{array} $	$     \begin{array}{r}       12_{7} \\       6_{9} \\       4_{e} \\       12_{7} \\       12_{7}     \end{array} $	4h	$     \begin{array}{r}       129 \\       620 \\       4a \\       129 \\       129     \end{array} $	$ \begin{array}{r} 12_{10} \\ 6_{20} \\ 4a \\ 12_{10} \\ 12_{10} \end{array} $	$ \begin{array}{r} 12_{11} \\ 6_{7} \\ 4d \\ 12_{11} \\ 12_{11} \end{array} $	$ \begin{array}{r} 12_{12} \\ 6_{14} \\ 4g \\ 12_{12} \\ 12_{12} \end{array} $	4h	$\begin{array}{r} 12_{14} \\ 6_5 \\ 4b \\ 12_{14} \\ 12_{14} \end{array}$	$ \begin{array}{r} 12_{15} \\ 6_{14} \\ 4i \\ 12_{15} \\ 12_{15} \end{array} $	$ \begin{array}{r} 12_{16} \\ 6_{9} \\ 4_{c} \\ 12_{16} \\ 12_{16} \end{array} $
X.1 X.2 X.3 X.4 X.5	1 1 1 1	1 1 1 1	1 1 1 1	1	$-1 \\ -1 \\ -1 \\ 1$	1 -1 -1 1	1 1 3	$ \begin{array}{r} 1 \\ -6 \\ -6 \\ -6 \end{array} $	1 1 3 3 6	1 1 3 3	1 3 3 -3	1 -1 1 -1	$-1 \\ -1 \\ -1 \\ -1$	$\begin{array}{c} 1 \\ 1 \\ 2 \\ 2 \\ -2 \end{array}$	1 1 3 3	1 1 3 3	1 -1 -3 3	1 1 1 1 -3	$\begin{array}{c} 1 \\ 1 \\ -1 \\ -1 \end{array}$	$\begin{array}{c} 1 \\ 1 \\ -1 \\ -1 \end{array}$	1 1 1 1 -3	-1 -3 -3
$X.6 \ X.7 \ X.8 \ X.9 \ X.10$	2	· · · 2	· · · 2	2			$     \begin{array}{r}       -1 \\       8 \\       8 \\       -1 \\       12     \end{array} $	8 8 8 -3	$     \begin{array}{r}       -1 \\       8 \\       8 \\       -1 \\       12     \end{array} $	$     \begin{array}{r}       8 \\       -1 \\       -1 \\       8     \end{array} $	$-1 \\ 8 \\ 8 \\ -1 \\ -6$	$     \begin{array}{c}       -1 \\       -2 \\       2 \\       1     \end{array} $	$     \begin{array}{c}       -1 \\       -2 \\       2 \\       1     \end{array} $	i	$     \begin{array}{r}       -1 \\       -1 \\       -1 \\       -1 \\     \end{array} $	$     \begin{array}{r}       -1 \\       -1 \\       -1 \\       -1 \\     \end{array} $	$     \begin{array}{r}       3 \\       -2 \\       2 \\       -3 \\     \end{array} $	3 3 3 -1	3	3 3 4	3 3 3 -1	$     \begin{array}{r}       3 \\       -2 \\       2 \\       -3 \\     \end{array} $
$X.11 \\ X.12 \\ X.13 \\ X.14 \\ X.15$	-i	-i	-i	-i	i	i	$   \begin{array}{c}     9 \\     9 \\     -4 \\     -4   \end{array} $	9 5 5 27	$   \begin{array}{c}     9 \\     9 \\     -4 \\     -4   \end{array} $	$   \begin{array}{c}     9 \\     9 \\     -4 \\     -4   \end{array} $	9 9 -4 -4	$ \begin{array}{r} -3 \\ 3 \\ 2 \\ -2 \\ \end{array} $	$-3 \\ 3 \\ -2 \\ -2 \\ .$	$ \begin{array}{c} 1 \\ -3 \\ -3 \\ 3 \end{array} $	5 5	5 5	$     \begin{array}{r}       -3 \\       3 \\       -2 \\       2     \end{array} $	$     \begin{array}{r}       -2 \\       -2 \\       4 \\       4 \\       5     \end{array} $	1	1 1	$     \begin{array}{r}       -2 \\       -2 \\       4 \\       4 \\       5     \end{array} $	$     \begin{array}{r}       -3 \\       3 \\       -2 \\       2     \end{array} $
$X.16 \\ X.17 \\ X.18 \\ X.19 \\ X.20$	-1	-1	-1	-1 : :	-1	-1	6 3 6 3	$ \begin{array}{r} 27 \\ -15 \\ 21 \\ -15 \\ 21 \end{array} $	6 3 6 3	3 3	$ \begin{array}{r}     -3 \\     3 \\     -3 \\     3 \\     -12 \end{array} $	-i -i i	-i -i i	$     \begin{array}{r}       3 \\       -3 \\       5 \\       1 \\       5     \end{array} $	3 3	3 3	3 -3	$     \begin{array}{r}       5 \\       -1 \\       -2 \\       -5 \\       -2     \end{array} $	$ \begin{array}{c}                                     $	$-1 \\ -1 \\ -1 \\ -1$	$     \begin{array}{r}       5 \\       -1 \\       -2 \\       -5 \\       -2     \end{array} $	3 -3
X.21 $X.22$ $X.23$ $X.24$ $X.25$			•				24 1 1 1	12 10 10 10 10	24 1 1 1	10 10 10 10	1 1 1 1	$\begin{array}{c} -1 \\ -1 \\ 1 \\ 1 \end{array}$	$\begin{array}{c} -\overset{\cdot}{1}\\-\overset{\cdot}{1}\\1\\1\end{array}$	$ \begin{array}{r} -2 \\ -2 \\ -2 \\ -2 \\ -2 \end{array} $	1 1 1 1 1	1 1 1 1 1	$     \begin{array}{r}       -1 \\       -1 \\       1 \\       1    \end{array} $	-1	1 1 1 1	$ \begin{array}{r} -2 \\ -2 \\ -2 \\ -2 \\ -2 \end{array} $	-1	$     \begin{array}{c}       -1 \\       -1 \\       1 \\       1    \end{array} $
X.26 X.27 X.28 X.29 X.30	3	-1	-1	3	1	1	18 -9 18 -9 18 18	-9 -9 -9 -9 3	18 -9 18 -9 -6 -6	-9 18 -9 18 -3 -3	18 -9 18 -9	-3 3 6 -6	-3 3 6 -6	-1 $-1$ $-1$ $-1$ $3$ $3$	•		-3 $3$ $2$ $-2$	-1 $-1$ $-1$ $-1$	$     \begin{array}{r}       -1 \\       2 \\       -1 \\       2 \\       2     \end{array} $	$     \begin{array}{r}       -1 \\       2 \\       -1 \\       2 \\       -3 \\       -3     \end{array} $	$     \begin{array}{r}       -1 \\       -1 \\       -1 \\       -1 \\     \end{array} $	$ \begin{array}{r}     -3 \\     \hline     3 \\     -2 \\     2 \end{array} $
X.32 X.33 X.34 X.35 X.36	3	-1	-1	3	-1	-1	6 6 30 20 2	$     \begin{array}{r}       -39 \\       -39 \\       \hline       -3 \\       20     \end{array} $	6 6 30 20 2	-3 6 6 2 2 2	$\begin{array}{r} & \overset{\cdot}{6} \\ & 6 \\ -15 \\ 20 \\ 2 \end{array}$	-6 2 -2	-6 2 -2	$\begin{array}{c} 3 \\ 1 \\ 1 \\ -4 \\ 2 \end{array}$	$-\frac{3}{3}$	-3 -3 -3 2 2	-2 -6 6	4 4 2	$-2 \\ -2$	-2	4 4 2	-6 6
X.37 X.38 X.39 X.40 X.41	2	2	2	2			$   \begin{array}{r}     2 \\     18 \\     12 \\     -18 \\     -18   \end{array} $	$^{-34}_{\  \   6}_{\  \   18}$	$     \begin{array}{r}       2 \\       18 \\       12 \\       -18 \\       -18     \end{array} $	2	-9 -6 9			$ \begin{array}{r}     2 \\     -4 \\     2 \\     -2 \\     -2 \end{array} $	2 2	2		-3 :	$ \begin{array}{r}     2 \\     2 \\     -4 \\     -2 \\     -2 \end{array} $	$     \begin{array}{r}       2 \\       -4 \\       -4 \\       4   \end{array} $	-3 :	
$X.42 \\ X.43 \\ X.44 \\ X.45 \\ X.46$	$     \begin{array}{r}       -1 \\       -1 \\       -8 \\       -2     \end{array} $	$-1 \\ -1 \\ -2 \\ \cdot$	$-1 \\ -1 \\ -2$	$-1 \\ -1 \\ -2$	-1 1	-1 1		-24	24 -12		-12 6							$     \begin{array}{r}       -3 \\       -3 \\       2 \\       \hline       -4    \end{array} $	:		$ \begin{array}{r} -3 \\ -3 \\ -2 \end{array} $	:
$X.47 \\ X.48 \\ X.49 \\ X.50 \\ X.51$	1 1	1 1	1 1	1 1	-1 1	-1 1	$     \begin{array}{r}     -4 \\     -4 \\     -18 \\     -16     \end{array} $	$\begin{array}{r} 32 \\ 32 \\ -9 \\ -16 \\ \end{array}$	$     \begin{array}{r}     -4 \\     -4 \\     -18 \\     -16 \\     \cdot   \end{array} $	$^{-4}_{-4}$ $^{-16}$	$^{-4}_{-4}$ $^{9}_{-16}$			3	$     \begin{array}{r}       -4 \\       -4 \\       \dot{2} \\       \vdots    \end{array} $	$     \begin{array}{r}       -4 \\       -4 \\       \dot{2} \\       \vdots    \end{array} $	$-4 \\ -4 \\ \vdots \\ \vdots$	-2 $-4$	6		$-\frac{1}{2}$	$\begin{pmatrix} 4 \\ -4 \\ \vdots \\ \vdots \end{pmatrix}$
$X.52 \ X.53 \ X.54 \ X.55 \ X.56$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{2}{2}$		-2 -2	$\begin{array}{c} \cdot \\ -27 \\ -27 \end{array}$	-27 :	-3 -3				1	-3 :	-3 -3	-3 -3	-3 3	$     \begin{array}{r}       -4 \\       3 \\       4 \\       \vdots \\     \end{array} $			4 3 4	
X.57 X.58 X.59 X.60 X.61	6 6 3 3	$ \begin{array}{r} -2 \\ -2 \\ -1 \\ -1 \end{array} $	$-2 \\ -1$	$     \begin{array}{r}       -2 \\       -2 \\       3 \\       3     \end{array} $		1 -1	8 8	$     \begin{array}{r}       27 \\       -4 \\       -4 \\       -9 \\       -9     \end{array} $	8 8	2 2 -9 -9	-1 -1	$-\frac{\dot{8}}{8}$	$-\frac{\dot{8}}{8}$	$     \begin{array}{r}       3 \\       -4 \\       -4 \\       3 \\       3     \end{array} $	2 2	2 2 2	4 -4	-1	-4 -4	2 2 3 3	-1	-4 4
X.62 X.63 X.64 X.65 X.66 X.67	$     \begin{array}{r}       -4 \\       -4 \\       -4 \\       \hline     \end{array} $				$     \begin{array}{r}       -2 \\       -2 \\       2 \\       2     \end{array} $	-2 $-2$ $-2$		3		6		-12 $-12$ .	$-12^{\circ}_{12}$	-9						-6		
X.68 X.69 X.70												-12 -12	-12 12					$     \begin{array}{r}       -2 \\       -2 \\       -2 \\       -2 \\       -2 \\       -2 \\     \end{array} $			2 2 2 2 2 2 2	
X.71 X.72 X.73 X.74 X.75 X.76							54	$ \begin{array}{r} -9 \\ 18 \\ 18 \\ -9 \\ -21 \end{array} $	-18 -18 -12 6 6	-9 -9 -6		6 -6	6 -6	3 6 6 3 3			$ \begin{array}{r} -2 \\ 4 \\ -4 \\ 2 \end{array} $	-2	$\begin{array}{c} 2 \\ 4 \\ 4 \\ 2 \\ -4 \end{array}$	3 -6		$\begin{array}{c} \cdot \\ \cdot \\ 2 \\ -4 \\ 4 \\ -2 \end{array}$
X.76 X.77 X.78 X.79 X.80 X.81 X.82	-6	· · · · 2	· · · ·	· · · ·			$^{-18}_{-18}$	6 6	6 6	8	-4	$ \begin{array}{c}     & \dot{6} \\     & -6 \\     & \dot{} \\     & -4 \\ \end{array} $	$ \begin{array}{c}     6 \\     -6 \\     \hline     -4 \end{array} $	-6 -6	· · ·	: : : 2	$-\frac{1}{2}$		2 2	-6 -3 -3	•	$ \begin{array}{c} 4 \\ -2 \\ \vdots \\ -2 \\ \vdots \\ \vdots \end{array} $
X.83 X.84 X.85 X.86 X.87 X.88	-6 :	2	2	2			$     \begin{array}{r}       32 \\       -54 \\       -54 \\       27 \\       27   \end{array} $	-16	$     \begin{array}{r}       8 \\       -6 \\       -6 \\       3 \\       3     \end{array} $	8	-4	$ \begin{array}{c} 4 \\ 2 \\ -2 \\ -1 \\ 1 \end{array} $	$ \begin{array}{c} 4 \\ 2 \\ -2 \\ -1 \\ 1 \end{array} $		2 3 3 -6 -6	3 3 -6 -6	-6 6 3 -3	-4	-6 -6 3		4	$\begin{array}{c} 2 \\ -2 \\ -1 \\ 1 \end{array}$

Character table of  $H(\mathrm{Fi}_{24}')$  (continued)

22 33 55 77 2P 3P 5P 12 7P 12	7 1  217 69 4f 217 217	$\begin{array}{c} 5\\2\\\vdots\\12_{18}\\6_{15}\\4g\\12_{18}\\12_{18}\end{array}$	$\begin{array}{r} 5\\2\\ \vdots\\12_{19}\\6_6\\4h\\12_{19}\\12_{19}\end{array}$	$\begin{array}{c} 5\\2\\ \vdots\\12_{20}\\6_{15}\\4i\\12_{20}\\12_{20}\end{array}$	$\begin{array}{c} 6\\1\\\vdots\\12_{21}\\6_{7}\\4k\\12_{21}\\12_{21}\end{array}$	$\begin{array}{c} 4\\2\\\vdots\\12_{22}\\6_{21}\\4d\\12_{22}\\12_{22}\end{array}$	$\begin{array}{c} 4\\2\\\vdots\\12_{23}\\6_{21}\\4h\\12_{23}\\12_{23}\end{array}$	$\begin{array}{c} 4\\2\\\vdots\\12_{24}\\6_{36}\\4e\\12_{24}\\12_{24}\end{array}$	$\begin{array}{c} 4\\2\\\vdots\\12_{25}\\6_{31}\\4g\\12_{25}\\12_{25}\end{array}$	$\begin{array}{c} 4\\2\\\vdots\\12_{26}\\6_{31}\\4i\\12_{26}\\12_{26}\end{array}$	$\begin{array}{c} 4\\2\\\vdots\\12_{27}\\6_{36}\\4e\\12_{27}\\12_{27}\end{array}$	$\begin{array}{c} 4\\2\\\vdots\\12_{28}\\6_8\\4b\\12_{28}\\12_{28}\end{array}$	5 1 12 <sub>29</sub> 6 <sub>35</sub> 40 12 <sub>29</sub> 12 <sub>29</sub>	1220	$\begin{array}{c} 4\\1\\\vdots\\12_{31}\\6_{17}\\4q\\12_{31}\\12_{31}\end{array}$	1222	$\begin{array}{c} 4\\1\\\vdots\\12_{33}\\6_{11}\\4p\\12_{33}\\12_{33}\end{array}$	1 1224	$\begin{array}{c} 1\\1\\\hline 1\\\hline 14a\\\hline 7a\\14a\\14a\\2a\\\end{array}$
X.2 X.3 X.4 X.5 X.7 X.8 X.9 X.10 X.11 X.12 X.13 X.14 X.15 X.16 X.17 X.18 X.20 X.21 X.22 X.24 X.25 X.27 X.28 X.28 X.28 X.29 X.30 X.31 X.41 X.42 X.43 X.44 X.44 X.44 X.44	$\begin{bmatrix} 1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1$	1	11 22 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 5 5 1 5 5 2 2 2 2 1 1 1 1 1 1 1 1 1 1 1	1	-11 -11 -11 -11 -11 -11 -11 -11 -11 -11	3 -3 -2 -2 -2 -3 -3 -1	11 -11 -22 -33 3 3 -3 -2 1 1 1 -1 1 -2 2 -2 1 1 -2 2 -2 2	-11-11-11-11-11-11-11-11-11-11-11-11-11	1 1 2 2 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	1 -22 -2 -2 -2 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1	-1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1				-1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	-2 -2 -2	1
X.445 X.446 X.447 X.488 X.500 X.551 X.553 X.554 X.557 X.558 X.611 X.622 X.632 X.644 X.632 X.648 X.702 X.711 X.723 X.745 X.757 X.758 X.758 X.702 X.712 X.723 X.745 X.745 X.758 X.758 X.758 X.758 X.758 X.759 X.	$\begin{array}{c} \cdot \cdot \cdot \\ -4 \\ \cdot \cdot \cdot \\ $	2 2 1 1 2 2 2 2 2 2 2 2 2 3 3 2 2 2 2 2	-1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -	-2 -2 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1	-22 -22 -22 -21 -11	-22 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	-11-11-11-11-11-11-11-11-11-11-11-11-11	-66 -22 -22 -11 -22 -22 -22 -22 -22 -22 -22	2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	-2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -		-11-11-11-11-11-11-11-11-11-11-11-11-11	-22 -11 11 -22 -22 -22 -22 -22 -11 -11 -	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		· · · · · · · · · · · · · · · · · · ·	-11	EE E E E E E E E E E E E E E E E E E E	-2 -2 

2 3 5 7	1 1 1	5 ·	5 4	3 4	1 4	4 2	2 3		2	:	:	:	1		3 i		1	<u>i</u>		:	32	3 2	4 1	4 1 :
2P 3P 5P 7P	$\frac{5a}{3a}$	16a 16a 16a 16a		$^{63}_{18b}$	$\frac{9c}{63}$ 18c	$^{66}_{18d}$	9e 6 <sub>4</sub> 18 f	9d 6 <sub>4</sub> 18e	$6_{22} \\ 18i$	$\begin{array}{c} 9d \\ 6_{24} \\ 18j \end{array}$	$^{9d}_{623}$ $^{18q}$	$^{9e}_{625} \\ 18h$	10a 20a 4a	$\begin{array}{c} 10b \\ 20d \\ 4c \end{array}$	$\begin{array}{c} 10d \\ 20c \\ 4d \end{array}$	$\frac{10c}{20b}$	21 <i>t</i> 7 <i>a</i> 21 <i>a</i>	21a 7a 21b	$\begin{array}{ccc} 12_{3} \\ 8a \\ 24a \end{array}$	$\frac{8a}{24b}$	$12_{10} \\ 8a \\ 24a$	$ \begin{array}{c} 24d \\ 129 \\ 8a \\ 24d \\ 24d \\ 24d \end{array} $	$     \begin{array}{r}     24e \\     \hline     12_{12} \\     8i \\     24i \\     24i   \end{array} $	$     \begin{array}{r}       24f \\       \hline       12_{14} \\       8c \\       24g \\       24g     \end{array} $
X.1 X.2 X.3 X.4 X.5	1 1 1	$-1 \\ -1 \\ -1$	1 1	1	1		1	1	- 1	1	1	1	1	$-1 \\ -1 \\ -1$	$-1 \\ -1$	$-1 \\ -1 \\ -1$	1	. 1	$ \begin{array}{ccc}  & 1 \\  & -1 \\  & 1 \\  & -1 \end{array} $	$-\frac{1}{1}$	-1 -1 -1	$-\frac{1}{1}$	$ \begin{array}{c} 1 \\ -1 \\ -1 \\ -1 \\ -1 \end{array} $	$ \begin{array}{c} 1 \\ -1 \\ 1 \\ -1 \end{array} $
X.6 X.7 X.8 X.9 X.10	-1	$-1 \\ -1 \\ 1 \\ 1$	2	$-1 \\ 2 \\ 2 \\ -1$	$-1 \\ 2 \\ 2 \\ -1$	-1 $2$ $2$ $-1$ $3$	$-\frac{1}{2}$	$-1 \\ -1$	$-1 \\ -1$	-1 -1	-1 -1	-1 -1							$     \begin{array}{c}       -1 \\       -2 \\       \hline       1     \end{array} $	$-\frac{2}{2}$	-1 -1 -1	$-1 \\ -1$	$     \begin{array}{r}       -1 \\       -1 \\       -1 \\       -1 \\       1     \end{array} $	-i -i
X.11 X.12 X.13 X.14 X.15		_1	-1 -1	-i -1	:	-i -1	- i	-1 -1		1 -1	i -1		-1	i	1		-1 -1	-1 -1		$-3 \\ 3 \\ 2 \\ -2$	-1 1			-1 -1
X.16 X.17 X.18 X.19		-1 1	-3 -3	3 3		-3 $-3$		-			:		-1 -1		-1 :	-1			-i		-1		-1 $i$	i
X.20 $X.21$ $X.22$ $X.23$ $X.24$			3 1 1 1	1 1 1		3 1 1 1		B B	$\bar{C}$		C	C <u>C</u> –C							. 1 1 1	$-1 \\ -1 \\ -1$	-1 -1	$-1 \\ -1$	-i	-1
X.25 X.26 X.27 X.28 X.29		$-1 \\ 1 \\ 1 \\ -1$	1	1	1	1		$\bar{B}$	-C	$-\bar{C}$	$-\bar{C}$	-C							. 1	-3	1		1 1 1 1	i −i
X.30 X.31 X.32 X.33			9 9			1 1							-1 -1		-1 -1	-1 1			2 -2	2	-1			-i 1
X.34 X.35 X.36 X.37 X.38			$     \begin{array}{r}       -1 \\       -1 \\       -1 \\       -1   \end{array} $	$ \begin{array}{c} 3 \\ -1 \\ -1 \\ -1 \end{array} $	$-1 \\ -1 \\ -1 \\ .$	$-1 \\ -1 \\ -1 \\ -1$		-1	E										· ·				1	
X.39 X.40 X.41 X.42 X.43	-1 $-1$ $-1$		-3		3	-3							-1 -1	-i	-1 1	- i		. 1					-1 -1	
X.44 X.45 X.46 X.47 X.48	$-1 \\ 1 \\ 1 \\ 1 \\ 1$		$-12$ $\vdots$ $3$ $-1$ $-1$	-3 $-1$ $-1$	$     \begin{array}{r}       3 \\       3 \\       -3 \\       -1 \\       -1     \end{array} $	3 -1 -1		- i - 1		1 -1	: : -1		-2 1 1	-i	-i -1	- i	1	1						
X.49 X.50 X.51 X.52 X.53			2 4 4	$\begin{array}{c} \dot{2} \\ -2 \\ -2 \end{array}$	2 1 1		2 1 1	1 1	E								-1 $-1$ $-1$ $-1$	-1						
X.54 X.55 X.56 X.57 X.58	-1	-1 1	6	3									$-\frac{1}{2}$ $-\frac{1}{2}$		$-\frac{1}{2}$				-i -1	-1 1	-1 1		-i	
X.59 X.60 X.61 X.62	1	:	6 · 4	3 -2	1	$-\frac{2}{2}$	$-2^{-\frac{1}{2}}$	-2					-1 $-1$		-1 -1	-1 1								-1 -1
X.63 X.64 X.65 X.66 X.67	1 1 1		$     \begin{array}{r}       -8 \\       -8 \\       4 \\       9 \\       -4     \end{array} $	$\begin{array}{c} 4 \\ 4 \\ -2 \\ \vdots \\ 2 \end{array}$	$-2 \\ 1 \\ -1$	i	$-\frac{1}{2}$ $-\frac{1}{2}$	$-\frac{1}{2}$	1 1	-i	-1 1	-1 -1							· · ·					
X.68 X.69 X.70 X.71			$     \begin{array}{r}       -4 \\       -4 \\       -4 \\       -4 \\    \end{array} $	2 2 2 2 2	$-1 \\ -1 \\ -1$		$-1$ $-B$ $-\bar{B}$ $-\bar{B}$ $-B$	$-1$ $-\bar{B}$ $-B$ $-\bar{B}$ $-\bar{B}$	-C	$\begin{array}{c} 1 \\ \bar{C} \\ C \\ -C \\ -\bar{C} \end{array}$	$-\bar{C}$	1 C C -C -C							· ·					
X.72 X.73 X.74 X.75 X.76 X.77 X.78 X.79 X.80			-4		-1	-i	-B	- D		-0		-U												- <u>i</u> - <u>i</u> 1
X.78 X.79 X.80 X.81			-9 -9			-1 $-1$ $-1$ $-1$ $-1$ $-1$											1	. 1		6				-i 1
X.81 X.82 X.83 X.84 X.85 X.86	:	-1 -1 1	6 6	3 3		-2 -2							-2 -2						$\begin{array}{c} -6 \\ -2 \\ 2 \\ 1 \end{array}$	$^{-6}_{-2}$	-1 $-1$ $-2$	$-1 \\ -2$		
X.84 X.85 X.86 X.87 X.88			4				-2	-2											$   \begin{bmatrix}     -2 \\     2 \\     1 \\     -1 \\     \vdots   \end{bmatrix} $	1	-1	$-1 \\ -2$		

2 3 5	4 1	4 1	4 1	1 1 1	3	4 3	3	1 1	1 1
7	24g	$\frac{1}{24h}$	24i	$\frac{.}{30a}$	$\frac{.}{36a}$	$\frac{.}{36b}$	$\frac{.}{36c}$	$\frac{1}{42a}$	$\frac{1}{42b}$
$\frac{2P}{3P}$	$12_{14} \\ 8c \\ 24f$	$12_{8} \\ 8b \\ 24h \\ 24h$	$12_{12} \\ 8i \\ 24e \\ 24e$	$15a \\ 10a \\ 6_1 \\ 30a$	$18a$ $12_2$ $36a$ $36a$	$18a$ $12_2$ $36b$ $36b$	$18b$ $12_{2}$ $36c$ $36c$	$\frac{21a}{14a}$	$\begin{array}{c} 21b \\ 14a \end{array}$
3P 5P 7P	$\begin{array}{c} 24f \\ 24f \end{array}$	$^{24h}_{24h}$	$\frac{24e}{24e}$	$^{6_1}_{30a}$	$\frac{36a}{36a}$	$\frac{36b}{36b}$	36c $36c$	$14a \\ 42a \\ 6_1$	$14a \\ 42b \\ 6_1$
X.1 $X.2$ $X.3$	-1	1	1 1	1 1	1	1 1	1	1	1
	-1	$-2 \\ -2$	$-1 \\ -1$	1 1	:	:	:	:	:
$X.4 \\ X.5 \\ X.6$			$-1 \\ -1$	:	-i	-i	$-3 \\ -1$	-1	-1
X.4 X.5 X.6 X.7 X.8 X.9 X.10	$-1 \\ 1$	:	$-1 \\ -1$	:	$\frac{2}{2}$	$\begin{array}{c} 2 \\ 2 \\ -1 \\ 3 \end{array}$	2	:	:
X.9 X.10		-1	$^{-1}_{1}$	-i	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$		•
$X.11 \\ X.12$	$-1 \\ -1$	1	:				-1	$-1 \\ -1$	$-1 \\ -1$
$X.13 \\ X.14$	:	1			$-1 \\ -1$	$-1 \\ -1$	-1	:	:
X.16 X.16	:	$-1 \\ -1 \\ -1$	$\begin{array}{c} 1 \\ 1 \\ -1 \end{array}$	$-1 \\ -1$	-3	-3	3		:
X.10 X.11 X.12 X.13 X.14 X.15 X.16 X.17 X.18 X.19	i	1	-1 i		-3 -3	-3 -3	3		:
X.20 X.21	-1	1 2	-i						:
X.22 X.23	:		-1		3 1 1 1	3 1 1	i 1		:
X.24 X.25		·			1 1	1 1	1		
X.26 X.27	i	$-1 \\ -1$	i 1						÷
X.28 X.29	-1	$-1 \\ -1$	1						÷
$X.29 \\ X.30 \\ X.31 \\ X.32 \\ X.33$	$-1 \\ 1 \\ 1$	$-1 \\ -1$			$-3 \\ -3$	$-3 \\ -3$			
$X.32 \\ X.33$		1 1					:	:	:
X.11 X.12 X.13 X.14 X.16 X.16 X.18 X.20 X.21 X.24 X.25 X.26 X.27 X.28 X.30 X.31 X.34 X.34 X.34 X.34 X.34 X.34 X.34		-1			-i	-i	$-1^{3}$	:	:
$X.36 \\ X.37$				:	$-1 \\ -1$	$-1 \\ -1$	$-1 \\ -1$	:	
$X.38 \\ X.39$	:	:	1	-i	-3	-3	:		
$X.40 \\ X.41$	:	:			:	:	:	G F 1	$\overset{\cdot}{F}$ $\overset{\cdot}{G}$ $\overset{1}{1}$
X.40 X.41 X.42 X.43 X.44 X 45		:	$-1 \\ -1$	$-1 \\ -1$	:	:	:	1	1
X.44 X.45			:	1 1			-3	$^{-1}_{1}$	$-1 \\ 1$
X.46 X.47	:	-1	:	i	3 -1	$-1 \\ -1 \\ -1$	-1	:	:
X.49	:	i		1	$-1 \\ \dot{2}$	-1 2	$-1 \\ \dot{2}$	- 1	
X.51 V 52	:	:	:	:				$-1 \\ 1 \\ 1$	$-1 \\ 1 \\ 1$
X.53 X.54	:	i	i	1 -1		:	:	-1	-1
X.55 X.56									÷
X.57 X.58		-1	-i		2	. 2	-1		÷
$X.59 \\ X.60$	1	1			2	2	-1		÷
X.466 X.47 X.49 X.50 X.51 X.52 X.53 X.556 X.562 X.62 X.63 X.64 X.65 X.668 X.668 X.70	-1	1		- i	:	:	:	:	:
$X.63 \\ X.64$		:		$-1 \\ -1 \\ -1$	:	:	:	:	:
X.64 X.65 X.66 X.67 X.68 X.69 X.70	:	i	:	-1	$-\dot{3}$	$-\dot{3}$	:	:	:
$X.67 \\ X.68$	:	:	:	:	:	:	:	:	:
$X.69 \\ X.70$	:		:	:	:	:	:	:	:
$X.71 \\ X.72 \\ Y.72$	:	1	:			:	:	:	:
X.74 X.74	$-\frac{1}{1}$					:			:
X.76 X.77		1	:	:			:		:
X.78 X 79	-1 1	-1			3	3	:		
X.80 X 81	-1 1							-1	$-\frac{1}{1}$
X.82 X.83		1 -1 -1	:		2	2	$-1 \\ -1$		
X.84 X.85		:				-	:	-1	÷
X.71 X.72 X.73 X.74 X.75 X.76 X.77 X.80 X.81 X.82 X.83 X.84 X.85 X.86 X.87 X.87		:		:	3 3 3 3 	3333	:		-1 -1
X.88									

2 3 5 7	21 7 1	21 7 1	19 4 1	$^{20}_{\ 2}_{\ 1}$	$^{14}_{\ 4}_{\ 1}$	$^{14}_{4}$	17 3	17 3	16 1	13 2	8 7 1	12 6	10 7	5 7	8 5	5 6	6 4
	1a	$\frac{1}{2a}$	2b	2c	2d	$\frac{1}{2e}$	2f	2g	2h	2i	3a	3 <i>b</i>	3c	3d	3e	3f	3g
$\frac{2P}{3P}$	$\frac{1a}{1a}$	$\frac{1a}{2a}$	$\frac{1a}{2b}$	$\frac{1a}{2c}$	$\frac{1a}{2d}$	$\frac{1a}{2e}$	$\frac{1a}{2f}$	$\frac{1a}{2g}$	$\frac{1a}{2h}$	$\frac{1a}{2i}$	$\frac{3a}{1a}$	$\frac{3b}{1a}$	$\frac{3c}{1a}$	$\frac{3d}{1a}$	$\frac{3e}{1a}$	$\frac{3f}{1a}$	$\frac{3g}{1a}$
5P	1a	2a	2b	2c	2d	2e	2 f	2a	2h	2i	3a	3b	3c	3d	3e	3f	3g
$\frac{7P}{X.89}$	$\frac{1a}{9072}$	$\frac{2a}{9072}$	$\frac{2b}{1392}$	$\frac{2c}{-144}$	$\frac{2d}{144}$	$\frac{2e}{144}$	$\frac{2f}{48}$	$\frac{2g}{48}$	2h 48	2 <i>i</i> 48	3a	$\frac{3b}{162}$	$\frac{3c}{162}$	3d	3e	3f	3g
$X.90 \\ X.91$	$9072 \\ 10080$	9072 10080	$1392 \\ -160$	$-144 \\ -160$	-144	-144	48 96	48 96	$^{48}_{-32}$	$-48 \\ -32$		$\frac{162}{-48}$	162 180		30 30	15	$-\dot{6}$
X.92	10080	10080	-160	-160	:	:	96	96	-32	$-32 \\ 32$	:	-48	180		30	15	-6
$X.93 \\ X.94$	$10752 \\ 11340$	-10752 $11340$	1740	$-180^{\circ}$	-360	-360	$\frac{128}{12}$	$-128 \\ 12$	-52	$2\dot{4}$	-84	$\frac{192}{162}$	$-120 \\ 81$	-30	$2\dot{7}$	12	
X.95	11340 12096	11340	$1740 \\ -192$	-180	360	360	12	12	-52	-24		162	81 216		$^{27}$	10	
$X.96 \\ X.97$	13608	$\frac{12096}{13608}$	-216	$^{-192}_{296}$	216	216	$-192 \\ 72$	$-192 \\ 72$	$\frac{64}{72}$	8	:	$^{72}_{-81}$	216		36	18	$-\dot{9}$
X.98 X.99	$\frac{13608}{13608}$	13608 13608	$-216 \\ -216$	296 296	$-216 \\ 216$	$-216 \\ 216$	$\frac{72}{72}$	$\frac{72}{72}$	$\frac{72}{72}$	$^{-8}_{8}$		$-81 \\ -81$					$-9 \\ -9$
X.100	13608	13608	-216	296	-216	-216	72	72	72	$-\frac{8}{-27}$		-81		÷	- 1	i.	$-\overset{\circ}{9}$
$X.101 \\ X.102$	15309 15309	15309 15309	$-243 \\ -243$	333 333	$-243 \\ 243$	$-243 \\ 243$	189 189	189 189	93	27			:				:
$X.103 \\ X.104$	$15360 \\ 16128$	$-15360 \\ -16128$					$-256 \\ -320$	$\frac{256}{320}$			$-120 \\ -126$	$\frac{480}{288}$	$-48 \\ 144$	$-12 \\ 36$		30 18	
X.105	17010	17010	$-270 \\ -270 \\ -270$		-270	$-270 \cdot 0.00$	-126	-126	66	26	120	81					9
$X.106 \\ X.107$	$\frac{17010}{20160}$	$\frac{17010}{20160}$	-320	$-370 \\ -320$	270	270	192	$-126 \\ 192$	-64	-26	:	81 480	36		-12·	$-15^{\circ}$	$^{9}_{-12}$
$X.108 \\ X.109$	$20160 \\ 20160$	20160 20160	$-320 \\ -320$	$-320 \\ -320$			$\frac{192}{192}$	$\frac{192}{192}$	$-64 \\ -64$			-168 $-168$	36 36		-12 - 12 - 12 - 12 - 12 - 12 - 12 - 12		6
X.110	$\frac{22680}{22680}$	22680 22680	3480 3480	-360	-360		$\frac{120}{120}$	$\frac{120}{120}$	-8 -8	$-\frac{24}{24}$		-168 $-162$ $-162$	162		$-\frac{1}{27}$		·
$X.111 \\ X.112$	22680	22680	3480	$-360 \\ -360$	360	360	$\frac{120}{216}$	216	-8 88	24			$^{162}_{-81}$	:	-54	:	:
$X.113 \\ X.114$	$24192 \\ 24192$	$24192 \\ 24192$	$\frac{3712}{3712}$	$-384 \\ -384$	$-576 \\ 576$	$-576 \\ 576$		:	:	:	:	$-108 \\ -108$	$-216 \\ -216$	:	36 36	:	:
$X.115 \\ X.116$	$25515 \\ 25515$	$25515 \\ 25515$	$-405 \\ -405$	555 555	$-135 \\ 135$	$-135 \cdot$		$-117 \\ -117$	$-21 \\ -21$	57 -57		$\frac{243}{243}$					
X.117	25515	25515	-405	555	135	135	315	315	27	15	:	243	:	÷	÷	:	:
$X.118 \\ X.119$	$25515 \\ 26880$	$25515 \\ -26880$	-405	555	-135		$\frac{315}{320}$	$-315 \\ -320$	27	-15	420	243 96	-336	$\dot{42}$	$2\dot{4}$	$-12^{-12}$	$2\dot{4}$
$X.120 \\ X.121$	$\frac{26880}{30240}$	$-26880 \\ 30240$	4640	$-480^{\circ}$			$-192 \\ -96$	$^{192}_{-96}$	-96		-210	-96 $108$	$-192 \\ -270$	-48	_36	-6	
$X.122 \\ X.123$	30618 30618	30618 30618	$\frac{4698}{4698}$	$-486 \\ -486$	324	$324 \\ -324$	$-54 \\ -54$	$-54 \\ -54$	$\frac{42}{42}$	$-36 \\ 36$							
X.124	32256	-32256	4098	-400	768	$-768 \cdot$	-128	128	42		504	-144	-144	18	72	18	:
$X.125 \\ X.126$	$\frac{32256}{32256}$	$-32256 \\ -32256$			$-768 \\ 192$	$-192 \cdot$	$-128 \\ -128$	$\frac{128}{128}$	:	:	504	$-144 \\ 288$	$-144 \\ -144 \\ -144$	18 18	$^{72}_{-36}$	$^{18}_{-36}$	:
$X.127 \\ X.128$	$32256 \\ 34020$	-32256 $34020$	5220	-540	-192	192	$-128 \\ -252$	$128 \\ -252$	$-60^{\circ}$		504	288	$-144 \\ 243$	18	-36 ·	-36	
X.129	34560	-34560					192	$-192 \\ -192$		÷	540		216	$-27^{-27}$	÷	÷	÷
$X.130 \\ X.131$	$\frac{34560}{34560}$	$-34560 \\ -34560$					192	-192	:	:	-270		$\frac{216}{216}$	54	:	:	:
$X.132 \\ X.133$	$\frac{34560}{35840}$	$-34560 \\ -35840$		:	-640	640	-256	$-192 \\ 256$	:	:	$-270 \\ 560$	32	$\frac{216}{272}$	-34	8	$-\dot{4}$	8
$X.134 \\ X.135$	$35840 \\ 40320$	$-35840 \\ 40320$	$-640^{\circ}$	-640	640	-640 ·	-256	256		-64	560	-120	272	-34	$^{8}_{-24}$	$\frac{-4}{24}$	8
X.136	40320	40320	-640	-640	:	:				-64	:	-120	72 72		-24	$\frac{1}{24}$	3
$X.137 \\ X.138$	$\frac{40320}{40320}$	$^{40320}_{-40320}$	-640	-640	$-480^{\circ}$	$480^{-}$	$22\dot{4}$	-224		64	630	$-120 \\ 144$		$-18^{\circ}$	$^{-24}_{36}$	$^{24}_{-18}$	3
$X.139 \\ X.140$	$\frac{40320}{40320}$	$\frac{40320}{40320}$	$-640 \\ -640$	$-640 \\ -640$	:	:	-384	-384	128	64	:	$-120 \\ 240$	-252		$-24 \\ 12$	24 6	3 12
$X.141 \\ X.142$	$\frac{40320}{40320}$	$-40320 \\ 40320$	-640	-640	480	-480	224	-224		$6\dot{4}$	630	144 204	$\frac{144}{72}$		$^{36}_{-24}$	$-18 \\ 24$	$-\dot{6}$
X.143	40320	40320	-640	-640	:	:			100	-64	:	204	$\frac{1}{72}$		-24	$\frac{24}{24}$	$-\ddot{6}$
$X.144 \\ X.145$	$\frac{40320}{43008}$	$^{40320}_{-43008}$	-640	-640	:	:	$\frac{384}{512}$	-512	-128	:	-336	192	$-252 \\ -48$	$-1\dot{2}$	12	$\frac{6}{12}$	12
$X.146 \\ X.147$	$\frac{49152}{51030}$	-49152 $51030$	$-810^{\circ}$	1110	-270	-270	198	198	6	42	-384	-384 $-243$	192	48		24	:
$X.148 \\ X.149$	$51030 \\ 53760$	$51030 \\ -53760$	-810	1110	270	270	198	198			_420		-168	_42		24	
X.150	57344 57344	-57344 $-57344$	:		512	-512	-304		:	:	896	$-64 \\ -64$	-256	32	-16	-8	-16
$X.151 \\ X.152$	60480	60480	$-960 \\ -960$	$-960^{\circ}$	-512	512	-192	-192	$6\dot{4}$	:	896	$-64 \\ 144$	$-256 \\ 108$	32	- 36 .	_45	-16
$X.153 \\ X.154$	60480 60480	60480 60480	$-960 \\ -960$	-960 -960 -960 -960 -960 -960			192 192	192 192	-64 $-64$	•	-420 896 896	$-72 \\ -72$	108 108		18 ·	$-18 \\ -18$	•
X.155	60480 60480	60480 60480	$-960 \\ -960$	-960			-192	-192	64			-72	108 108		18 · 18 ·	-18	
X.156 X.157	76545	76545	-1215						-03	41	:	- 12		:	10.	-10	÷
$X.158 \\ X.159$	$76545 \\ 76545$	76545 76545 76545	$-1215 \\ -1215$	1665	$-405 \\ 405$	405 -	$^{81}_{-351}$	$^{81}_{-351}$	$-15 \\ -63 \\ -15$	$-45 \\ -27$				:	:	:	:
$X.160 \\ X.161$	76545 80640	$76545 \\ -80640$	-1215	1665	405	405	$^{81}_{-576}$	81 576	-15	45	-630	-288	72	18	٠.	-18	
X.162 X.163	80640 80640	80640 -80640	-1280	-1280	:		-64	64	:			-168	$-504 \\ 72$	18	24	12 -18	$-12^{-12}$
X.164	80640	-80640			:		-64	64	:		-630	-288	72	18		-18	
$X.165 \\ X.166$	$81920 \\ 107520$	$-81920 \\ -107520$					256	-256			-840	$-256 \\ -96$	$^{128}_{-336}$	$-16 \\ -84$	-64	$\frac{32}{-6}$	8
X.167	107520	-107520			•		256	-256			-840	192	96	24		12	

2 3 5 7	15 5 1	15 2	13 1 1	11 2 1	13 2	14 1	11 2	11 2	11 2	14	11 1	12	10 1	10 1	10 1	9	9	10	10	9	6 1 1
2P 3P 5P 7P X.89 X.90 X.91 X.92 X.93	$ \begin{array}{r} 4a \\ 2a \\ 4a \\ 4a \\ 4a \\ -144 \\ -160 \\ 160 \end{array} $	4b 4b 4b 48 48 32 32	4c 4c 4c 4c 4c 16 -16	4d 2b 4d 4d 4d -16 16	4e 4e 4e 4e 48 -48 32 -32	$ \begin{array}{r} 4f \\ 2c \\ 4f \\ 4f \\ 4f \\ -16 \\ -16 \\ -16 \\ \end{array} $	4g 2f 4g 4g 4g 	2b 4h 4h 4h 16 16	4i 2f 4i 4i 4i	4j $2c$ $4j$ $4j$ $4j$ $-16$ $-16$	$     \begin{array}{r}       4k \\       2b \\       4k \\       4k \\       4k \\       \hline       16 \\       -16 \\       \vdots     \end{array} $	$ \begin{array}{r} 4l \\ 2c \\ 4l \\ 4l \\ 4l \\ -16 \\ 16 \end{array} $	2h 4m 4m 4m -8 8	4n 2f 4n 4n 4n	40 2h 40 40 40 	4p 4p 4p 4p 	$\begin{array}{c} 2g \\ 4q \\ 4q \end{array}$	$\frac{4r}{2h}$ $\frac{4r}{4r}$ $\frac{4r}{4r}$ $\frac{1}{4r}$	4s 2h 4s 4s 4s	2h 4t 4t 4t 4t	5a 5a 5a 1a 5a -3 -3
X.94 X.95 X.96 X.97 X.98 X.100 X.101 X.102 X.103	-180 -180 192 -216 -216 -216 -216 -243 -243	-84 -84 -64 72 72 72 72 45 45	$ \begin{array}{r} 40 \\ -40 \\ 40 \\ -40 \\ 40 \end{array} $	$^{24}_{-24}$	24 -24 -8 -8 -8 -8 -27 27	44 44 8 8 8 8 8 -3 -3	-12 -12	$ \begin{array}{r} -20 \\ -20 \\ -24 \\ -24 \\ -24 \\ -24 \\ -27 \\ -27 \\ -27 \\ -27 \end{array} $	-12	12 12 8 8 8 8 13 13	-8 -8 -8 -8 -3 -3	-8 8 -8 -8 -11 11	-4 -4 4 	-15 15	-4 -4	99	-4 -4 3 -3	4 4 		1	6 3 3 3 -6 -6
X.104 $X.105$ $X.106$ $X.107$ $X.108$ $X.109$ $X.110$ $X.111$ $X.112$	-270 -270 320 320 320 -360 -360 -360	162 162 64 64 64 -72 -72 24	-50 :	40	26 -26 -26 	34	-16 -18 -18 -18	-30 -30 -30 -8 -8 -8 24	16 -18 -18 -18	2 2 2 	-22 22  -8 8	-6 6	6 -6	6 -6	6 -6	-6 -6 -6	6 -6 - -6 - -8 -8	-2 -2 -2	6 -6	-6 -6	8
X.113 X.114 X.115 X.116 X.117 X.118 X.119 X.120	-384 -384 -405 -405 -405 -405	-117 -117	$\begin{array}{r} 64 \\ 25 \\ -25 \end{array}$	$^{-64}_{-15}$ $^{15}_{15}$	57 -57 15 -15	11 11 -37 -37	27 27 -9 -9 -16 -48	3 3 3	27 27 -9 -9 16 48	-5 -5 11 11	-15 15 -9 9	9 -9 -1 1	-3 3 3 -3	-15 $15$ $-21$ $21$	-3 3 -3 -3	-9 -9 3 3	-3 3 3 -3	3 3 -1 -1	$\begin{array}{c} \cdot \\ \cdot \\ -1 \\ -5 \\ 5 \\ \cdot \end{array}$	-1 -1 -5 -5	-3 -3
X.121 $X.122$ $X.123$ $X.124$ $X.125$ $X.126$ $X.127$ $X.128$	-480 -486 -486	-96 90 90		-36 36	-36 36	-54	12	-32 18 18 	-18 -18 -18	32 -6 -6	-12 12	-12 -12	-16 16		16 -16	$ \begin{array}{c} -6 \\ -6 \\ \vdots \\ \vdots \\ 12 \end{array} $		6 6		2 2	$\begin{array}{c} 3\\ 3\\ -4\\ -4\\ -4\\ -4\end{array}$
X.129 X.130 X.131 X.132 X.133 X.134 X.135 X.136	640 640				64 64		-16 -16 -16 -16		16 16 16 16												
X.137 $X.138$ $X.139$ $X.140$ $X.141$ $X.142$ $X.143$ $X.144$	640 640 640 640 640 640	-128 -128			-64 -64 -64 64		24 24 24		-24 -24 -24				8 -8		-8						
X.145 $X.146$ $X.147$ $X.148$ $X.149$ $X.150$ $X.151$	-810 -810	-90 -90	50 -50 -50	-30 30	42 -42 -42	-26 -26 -26	18 18 18 32	6 6	18 18 18 -32	6 6	-6 6	10 -10 -10	-6 6	6 -6	-6 6	-6 -6 :	-6 6	· 2 2 · ·	6 -6	-6 -6 -6	-8 -8 -4 4
X.152 X.153 X.154 X.155 X.156 X.157 X.158 X.159 X.160 X.161	960 960 960 960 960 -1215 -1215 -1215	-64 64 -64 -64 81 -63 81	75 75 75 -75 -75	-45 -45 45 45	27 -45 -27 45	33 -15 33 -15	$\begin{array}{c} \cdot \\ \cdot \\ \cdot \\ \cdot \\ 9 \\ -27 \\ 9 \\ -27 \\ -16 \end{array}$	9 9 9	9 -27 9 -27 16	-15 -15 -15 1	3 27 -3 -27	11 3 -11 -3	16 -16 -3 -3 -3		16 -16 -3 -3		-16 16  3  -3 -3		-13 7 13 -7		
X.162 X.163 X.164 X.165 X.166 X.167	1280						16 16 16		-16 -16												

2 3 5 7	8 12 7 6	3 7 · · ·	5 8 7 5 	9 10 4 3	5 6	11 2	8 3	8 3	8 3	8 3	8 3	8 3	8 3	8 3	8 3	8 3	6 4 :	7 3
2P 3P 5P 7P X.89	$egin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{ccc} & 2a \\ & 6_3 \\ & 6_3 \end{array}$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccc} 6_6 & 6_7 \\ 3c & 3b \\ 2b & 2b \\ 6_6 & 6_7 \\ 6_6 & 6_7 \\ -30 & -6 \end{array} $	$     \begin{array}{r}       6_8 \\       3f \\       2a \\       6_8 \\       6_8    \end{array} $	69 $3b$ $2c$ $69$ $69$ $18$	$     \begin{array}{r}       6_{10} \\       3_{0} \\       2_{f} \\       6_{10} \\       \hline       -6     \end{array} $	$     \begin{array}{r}       6_{11} \\       3a \\       2g \\       6_{11} \\       6_{11}    \end{array} $	$     \begin{array}{r}       6_{12} \\       3b \\       2e \\       6_{12} \\       6_{12}     \end{array} $	$     \begin{array}{r}       6_{13} \\       3b \\       2d \\       6_{13} \\       6_{13}     \end{array} $	$6_{14} \\ 3a \\ 2f \\ 6_{14} \\ 6_{14}$	$ \begin{array}{r} 6_{15} \\ 3c \\ 2f \\ 6_{15} \\ 6_{15} \end{array} $	$ \begin{array}{r} 6_{16} \\ 3c \\ 2g \\ 6_{16} \\ 6_{16} \end{array} $	$6_{17} \\ 3e \\ 2g \\ 6_{17} \\ 6_{17}$	$6_{18}$ $3e$ $2f$ $6_{18}$ $6_{18}$	$ \begin{array}{r} 6_{19} \\ 3b \\ 2g \\ 6_{19} \\ 6_{19} \\ -6 \end{array} $	$6_{20}$ $3g$ $2a$ $6_{20}$ $6_{20}$	621 $3e$ $2b$ $621$ $621$
X.99 X.91 X.92 X.93 X.94 X.95 X.96 X.97 X.98 X.100 X.101	. 162 48 48 84 -192 . 162 . 163 . 72 83 83	2 162 180 180 180 2 120 2 81 2 81 2 216 1	30 30 30 27 27 36	$ \begin{array}{cccc} -30 & -6 \\ 20 & 32 \\ 20 & 32 \\ -15 & -6 \end{array} $	$^{15}_{-12}$	$^{18}_{-16}$	-6 -6 -16 -6 -6 -9 -9 -9	4	-6 -6 -6 6 -9 9 -9	-6 -6 -6 -9 -9 -9	-4 -4	6 12 12 -4 -3 -3	6 12 12 4 -3	6 6 16 -9 -9 -12	6 6 -16 -9 -9 -12	-6 -6 -6 -6 -6 -9 -9 -9		-10 -10 -10 3 3 -12
X.102 X.103 X.104 X.105 X.106 X.107 X.108 X.110 X.111 X.112 X.113 X.114 X.115 X.117 X.118 X.117	108 . 243 . 243 . 243 . 243 -420 -96	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		4 40	12	-32 -8 -8 -18 -18 -12 -12 -12	-8 -9 9 · · · · · · · · · · · · · · · · · ·	-20	-9 9 -6 6 12 -12 -9 -9	-9 9 -6 6 12 -12 -12 9 -9	. 8 10 	. 8 -8 -12 -12 -12 -12 -6 -9 		16 -16	-16 16	-8 9 9 -6 -6 -6 -9 -9 -9 -8	9 9 -12 6 6	4 4 4 4 3 -3 -6 4 4
X.120 X.121 X.122 X.123 X.124 X.125	210 96 . 108 	3 192 3 -270 	48 36 	50 -4 	6 -18 -18	12	24 12 4 4	-6 8	12 -12	-12 12	-8 -8	6 -8 -8	6		-8 -8	-24 12 -4 -4		-4 :
X.126 X.127 X.128 X.129 X.131 X.133 X.134 X.135 X.136 X.137 X.138 X.140 X.141 X.142 X.143 X.144 X.145 X.145 X.147 X.149 X.150	-504 -288 -504 -288 -540 -540 270 270 -560 -33 -560 -32 -120 -120 -630 -142	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	27 27 54 54 54 34 -8 34 -8 34 -24 -24 18 -36 -24 -24 -24 -24 -24 -24 -24 -24 -24 -24	8 - 28 8 - 28 - 28 - 16 	36 36 36  4 4 4 24 24 24 18 24 24 6 -12 -24 -8 -8 -8 -8 -8 -18 -18		-8 -8   8 8  -4 16  9	8 8 -12 -12 6 6 16	16 -16	-16	-8 -8	-8 -8 -8 -9 -12 -12 -12 -12 -12 -12 -12 -12 -12 -12	8 8 8 8 9 9 12 12 12 12 12 12 12 12 12 12 12 12 12	-4 -4	-34 44 44 -3 -3 -4 -4 -12 -16 -6 -6 -6 -6 -6 -6 -6 -6 -6 -6 -6 -6 -6	-8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 4 4 4 8 16 16 16 16 16 16 16 16 16 16 16 16 16	-8 -8 3 3 3 3 12 -6 -6 12 · · · · · · · · · · · · · · · · · ·	
X.158 X.159 X.160 X.161 X.162 X.163 X.164 X.165 X.166 X.167	630 288 -168 630 288 630 288 -1280 256 840 96 840 -192	$     \begin{array}{r}       3 - 504 \\       3 - 72 - \\       3 - 72 - \\       3 - 128 \\       336     \end{array} $	-18	-56 40 	$ \begin{array}{c}                                     $	-8 -8		-18 -2 -2 -2 8 8			18 2 2 2 -8 -8	$     \begin{array}{r}       -4 \\       -4 \\       \hline       -8    \end{array} $	-12 -4 4 -16	$-16^{\circ}$	-16 -16 -16 16	24 -8 -8 -8 -8	-12 -8 :	-8 -8 -

Character table of  $H(\mathrm{Fi}_{24}')$  (continued)

2 3 5 7	5 4	5 4	5 4	5 4	7 2	5 3	5 3	5 3	5 3	5 3	5 3	5 3	5 3	7 1	5 2	3 2	1 1 1	8 2	9 1	8	8	8	8	8	8	6 1
2P 3P 5P 7P	$6_{22} \\ 3d \\ 2d \\ 6_{23} \\ 6_{22}$	$6_{23}$ $3d$ $2d$ $6_{22}$ $6_{23}$	624 $3d$ $2e$ $625$ $624$	625 $3d$ $2e$ $624$ $625$	$6_{26}$ $3b$ $2i$ $6_{26}$ $6_{26}$	$6_{27}$ $3e$ $2d$ $6_{27}$ $6_{27}$	$6_{28} \\ 3d \\ 2g \\ 6_{28} \\ 6_{28}$	$6_{29} \\ 3e \\ 2g \\ 6_{29} \\ 6_{29}$	$\begin{array}{c} 6_{30} \\ 3e \\ 2f \\ 6_{30} \\ 6_{30} \end{array}$	$6_{31} \\ 3d \\ 2f \\ 6_{31} \\ 6_{31}$	632 $3e$ $2e$ $632$ $632$	$6_{33}$ $3f$ $2g$ $6_{33}$ $6_{33}$	$634 \\ 3f \\ 2f \\ 634 \\ 634$	$6_{35} \\ 3e \\ 2h \\ 6_{35} \\ 6_{35}$	$6_{36} \\ 3g \\ 2c \\ 6_{36} \\ 6_{36}$	$\frac{6_{37}}{3g}$ $\frac{3g}{2i}$ $\frac{6_{37}}{6_{37}}$	7a 7a 7a 7a 1a	8a 8a 8a 8a	8b 8b	8c	8d 8d 8d 8d 8d	8e 8e 8e 8e	8f 4f 8f 8f 8f	8g 4j 8g 8g 8g 8g	$\frac{8h}{4f} \\ \frac{8h}{8h} \\ \frac{8h}{8h} \\ \frac{8h}{8h}$	$\begin{array}{c} 8i \\ 4g \\ 8i \\ 8i \\ 8i \end{array}$
X.99 X.91 X.92 X.93 X.94 X.95					-6 -8 8 -6	-3	-2	6 -4 -3 -3	6 -4 -3 -3	· · ·	-6 -6 -3 3	3	3 3 -4	$ \begin{array}{c}     -2 \\     -2 \\     -1 \\     -1 \end{array} $	2 2	-2 2			-4 -4	-4 4		-4 4			4 -4	
X.96 X.97 X.98 X.99 X.100 X.101					-1 1 -1 1							-6	-6	4	-1 -1 -1 -1	-1 -1 -1 1		-8 -8 -8 9	-3		1	i	-3		· · · ·	
X.102 $X.103$ $X.104$ $X.105$ $X.106$					-i		$\begin{array}{c} \overset{\cdot}{4} \\ -\overset{\cdot}{4} \\ \vdots \\ \vdots \\ \end{array}$	$\begin{matrix} \overset{\cdot}{4} \\ -4 \\ \vdots \\ \vdots \\ \end{matrix}$	$\begin{array}{c} -\overset{\cdot}{4}\\ \overset{\cdot}{4}\\ \vdots\\ \vdots\\ \end{array}$	$-\frac{1}{4}$		$-\frac{1}{2}$	$-\frac{1}{2}$	-4	1 1 4	-i -i 1		-9 10 -10	6	3 6 -6	-1 $-2$ $2$	-1 $-2$ 2	-3 2 2	1 2 2	-1 $-2$ $2$	3
X.108 X.109 X.110 X.111 X.112 X.113					6 -6	3 6		-3 -3 -3	-3 -3 -3		-3 3	:	-3 -3 -3	$     \begin{array}{r}     -4 \\     -4 \\     1 \\     -2 \\     \end{array} $	-2 -2				8	-8 8						
X.114 $X.115$ $X.116$ $X.117$ $X.118$ $X.119$					-3 $3$ $-3$	-6		4	-4	2	-6	4	-4					-3 3 3 -3	-9 -9 3 3	$-\frac{3}{3}$ $-3$	$-1 \\ -1 \\ 3 \\ -3 \\ .$	$-1 \\ -1 \\ -1 \\ 1$	$\begin{array}{c} 3\\ 3\\ -1\\ -1\\ \end{array}$	-1	$-1 \\ -1 \\ -1 \\ 1 \\ .$	3 3 -3 -3
X.120 $X.121$ $X.122$ $X.123$ $X.124$ $X.125$ $X.126$	-6 6 -6	-6 -6 -6	6 -6 6			6	2 2 2 2 2	6 -4 -4 2 2	6 4 4 -2	-2 -2 -2 -2	-6	-6	6 -2 -2 4						-6 -6			-4 4	$-\frac{1}{2}$	2 2 2	-4 -4	
X.127 $X.128$ $X.129$ $X.130$ $X.131$		-6 -6 -M 	-6 <u>M</u> <u>M</u>	-6 -6 -6 -6 -6 -7 -8 -8 -8 -8 -8 -8 -8 -8 -8 -8		-6	3 -6 -6	2	-2	$-3 \\ -3$		-4	4				1 1		4	•			4	-4 :		
X.132 X.133 X.134 X.135 X.136 X.137	10	:	10 -10 -10	10 -10 -10		-4 4	-2 -2	4 4	$ \begin{array}{c} -4 \\ -4 \\ -4 \end{array} $	6 6 2 2		4	-4 -4 -4		-1 -1 -1	-1 -1 1										
X.138 X.139 X.140 X.141 X.142 X.143 X.144	6 -6	6 -6	-6 6	-6	$-\frac{1}{8}$	6 -6	-2 -2 :	-2 -2 -2	2	2	-6 6	6	6 2 -6	-4 -4	$ \begin{array}{c} -1 \\ -4 \\ \vdots \\ 2 \\ 2 \\ -4 \end{array} $	$\begin{array}{c} \dot{1} \\ \cdot \\ -\dot{2} \\ 2 \end{array}$										
X.145 X.146 X.147 X.148 X.149 X.150			-8	-8	-3 -3	-4	4 -6	4	-4 :	-4 6	4	4	-4				-2 :	-6 6	-6 -6	-6 6	-2 2	-2 -2	2 2	2 2	-2 -2	
X.151 X.152 X.153 X.154 X.155 X.156	-8	-8	-8 8	8		4					-4		3 6 6 -6 -6	4 2 2 -2 -2												
X.157 X.158 X.159 X.160 X.161 X.162							6			-6		6	-6		4			-9 -9 9	-3 -3 -9	$\begin{array}{c} 3 \\ 3 \\ -3 \\ -3 \\ \end{array}$	$ \begin{array}{c} 3 \\ -1 \\ -3 \\ 1 \\ \vdots \end{array} $	$     \begin{array}{r}       -1 \\       -1 \\       1 \\       1 \\       \cdot \\       \cdot    \end{array} $	$     \begin{array}{c}       -3 \\       1 \\       -3 \\       1 \\       \vdots     \end{array} $	$\begin{array}{c} 1\\-3\\1\\-3\\ \end{array}$	-1 -1 1 1	-3 -3 -3
X.163 X.164 X.165 X.166 X.167	•	•					$ \begin{array}{c} -2 \\ -2 \\ -4 \\ 8 \end{array} $	4 4 -4 -4	-4 -4 4 4	2 2 4 -8		$ \begin{array}{c} -2 \\ -2 \\ -2 \\ -4 \end{array} $	$-\frac{1}{2}$				-i									•

2 3 5	6	6	6	5 4	3 4	$\overset{1}{\overset{4}{\cdot}}$	2 3	2 3	6 1 1	5 1	5 1	5 1	4 i	4 1	9 5	7 5	9 3	6 4	4 5	8 2	8 2	7 2	5 3	5 3
2P 3P 5P 7P X.89	8 <i>j</i> 4 <i>r</i> 8 <i>j</i> 8 <i>j</i> 8 <i>j</i>	8k 4r 8k 8k 8k	8l 8l 8l 8l 8l	9a 9a 3c 9a 9a	9b 9b 3c 9b 9b	9c 9c 3c 9c 9c	9d 9e 3d 9e 9d	9d 3d 9d	$     \begin{array}{r}       5a \\       10a \\       2a \\       10a   \end{array} $	5a $10c$ $2c$	5a $10b$ $2c$	10d 5a 10d 2b 10d -3	$\begin{array}{c} 5a \\ 10e \\ 2d \end{array}$	$ \begin{array}{c} 10f \\ 5a \\ 10f \\ 2e \\ 10f \end{array} $	$12_{1}$	$12_{2}$ $6_{3}$ $4a$ $12_{2}$ $12_{2}$ $18$	$12_{3}$ $6_{2}$ $4a$ $12_{3}$ $12_{3}$ $-18$	$ \begin{array}{r} 12_4 \\ 6_5 \\ 4a \\ 12_4 \\ 12_4 \end{array} $	$ \begin{array}{r} 12_{5} \\ 6_{8} \\ 4a \\ 12_{5} \\ 12_{5} \end{array} $	12 <sub>6</sub> 6 <sub>9</sub> 4 <sub>e</sub> 12 <sub>6</sub> 12 <sub>6</sub>	12 <sub>7</sub> 6 <sub>9</sub> 4e 12 <sub>7</sub> 12 <sub>7</sub>	$ \begin{array}{r} 12_8 \\ 6_3 \\ 4b \\ 12_8 \\ 12_8 \end{array} $	$12_9$ $6_{20}$ $4a$ $12_9$ $12_9$	$ \begin{array}{c} 12_{10} \\ 6_{20} \\ 4a \\ 12_{10} \\ 12_{10} \end{array} $
X.90 X.91 X.92 X.93 X.94			•	-6 -6	-3 -3 -6	-3 -3			-3 -3 8	1	1	-3 -3	1	1	54	$^{18}_{-20}$ $^{-20}$	-18	10 10 -9	-5 -5	-6 8 -8 -6	$-8 \\ -6$	$ \begin{array}{r} 6 \\ -4 \\ -4 \\ -3 \\ -3 \\ 8 \end{array} $	$ \begin{array}{c}     -2 \\     -2 \\     -2 \end{array} $	$ \begin{array}{c}     \cdot \\     -2 \\     -2 \\     \vdots \\     \vdots \end{array} $
X.95 X.96 X.97 X.98 X.100 X.101 X.102	-1 1	-i -1	-1 1						6 3 3 3 -6 -6	$-2 \\ J \\ K \\ K \\ J \\ -2 \\ -2$	-2 K J K -2 -2	_	$\begin{array}{c} \cdot \\ \cdot \\ -1 \\ -1 \\ -1 \\ -2 \\ -2 \end{array}$	$\begin{array}{c} 1 \\ -1 \\ -1 \\ -1 \\ -2 \\ -2 \end{array}$	54 48 27 27 27 27 27	-24	-18 3 3 3	-9 -9 12	-6 : :	6 -1 1 -1 1	6 -1 1 -1 1	-3 8	3 3 3 3	3 3 3
X.103 $X.104$ $X.105$ $X.106$ $X.107$ $X.108$ $X.109$ $X.110$				12	6 -6 3 3				-8						$ \begin{array}{r} -27 \\ -27 \\ -27 \\ 32 \\ 32 \\ 32 \\ -54 \end{array} $	-4 -4 -4 18	$^{-16}_{18}$	-4 -4 -4 -9 9		-1 1	-1 1	4 4 4 6	$ \begin{array}{c}     -3 \\     -3 \\     -4 \\     -10 \\     14 \end{array} $	-3 -3 -4 14 -10
X.111 X.112 X.113 X.114 X.115 X.116 X.117 X.118	1 -1 -1 -1	-1 -1 -1 1		9 9					-3 -3 -3	1 1 1	1 1 1	-3 -3 -3	-i 1 :	-1 1	-54 $-36$ $-36$ $-81$ $-81$ $-81$	18 -9 -24 -24	18 12 12 -9 -9 -9	9 18 -12 -12		-6 -3 3 -3	-6 -3 3 -3	-9 - - - -		
X.119 X.120 X.121 X.122 X.123 X.124 X.125	-1 -2 -2 -2		-1 -2 -2 -2	-12 -9		3			3 3 4 4	-i -1 -1	-1 -1	3	-1 -1 -2 2 2 -2	-1 1 2 -2		-30	:	12		-12 12	-3	6		
X.126 X.127 X.128 X.129 X.130 X.131 X.132									4				$-\frac{2}{2}$	-2 2		27						-9 -9		
X.133 X.134 X.135 X.136 X.137 X.138 X.139				$     \begin{array}{r}       -4 \\       -4 \\       3 \\       3 \\       3     \end{array}   $	$ \begin{array}{c} 2 \\ -3 \\ -3 \\ -3 \\ -3 \end{array} $	-1 -1	-1 -1	-1 -1							-8 -8 -8	-8 -8 -8	-8 -8 -8	-8 -8 -8 -8	$ \begin{array}{r}     -8 \\     -8 \\     -8 \\     -8 \\     -2 \end{array} $	-8 -8 -8 -12 8	-8 -8 -8 12 8		$\begin{array}{c} . \\ . \\ . \\ . \\ . \\ . \\ . \\ . \end{array}$	$ \begin{array}{c}     -5 \\     7 \\     -5 \\     \hline     7 \end{array} $
X.140 $X.141$ $X.142$ $X.143$ $X.144$ $X.145$ $X.146$				-6 -6 -6 -6 -12	-3 6 6	3 3			-8 8						16 -8 -8 16	28 -8 -8 28	16 16 16 16	4 -8 -8 4	$ \begin{array}{r} -2 \\ -8 \\ -8 \\ -2 \\ \vdots \end{array} $	12 -4 4	-12 -4 4	4  -4 	$ \begin{array}{c} 4 \\ -2 \\ -2 \\ 4 \\ \vdots \end{array} $	$ \begin{array}{c} 4 \\ -2 \\ -2 \\ 4 \\ \vdots \end{array} $
X.147 $X.148$ $X.149$ $X.150$ $X.151$ $X.152$ $X.153$				-12 -4 -4	2	-1 -1 -1	-1 -1 -1	-i -1 -1	-4 -4					-2 -2 2	-48	-12 -12	9 9	-12 6	6	3 -3 .	3 -3			
X.154 $X.155$ $X.156$ $X.157$ $X.158$ $X.159$ $X.160$	-i -i 1 1 -1		-1 -1 1 -1												-48	-12 -12 -12		6 6 6	6 6 			$\begin{array}{c} 4 \\ -4 \\ -4 \\ \end{array}$		
X.161 X.162 X.163 X.164 X.165 X.166 X.167				6	3 -4 -6 -6		2	· · · ·							32	56	-16 :	8	-4 : : :				-4 : : :	-4 : :

Character table of  $H(\mathrm{Fi}_{24}')$  (continued)

$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$\begin{array}{c} 12_1  12_{12}  12_{13}  12_{14}  12_{15}  12_{16}  12_{17}  12_{18}  12_{19}  12_{20}  12_{21}  12_{22}  12_{23}  12_{2} \\ 2P  67  614  67  65  61_4  69  69  61_5  66  61_5  66_21  62_1 $	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
X.89 222 222 2 -2 $X.90$ -222 -2 -2	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$X.101 \ X.102 \ .$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	
X.113 4	
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	
X.120	
X.124 X.125 X.126 X.127	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
X.132	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
X.136	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
X.1444	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
X.156 2	
X.160	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$

2 3 5 7	4 1	4 1	4	4 1	1 1 i	1 1 1	5	5 4	3 4	$\overset{1}{\overset{4}{\cdot}}$	4 2	2 3	2 3	2 2	2 2	2 2	2 2	4 i	3 i	3 i	3 i	1 1 i	1 1 i	5 2
$\frac{1}{2P}$	$\frac{12_{31}}{6_{17}}$	$\frac{12_{32}}{6_{31}}$	$\frac{12_{33}}{6_{11}}$	$\frac{12_{34}}{6_{31}}$	14a	15a $15a$	16a 8b	18a 9a	18b 9b	$\frac{18c}{9c}$	$\frac{18d}{9a}$	18e 9e	18f 9d	18 <i>g</i> 9 <i>e</i>	18h 9d	18i 9d	18 <i>j</i> 9 <i>e</i>	$\frac{20a}{10a}$	$\frac{20b}{10b}$	$\frac{20c}{10d}$	$\frac{10c}{20d}$	$\frac{21a}{21b}$	$\frac{21b}{21a}$	$\frac{.}{12_3}$
3 <i>P</i> 5 <i>P</i> 7 <i>P</i>	$\frac{4q}{1231}$	$\frac{4n}{1234}$	$\frac{4p}{1233}$	$\begin{array}{r} 6_{31} \\ 4n \\ 12_{32} \\ 12_{34} \end{array}$	$\frac{14a}{14a}$	$\frac{5a}{3a}$	$\frac{16a}{16a}$	$^{6_3}_{18a}$	$\frac{63}{18b}$	63 18c	$^{66}_{18d}$	$^{64}_{18f}$	64 18e	$\frac{622}{18i}$	$624 \\ 18j$	$\frac{623}{18g}$	$625 \\ 18h \\ 18i$	$\frac{20a}{4a}$	20d	20c 4d	20b 4c	7a $21a$	7a $21b$	$     \begin{array}{c}       8a \\       24a \\       24a   \end{array} $
X 90		1232	1233	1234	2a	13 <i>a</i>	10 <i>a</i>	:	:	:	:	106	10 <i>j</i>	109	1011	101	103	20 <i>a</i> 1	-1	-1 1	1 -1	- Su	- Sa	
$X.91 \\ X.92$	$-\frac{2}{2}$	:		:				$^{-6}_{-6}$	$-3 \\ -3$		2 2									:				
X.93 X.94 X.95	1	:		:	:	1		:	6	3	:	:	:	:	:					:	:	:		
X.95 X.96 X.97				:	:						:							$-1^{2}$	Ĺ	-i	$-\dot{L}$	:		1
$X.98 \\ X.99$	:	:	:	:	:			:	:			:	:					$-1 \\ -1$	-L	-1 $1$	$-L \\ L \\ L$			$-1 \\ 1 \\ 1$
$X.100 \\ X.101 \\ X.102$							$-1^{i}$	:			:		:					2	$\begin{array}{c} \stackrel{L}{\stackrel{L}{\stackrel{L}{}{}{}{}{$	$-\frac{1}{2}$				-1
X.103 X.104	:	:		:	-2	-i	:	-12	-6	:	:	:	:	:	:	:	:	:	:	:	:	-1	-1	
$X.105 \\ X.106 \\ X.107$	:	:	:	:				6	-6		$-\dot{2}$						:	:	:					$-\frac{1}{1}$
$X.108 \\ X.109$		:		:	:			$\begin{array}{c} -6 \\ -3 \\ -3 \end{array}$	$-6 \\ 3 \\ 3$	:	$\frac{1}{1}$		:				:	:	:	:	:			
$X.110 \\ X.111 \\ X.112$	-1 1																							
$X.113 \\ X.114$			:					9			1							1	$-1^{1}$	$-\frac{1}{1}$	$-1^{1}$			
$X.115 \\ X.116 \\ X.117$		:	:	:			-1 1 -1	:			:	:		:		:	:	:	:	:	:	:		$     \begin{array}{r}       3 \\       -3 \\       -3     \end{array} $
$X.118 \\ X.119$							1																	3
$X.120 \\ X.121 \\ X.122$		:	:	:				$^{12}_{-9}$		-3 ·	-i	:	:	:	:	:		-i	i	-i		:	:	
$X.123 \\ X.124$		:				-1												-1	-1	1	-1			
$X.125 \\ X.126 \\ X.127$	:	:	:	:		$-1 \\ -1 \\ 1$		:												:	:			:
X.127 X.128 X.129	:	E		Ē	-1	-1 ·									:		:	:	:			1	1	
$X.130 \\ X.131$		$\bar{E}$		Ē	$-1 \\ -1$																	$-\stackrel{1}{F}$ $-G$	-G	
X.132 X.133 X.134	:	:	:	:	-1	•		4	$-\frac{1}{2}$	1 1		1	1 1	$-\overset{\cdot}{\underset{1}{1}}$	1 -1		1 -1					-G	-F	:
X.134 X.135 X.136		:						3 3	$     \begin{array}{r}       -2 \\       -2 \\       -3 \\       -3 \\       -3     \end{array} $		$-1 \\ -1$				-1		-1							
$X.137 \\ X.138 \\ X.139$		:	:	:					-3		-1 i	:	:	:	:	:				:	:	:	:	
X.140 X.141		:						-6	$-3 \\ -3 \\ .$		2													
$X.142 \\ X.143 \\ X.144$	:	:	:	:		:	:	$ \begin{array}{r} -6 \\ -6 \\ -6 \\ 12 \end{array} $	6		2 2 2									:	:		:	$^{6}_{-6}$
X.144 X.145 X.146		:			2	$-\frac{1}{1}$		12	6 6 -3 -6	$-3 \\ -3$												1	1	
$X.147 \\ X.148 \\ X.149$	:	:	:	:		:	:	12												:	:		:	$-\frac{3}{3}$
X.149 X.150 X.151						1 1		4 4	$\begin{array}{c} 6 \\ -2 \\ -2 \end{array}$	i 1		1 1	i 1	$-1 \\ 1 \\ 1$	$-1 \\ -1$	$-1 \\ 1 \\ 1$	$-1 \\ -1$	:						
$X.152 \\ X.153$	2	:		:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:
$X.154 \\ X.155 \\ X.156$	-2	:	:	:				:									:	:	:			:		:
$X.157 \\ X.158$	:						$-\frac{1}{1}$																	:
$X.159 \\ X.160 \\ X.161$							-1 1																	:
$X.162 \\ X.163$		:		:				6	3		$-\frac{i}{2}$		:			:				:		:		
$X.164 \\ X.165 \\ X.166$					i	:	:	$-8 \\ -12$	$-6^{\dot{4}}$	$-\dot{2}$		$-\dot{2}$	$-\dot{2}$		:		:		:		:	-i	-i	:
X.167									6	3														

Character table of  $H(Fi'_{24})$  (continued)

2 3 5 7	5 2	3 2	3 2	$^4_1$	4 1	4 1	4 1	4 1	$\begin{array}{c} 1 \\ 1 \\ 1 \end{array}$	4 3	4 3	3	1 1	1 1
7	$24\dot{b}$	24c	$\frac{1}{24d}$	24e	24 f	24a	24h	24i	30a	$\frac{.}{36a}$	36b	36c	$\frac{1}{42a}$	$\frac{1}{42b}$
$\frac{2P}{2P}$	$\frac{12_{1}}{8a}$	$^{12}_{\stackrel{10}{8a}}$	$^{129}_{8a}$	$12_{12} \\ 8i$	$^{12}_{\overset{14}{8c}}$	$12_{14}$	$12_{8} \\ 8b \\ 24h \\ 24h$	$^{12}_{\overset{12}{8}i}$	15a	$18a$ $12_{2}$ $36a$ $36a$	18a	$18b_{10}$	$     \begin{array}{r}       42a \\       14a \\       42a \\       \hline       6_1     \end{array} $	$\frac{21b}{14}$
3P 5P 7P	$     \begin{array}{r}       8a \\       24b \\       24b     \end{array} $	$\begin{array}{c} 8a \\ 24c \\ 24c \end{array}$	24d	24i	24a	$\frac{8c}{24f}$	24h	$\begin{array}{c} 8i \\ 24e \\ 24e \end{array}$	$6_1$	$\frac{122}{36a}$	$\frac{122}{36b}$	$\frac{122}{36c}$	$\frac{14a}{42a}$	42b
$\frac{7P}{X.89}$	24b	24c	24d	24i	24g	24f	24h	24e	30a	36a	36b	36c	$6_{1}$	$6_1$
X.90			:	:							j			
$X.91 \\ X.92$	1 :		:			:		:		$-\frac{1}{2}$	$-2 \\ -2$	1 1		
$X.93 \\ X.94$					i	$-\dot{1}$	-i		-1					
X.95		:	:	:	$-1 \\ 1$	1	$-1 \\ -1$			:				:
$X.96 \\ X.97$	i	i	i			:		:		:				
X 98	-1	-1 $1$	-1											
$X.99 \\ X.100$	1 -1	-1	-1	:	:	:	:	:		:			:	:
- x = 101														
$X.102 \\ X.103 \\ X.104$													1	1
X.105	i	$-\overset{i}{{1}}$	i	:	:	:	:	:		:			:	:
$X.106 \\ X.107$	-1	-1	-1							2	2	2	•	
X.108	Ė	- 1		·	÷					$-{7 \atop 5}$	$^{2}_{5}$	-1		
$X.109 \\ X.110$			:	:	i	i	:	:		5	-7	-1		:
$X.111 \\ X.112$					-1	-1	_ i							
$X.112 \\ X.113 \\ X.114$										$-3 \\ -3$	$-3 \\ -3$			
$X.114 \\ X.115$	3	:	:	:	:	:	:	:		-3	-3			:
X.115 X.116 X.117	$\begin{array}{c} 3 \\ -3 \\ -3 \\ 3 \end{array}$													
X 118	3													
$X.119 \\ X.120$		:	:	:	:		:		:	:	:	:	:	
$X.121 \\ X.122$										3	3	•	:	
X.123														
X.119 X.120 X.121 X.122 X.123 X.124 X.125 X.126 X.127 X.128	:	:	:	:	:	:	:	:	$\frac{1}{1}$	:			:	:
$X.126 \\ X.127$	:		:	:	:	:	:	:	1 1	:				:
X.128							1						i	i
X.129 X.130 X.131			:	:									-1 -1 G F	-1 $-1$ $F$ $G$
X.132		:	:		:	:	:	:	:	:	:	:	F	G
$X.133 \\ X.134$														
X.135 X.136 X.137 X.138 X.139		3	-3	:	:		:			$-\frac{1}{5}$	7	1		:
$X.136 \\ X.137$	:	$\begin{array}{c} 3 \\ -3 \\ -3 \end{array}$	$-3 \\ 3 \\ 3$	:	:	:	:	:		$-5 \\ 7 \\ -5$	$-5 \\ 7$	1 1	:	:
X.138 X 139		3	$-\dot{3}$							ż		i		
X.140				:	:		:			$-\overset{\cdot}{2}$	$-5 \\ -2$	1		
$X.141 \\ X.142$	$-\dot{6}$	:	:	:	:	:	:	:		$-\dot{2}$	$-\dot{2}$	$-\dot{2}$	:	:
$X.143 \\ X.144$	6									$     \begin{array}{r}       -2 \\       -2 \\       -2     \end{array} $	$     \begin{array}{r}       -2 \\       -2 \\       -2     \end{array} $	$-\frac{1}{2}$		
X.145			:	:					i	-2	-2			
$X.146 \\ X.147$	$-\overset{\cdot}{\overset{\cdot}{3}}$	:	:		:	:	:	:	-1	:	:		-1	-1
X 148	3													
X.149 X.150			:	:	:		:		-1					
$X.151 \\ X.152 \\ X.153$		:	:	:	:				-1	:	:	:	:	
$X.153 \\ X.154$														
X.155			÷	:	$-\stackrel{\dot{N}}{N}$	$-\stackrel{\dot{N}}{N}$	:					:	:	:
$X.156 \\ X.157$	:	:	:	:	- N	N.	:	:	:	:	:	:	:	
$X.158 \\ X.159$														
X.160			÷	:			:		:			:	:	:
$X.161 \\ X.162$	:	:	:	:		:	:	:	:	$\dot{2}$	2	-i	:	
$X.163 \\ X.164$				$-N \atop N$				$-\stackrel{\dot{N}}{N}$						
X.165			÷	٠.			:	1.4	:				i	i
$X.166 \\ X.167$	:	:	:	:	:	:	:	:	:	:	:	:	:	:

, where  $A=12\zeta(3)+2$ ,  $B=-3\zeta(3)-2$ ,  $C=-\zeta(3)$ ,  $D=-12\zeta(3)-6$ ,  $E=-2\zeta(3)-1$ ,  $F=2\zeta(21)_3\zeta(21)_7^4+2\zeta(21)_3\zeta(21)_7^2+2\zeta(21)_3\zeta(21)_7+\zeta(21)_3+\zeta(21)_7^4+\zeta(21)_7^2+\zeta(21)_7+1$ , G=-F+1, H=-3E,  $I=6\zeta(3)+1$ ,  $J=-4\zeta(5)^3-4\zeta(5)^2-1$ ,  $K=4\zeta(5)^3+4\zeta(5)^2+3$ ,  $L=2\zeta(5)^3+2\zeta(5)^2+1$ ,  $M=18\zeta(3)+9$ ,  $N=-4\zeta(12)_4\zeta(12)_3-2\zeta(12)_4$ .

**B.5.** Character table of  $A_1(\text{Fi}'_{24}) = \langle y, q, s \rangle$ 

$\begin{array}{cccccccccccccccccccccccccccccccccccc$	14 13 1 3 1 .
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c cc} 4c & 4d \\ \hline 2e & 2e \\ 4c & 4d \\ \end{array}$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	50 - 14 
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c} 64 \\ -64 \\ 1 \\ 29 \\ -1 \\ 29 \\ \cdot -16 \\ \cdot -16 \\ -30 \\ -14 \\ 10 \\ 10 \\ \end{array}$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	30 - 14 $ -10 10 $ $ 5 - 5 $ $ 65 83 $ $ -65 83 $ $ 5 - 5 $ $ -5 - 5$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	-5 -5 75 33 -75 33 - 25 33
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	64 64 . 64 
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	50 - 34 $64 - 64$ $-64 - 64$ $-15 - 15$ $-15 - 15$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
$\begin{array}{c} X.76 \ 370656 \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ $	$\begin{array}{cccc}  & & & & \\  & 10 & -6 \\  & 10 & 130 \\  & -10 & 130 \\  & 10 & -6 \\  & & & \\  & -64 & & \\  & & & \\  & 64 & & \\ \end{array}$

Character table of  $A_1(\mathrm{Fi}_{24}')$  (continued)

2 3 5 7 11 13	11 2 1	13 1	11 2	11 2	11 2	12 1	11 1	10 1	5 1 2	10 7 1 1	9 9	8 7	8 5 1	8 5 1	8 6	8 4	8 4 i	5 7	9 4	9 4	10 3
2P 3P 5P 7P 11P 13P	4e 2d 4e 4e 4e 4e 4e 4e	$\begin{array}{c} 4f \\ 2e \\ 4f \\ 4f \\ 4f \\ 4f \\ 4f \\ 4f \end{array}$	$\frac{4g}{2d}$ $\frac{4g}{4g}$ $\frac{4g}{4g}$ $\frac{4g}{4g}$ $\frac{4g}{4g}$	2g 4h 4h 4h 4h 4h 4h	$\begin{array}{c} 4i \\ 2g \\ 4i \\ 4i \\ 4i \\ 4i \\ 4i \\ 4i \end{array}$	$\begin{array}{c} 4j \\ 2e \\ 4j \\ 4j \\ 4j \\ 4j \\ 4j \\ 4j \end{array}$	4k $2d$ $4k$ $4k$ $4k$ $4k$ $4k$ $4k$	2g 4l 4l 4l 4l 4l 4l 4l	5a 5a 1a 5a 5a 5a 5a 5a	$ \begin{array}{c} 6_1 \\ 3_a \\ 2_a \\ 6_1 \\ 6_1 \\ 6_1 \\ 6_1 \end{array} $	$ \begin{array}{r} 6_2 \\ 3b \\ 2a \\ 6_2 \\ 6_2 \\ 6_2 \\ 6_2 \end{array} $	63 3c 2a 63 63 63 63	$ \begin{array}{c} 64 \\ 3a \\ 2b \\ 64 \\ 64 \\ 64 \\ 64 \end{array} $	$\begin{array}{c} 6_{5} \\ 3a \\ 2c \\ 6_{5} \\ 6_{5} \\ 6_{5} \\ 6_{5} \end{array}$	66 3b 2b 66 66 66	67 3a 2c 68 67 68	68 3a 2c 67 68 67 68	69 3d 2a 69 69 69	$6_{10}$ $3b$ $2e$ $6_{10}$ $6_{10}$ $6_{10}$ $6_{10}$	$\begin{array}{c} 6_{11} \\ 3b \\ 2d \\ 6_{11} \\ 6_{11} \\ 6_{11} \\ 6_{11} \end{array}$	$\begin{array}{c} 6_{12} \\ 3a \\ 2d \\ 6_{12} \\ 6_{12} \\ 6_{12} \\ 6_{12} \end{array}$
X.1 X.2 X.3 X.4 X.5	1 -6 -6 -6	$\begin{array}{c} 1 \\ -2 \\ -2 \\ \end{array}$	$ \begin{array}{c} 1 \\ -1 \\ 2 \\ -2 \end{array} $	1 1 2 2 -8 -8	1 1 2 2 8 8	$-\frac{1}{-2}$	1 1 2 2	$ \begin{array}{c} 1 \\ -1 \\ 2 \\ -2 \end{array} $	1 3 3 2 2	1 15 15 15 8 8	$ \begin{array}{r} 1 \\ -3 \\ -3 \\ -28 \\ -28 \end{array} $	1 6 6 -10 -10	1 -7 -7 -7	-1 -5 5	1 -7 -7 -7	$ \begin{array}{ccc}  & 1 & \\  & 1 & \\  & 1 & \\  & -1 & \\  & A & \\  & \bar{A} & \\  & &$	$ \begin{array}{c} 1 \\ -1 \\ 1 \\ -\frac{1}{A} \\ A \end{array} $	$ \begin{array}{c} 1 \\ -3 \\ -3 \\ -1 \\ -1 \end{array} $	1 5 5 4 4	$     \begin{array}{c}       1 \\       5 \\       5 \\       -4 \\       -4    \end{array} $	$ \begin{array}{c} 1 \\ -1 \\ -1 \\ 8 \\ 8 \end{array} $
X.7 X.8 X.9 X.10 X.11 X.12 X.13	5 $ -11 $ $ -11 $ $ -6 $ $ -6 $ $ 11$	-3 -3 9 6 6 -5	$     \begin{array}{r}       7 \\       -7 \\       -3 \\       3 \\       -6 \\       6 \\       5     \end{array} $	5 5 5 10 10 3	5 5 5 10 10 3	$ \begin{array}{c} -1 \\ 9 \\ -9 \\ -2 \\ 2 \\ 5 \end{array} $	5 -3 -3 2 -5	$     \begin{array}{r}       -1 \\       1 \\       -3 \\       3 \\       2 \\       -2 \\       -3     \end{array} $	4 1 1 5 5 3	$     \begin{array}{r}       6 \\       6 \\       56 \\       56 \\       -1 \\       -1 \\       105     \end{array} $	24 24 29 29 -28 -28 6	15 15 2 2 26 26 15	$ \begin{array}{r} 14 \\ -6 \\ -6 \\ -19 \\ -17 \end{array} $	$ \begin{array}{r} 16 \\ -16 \\ 4 \\ -4 \\ 19 \\ -19 \\ -5 \end{array} $	$ \begin{array}{r} -4 \\ -4 \\ -15 \\ -15 \\ 8 \\ 8 \\ 26 \end{array} $	$     \begin{array}{r}       -8 \\       8 \\       -14 \\       \hline       14 \\       \hline       -1 \\       -1 \\       7     \end{array} $	$     \begin{array}{r}       -8 \\       8 \\       -14 \\       14 \\       1 \\       -1 \\       7     \end{array} $	$ \begin{array}{r} -3 \\ -3 \\ 2 \\ -1 \\ -1 \\ 6 \end{array} $	5 5 -4 -4 14	5 5 -4 -4 14	$     \begin{array}{r}       6 \\       8 \\       8 \\       -1 \\       -1 \\       -7     \end{array} $
X.14 X.15 X.16 X.17 X.18 X.19	$\begin{array}{c} 11 \\ 24 \\ 24 \\ -15 \end{array}$	$ \begin{array}{r} -5 \\ 8 \\ 8 \\ -11 \\ -11 \end{array} $	-5 -8 8 -3 3	3 -16 9 9	3 16 9 9	-5 -8 8 -9 -9	$   \begin{array}{r}     -5 \\     8 \\     8 \\     \hline     -7 \\     -7 \\     \end{array} $	3	3 5 5 10	105 $119$ $119$ $-128$ $15$ $15$	$\begin{array}{c} 6 \\ 2 \\ 2 \\ 52 \\ 114 \\ 114 \end{array}$	$ \begin{array}{r} 15 \\ 20 \\ 20 \\ -38 \\ 24 \\ 24 \end{array} $	17 31 31 5 5	-19 19 -35 35	26 22 22 14 14	$-7 \\ -7 \\ 7 \\ -29 \\ 29$	-7 $-7$ $7$ $-29$ $29$	$\begin{array}{c} 6 \\ 2 \\ -2 \\ 6 \\ 6 \end{array}$	14 $10$ $10$ $-12$ $-6$ $-6$	14 $10$ $10$ $12$ $-6$	$   \begin{array}{r}     -7 \\     7 \\     7 \\     \hline     15 \\     15 \\     \end{array} $
X.20 X.21 X.22 X.23 X.24 X.25 X.26	10 10 10	2 2 2	-6 6	10 10 -16 -16	10 10 16 16 16	-14 14	10 10 10	-2 -2 -2	-2 -2 6 6 6	$     \begin{array}{r}       -308 \\       -308 \\       105 \\       105 \\       -240 \\       120 \\       120 \\    \end{array} $	16 16 123 123 -240 84 84	16 16 33 33 -78 -60 -60	25 25 25	-35 35	7 7	$-35$ $35$ $\dot{\bar{A}}$ $A$	$-35$ $35$ $A$ $\bar{A}$	$ \begin{array}{r} 16 \\ 16 \\ 15 \\ 15 \\ -24 \\ 3 \\ 3 \end{array} $	$ \begin{array}{r} 16 \\ 16 \\ 3 \\ 3 \\ -16 \\ 20 \\ 20 \end{array} $	$ \begin{array}{r} -16 \\ -16 \\ 3 \\ 3 \\ 16 \\ -20 \\ -20 \end{array} $	-4 -4 9 9 16 -8 -8
X.27 X.28 X.29 X.30 X.31 X.32	26 26 -24 -24 -24 -24	-2 -2	6 -6 8 -8 -8 8	18 18	18	10 -10 -10	2 8 8 8 8	6 -6	5 7 7 5 5	-21 -21 91 91 399 399	$ \begin{array}{r} -102 \\ -102 \\ -44 \\ -44 \\ -60 \\ -60 \\ 90 \end{array} $	60 64 64 48 48	29 29 -79 -79 -71 -71	-11 - 11 - 61 - 59 -	-34 -34 20 20 -44 -44	7 -7 -7 7 -7 -7 21	7 -7 -7 7 7 -7 21	6 6 10 10 -6 -6	-14 $-14$ $4$ $20$ $20$	$     \begin{array}{r}     -14 \\     -14 \\     4 \\     20 \\     20   \end{array} $	$     \begin{array}{r}       -5 \\       -5 \\       -5 \\       -1 \\       -1     \end{array} $
X.33 X.34 X.35 X.36 X.37 X.38	41 41	$     \begin{array}{r}       5 \\       5 \\       -16 \\       -16 \\       18 \\       -14 \\       18 \\    \end{array} $	$ \begin{array}{c} 21 \\ -21 \\ 16 \\ -16 \\ 10 \\ 6 \\ -10 \end{array} $	$ \begin{array}{c} 1 \\ 16 \\ 16 \\ 18 \\ -2 \\ 18 \end{array} $	1 16 16 18 -2 18	$ \begin{array}{c} -9 \\ \vdots \\ 2 \\ -14 \\ -2 \end{array} $	1 1 -6 6 -6	-5 $-6$ $-2$ $6$	-5 -5 -2 -2	441 441 -84 -84 -35 595 -35	90 258 258 235 73 235	$   \begin{array}{r}     -18 \\     6 \\     6 \\     19 \\     10 \\     19 \\   \end{array} $	$     \begin{array}{r}       -21 \\       -21 \\       -44 \\       -44 \\       5 \\       -35 \\       5    \end{array} $	15 · 35 ·	$\begin{array}{r} 42 \\ 42 \\ 10 \\ 10 \\ -13 \\ -71 \\ -13 \end{array}$	$ \begin{array}{r} -21 \\ -56 \\ 56 \\ 49 \\ 21 \\ -49 \end{array} $	$     \begin{array}{r}       -21 \\       -56 \\       \hline       49 \\       21 \\       -49 \\    \end{array} $	$     \begin{array}{r}       9 \\       -12 \\       -12 \\       -8 \\       19 \\       -8     \end{array} $	$     \begin{array}{r}       -6 \\       -6 \\       18 \\       18 \\       -5 \\       9 \\       -5 \\    \end{array} $	$     \begin{array}{r}     -6 \\     -6 \\     18 \\     18 \\     -5 \\     9 \\     -5 \\   \end{array} $	9 9 12 12 13 3 13
X.40 X.41 X.42 X.43 X.44 X.45 X.46	$     \begin{array}{r}       -50 \\       55 \\       -5 \\       -5 \\       15 \\       \hline       15     \end{array} $	$     \begin{array}{r}       -14 \\       -13 \\       19 \\       19 \\       3 \\       -13 \\       3     \end{array} $	$     \begin{array}{r}       -6 \\       1 \\       25 \\       -25 \\       9 \\       -1 \\       -9 \\    \end{array} $	$     \begin{array}{r}       -2 \\       7 \\       3 \\       3 \\       -1 \\       7 \\       -1     \end{array} $	$     \begin{array}{r}       -2 \\       7 \\       3 \\       3 \\       -1 \\       7 \\       -1     \end{array} $	$     \begin{array}{r}       14 \\       13 \\       1 \\       -1 \\       -3 \\       -13 \\       3     \end{array} $	$     \begin{array}{r}       6 \\       -1 \\       11 \\       11 \\       -9 \\       -1 \\       -9 \\    \end{array} $	$ \begin{array}{c} 2 \\ 9 \\ -9 \\ -7 \\ -9 \\ 7 \end{array} $		595 $735$ $-210$ $-210$ $420$ $735$ $420$	73 $-93$ $150$ $150$ $-12$ $-93$ $-12$	$     \begin{array}{r}       10 \\       -3 \\       51 \\       51 \\       42 \\       -3 \\       42     \end{array} $	$ \begin{array}{c} -35 \\ 25 \\ 30 \\ 30 \\ -10 \\ 25 \\ -10 \end{array} $	$\begin{array}{c} 25 \\ 60 \\ -60 \\ -20 \\ -25 \\ 20 \end{array}$	-71 $-79$ $-6$ $-64$ $-64$ $-64$	$ \begin{array}{c} -21 \\ 7 \\ -14 \\ -7 \\ 14 \end{array} $	$     \begin{array}{r}       -21 \\       7 \\       \hline       -14 \\       \hline       -7 \\       \hline       14    \end{array} $	$     \begin{array}{r}       19 \\       -12 \\       -12 \\       -12 \\       15 \\       -12 \\       15     \end{array} $	$   \begin{array}{r}     9 \\     11 \\     -2 \\     -2 \\     28 \\     11 \\     28 \\   \end{array} $	9 11 -2 -2 28 11 28	$ \begin{array}{r}     3 \\     -1 \\     -2 \\     -2 \\     4 \\     -1 \\     4 \end{array} $
X.47 X.48 X.49 X.50 X.51 X.52	5	-7 -7	-1 1	-3 -8 -8	-3 -3 8	-13 13	-3 -3	-9 9	$\begin{array}{c} 6 \\ 6 \\ -4 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{array}$	-1092 $-120$ $-120$ $-120$	$     \begin{array}{r}       162 \\       162 \\       -192 \\       24 \\       24 \\       24    \end{array} $	$     \begin{array}{r}       81 \\       81 \\       6 \\       -48 \\       -48 \\     \end{array} $	90	90 - -90 - -	-18 -18	; Ā A A	$egin{array}{c} \dot{A} \\ ar{A} \\ ar{A} \end{array}$	-30 24 24 24 24	$     \begin{array}{r}       -6 \\       -6 \\       32 \\       -8 \\       -8 \\     \end{array} $	$     \begin{array}{r}     -6 \\     -6 \\     -32 \\     8 \\     8   \end{array} $	-20 8 8 8
X.53 X.54 X.55 X.56 X.57 X.58	-40 -40 -40		-8 -8	-16 -16 -16	16 16 16		-8 -8		6		$ \begin{array}{r} 24 \\ 28 \\ 28 \\ -404 \\ -404 \\ 108 \\ \end{array} $	$     \begin{array}{r}     -48 \\     -8 \\     -8 \\     -62 \\     -62 \\     -54 \\   \end{array} $	-25 -25 -25	15 - -15 -	-52 -52 -52	$A$ $-15$ $15$ $G$ $\bar{G}$	A $-15$ $15$ $G$ $G$	$ \begin{array}{r} 24 \\ -26 \\ -26 \\ 28 \\ 28 \\ -54 \end{array} $	12	-12	$ \begin{array}{r}     8 \\     -3 \\     -3 \\     24 \\     24 \\     \cdot \end{array} $
X.59 X.60 X.61 X.62 X.63 X.64 X.65	-30 -30 24 24	22 22 	$ \begin{array}{c} -6 \\ 6 \\ 24 \\ -24 \end{array} $	-16 -6 -6 -16 -16	16 -6 -6 : 16 16	6 -6	10 10 -8 -8	-6 6	6 -5 -5	-279 $160$	108 57 57 -144 -144 -380 -380	72 52	55 -69 -69	-55 55 -99 99	73 73 - -24 -24 -	$\begin{array}{c} 35 \\ -35 \\ 15 \\ -15 \\ J \\ ar{J} \end{array}$	$ \begin{array}{c} 35 \\ -35 \\ 15 \\ -15 \\ \bar{J} \end{array} $	-54 $-24$ $-24$ $18$ $18$ $-29$ $-29$	12 25 25 4 4	-12 25 25 -4 -4	13 13 9 9 32 32
X.66 X.67 X.68 X.69 X.70 X 71	35 35 -20 -20	23 23 -20 -20	$ \begin{array}{c}     \vdots \\     5 \\     -5 \\     -4 \\     4 \\     -16 \end{array} $	16 3 3	16 3	$-23 \\ 23 \\ . \\ . \\ 12 \\ -12$	$-5 \\ -4$	$-\overset{\cdot}{\overset{\cdot}{3}}$	14	-210 - 210	$     \begin{array}{r}       162 \\       32 \\       -291 \\       -291     \end{array} $	$-81 \\ -112 \\ 114$	$     \begin{array}{r}       60 \\       -45 \\       -45 \\       \hline       -50 \\     \end{array} $	50	-18 -49	$-27 \\ 27 \\ 14 \\ 14$	27 14	18	$ \begin{array}{r} -18 \\ -6 \\ -6 \\ -32 \\ -11 \\ \end{array} $	$     \begin{array}{r}     -18 \\     -6 \\     -6 \\     32 \\     -11 \\     \hline     11     \end{array} $	$ \begin{array}{c} 12 \\ 21 \\ 21 \\ -16 \\ -2 \end{array} $
X.72 X.73 X.74 X.75 X.76 X.77	$-40 \\ 30$	-9 -9	16 3 -3 8 -8 26	-9 -9	-9 -9 -2	3 -3 -18	$ \begin{array}{r} 16 \\ 7 \\ 7 \\ -8 \\ -8 \\ -10 \end{array} $	3 -3	-5 5 6 6	406 406 729 729 405 405 1575	324 324 171	-18						9	- 3	- 3	-9
X.79 X.80 X.81 X.82 X.83 X.84	10 30	-30 -30 2	$ \begin{array}{c}     2 \\     -26 \\     -26 \\     \hline     -16 \end{array} $	-14 - -14 - -2 -48	$^{-14}_{-2}$	$-10_{18}$	$^{-6}_{-10}$	$     \begin{array}{r}       2 \\       -2 \\       14 \\                           $	-6 1 1	-315 -315 1575 126 126	-72 $-72$ $171$ $648$ $180$ $180$	$   \begin{array}{c}     9 \\     9 \\     -18 \\     -162 \\     72 \\     72   \end{array} $	45 45 -15 114					9	$\begin{array}{c} .\\ .\\ .\\ .\\ .\\ .\\ .\\ .\\ .\\ .\\ .\\ .\\ .\\ $		$^{21}_{-9}$

2	10 3	5 6	5 5	5 5	8	8 3	8	8	8 3	7	7 3	5 4	$\frac{5}{4}$	$\frac{5}{4}$	5 4	5 3	5 3	5 3	5 3	3	8	8	8	8	8
7 11 13		:	:	:	:	:	:	:					:	:	:			:	:	i	:	:	:	:	: :
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11P	6133 6133 6133 6133 6133 6133 6133 6133	614 614 614 614 614 614 614 614 614 614			$ \begin{array}{c} 617 \\ 617 \\ 617 \\ 1 \\ 1 \\ 2 \\ -66 $	$\begin{array}{c} 6_{18} \\ 6_{18} \\ 1 \\ -2 \\ -2 \\ 6_{6} \\ 6_{7} \\ 7_{7} \\ 2_{2} \\ -6_{6} \\ -1_{-1} \\ -1_{4} \\ 4_{4} \\ -6 \end{array}$		6200 620 1 -1 -1 -3 -3 -3 -3 -3 -3 -11 -11 -3 -3 -3 -3 -3 -3 -1 -1 -1 -2 -1 -23 -1 -23	$ \begin{array}{c} 021 \\ 621 \\ 621 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 2 \\ 2 \\ 2 \\ 2 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 5 \\ 5 \\ 5 \\ 5 \\ 5 \\ 5 \\ 5 \\ 5 \\ 5 \\ 5 \\ 1 \\ $	$6_{22}$	$6_{23}$	$\begin{array}{c} 625 \\ \underline{624} \\ 1 \\ -1 \\ -33 \\ \underline{8} \\ \bar{B} \\ \bar{B} \\ 3 \\ -44 \\ -33 \\ -66 \\ 6 \\ -22 \\ -66 \\ \cdot \\ \cdot \\ -33 \\ \cdot \\ F \\ \bar{F} \\ -66 \\ 66 \\ -66 \\ -66 \\ -66 \\ -66 \\ -66 \\ 64 \\ 4 \end{array}$	624	627	$6_{26}$	628	$\begin{array}{c} 629 \\ 629 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 3 \\ 3 \\ 1 \\ 1 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2$	630	631	7a	$     \begin{array}{r}       8a \\       8a \\       \hline       1 \\       -1 \\       2 \\       -2 \\       \hline       3 \\       -3 \\       \hline       1 \\       -1 \\       -6 \\     \end{array} $	8b	8c	8d 8d	
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2 3 5 7 11	8	6	6	4 4	3 4	2 3	5 1 2	4 1 2	5 1 :	5 1 :	3 1 1	4 1	2 i	6 3 1	8 3	5 4	5 4	8 2	8 2	6 3	7 2	7 2	5 3	8 1
2P 3P 5P 7P 11P 13P	8f 4f 8f 8f 8f 8f 8f	8g 8g 8q	8h $8h$		9b 3b 9b 9b 9b 9b	9c 3d 9c 9c 9c 9c	10a 5a 10a 2a 10a 10a 10a	$     \begin{array}{r}       10b \\       5a \\       10b \\       2c \\       10b \\       10b \\       10b \\       10b \\     \end{array} $	$     \begin{array}{r}       10c \\       5a \\       10c \\       2d \\       10c \\       10c \\       10c \\       10c \\     \end{array} $	10d 5a 10d 2e 10d 10d 10d	10e 5a 10e 2b 10e 10e 10e	$     \begin{array}{r}       10f \\       5a \\       10f \\       2f \\       10f \\       10$	11a 11a 11a 11a 11a 11a 11a	$ \begin{array}{c} 12_1 \\ 6_1 \\ 4a \\ 12_1 \\ 12_1 \\ 12_1 \\ 12_1 \end{array} $	$12_2$ $6_{13}$ $4b$ $12_2$ $12_2$ $12_2$ $12_2$ $12_2$	$ \begin{array}{c} 12_{3} \\ 6_{10} \\ 4b \\ 12_{3} \\ 12_{3} \\ 12_{3} \\ 12_{3} \end{array} $	$ \begin{array}{r} 12_4 \\ 6_2 \\ 4a \\ 12_4 \\ 12_4 \\ 12_4 \\ 12_4 \end{array} $	$ \begin{array}{r} 125 \\ 6_{13} \\ 4d \\ 125 \\ 125 \\ 125 \\ 125 \\ 125 \end{array} $	$12_{6}$ $6_{13}$ $4d$ $12_{6}$ $12_{6}$ $12_{6}$ $12_{6}$	$6_{10} \\ 4d \\ 12_{7} \\ 12_{7} \\ 12_{7}$	$\frac{4d}{128}$ $\frac{128}{128}$	$6_{13} \\ 4b \\ 12_{9} \\ 12_{9} \\ 12_{9}$	$\begin{array}{c} 12_{10} \\ 6_{22} \\ 4b \\ 12_{10} \\ 12_{10} \\ 12_{10} \\ 12_{10} \\ 12_{10} \end{array}$	1211
X.1 X.2 X.3 X.4 X.5 X.6	1 1 2 2			$\begin{array}{c} 1 \\ 1 \\ 3 \\ 3 \\ -2 \\ -2 \end{array}$	1 4 4	1 1 1 1	$\begin{array}{c} 1 \\ 3 \\ 3 \\ -2 \\ -2 \end{array}$	$-1 \\ -3 \\ 3 \\ \vdots$	$ \begin{array}{c} 1 \\ -1 \\ -1 \\ 2 \\ 2 \end{array} $	$     \begin{array}{r}       1 \\       -1 \\       -1 \\       -2     \end{array} $	1 1 1	$ \begin{array}{c} 1 \\ -1 \\ 1 \\ -1 \end{array} $	1	1 -1 -3 3	1 -1 5 -5	$     \begin{array}{c}       -1 \\       -1 \\       1     \end{array} $	$-1 \\ -3 \\ -3 \\ \vdots$	1 3 3	1 3 3	$ \begin{array}{c} 1 \\ -3 \\ -3 \\ \end{array} $	$\begin{array}{c} 1 \\ 1 \\ -3 \\ -3 \\ \end{array}$	$ \begin{array}{c} 1 \\ -1 \\ -1 \\ 1 \end{array} $	-1 -2 -2 -2	1 -1 -3 3
X.7 X.8 X.9 X.10 X.11 X.12 X.13 X.14 X.15 X.16	1 1 1 1 2 2 3 3	$\begin{array}{c} 1 \\ 3 \\ -3 \\ \end{array}$	$     \begin{array}{r}       1 \\       -1 \\       -1 \\       \hline       \cdot \\       -1 \\    \end{array} $	$\begin{array}{c} \cdot \\ \cdot \\ 2 \\ -1 \\ -1 \\ 3 \\ 3 \\ 5 \\ 5 \end{array}$	3 2 2 -1 -1 -1 -1	$-1 \\ 2 \\ 2 \\ -1$	$\begin{array}{c} 4 \\ 4 \\ 1 \\ 5 \\ 5 \\ 3 \\ 5 \\ 5 \\ \end{array}$	$ \begin{array}{r} 4 \\ -4 \\ -1 \\ 5 \\ -5 \\ -3 \\ 3 \\ -5 \\ 5 \end{array} $	1 1 1 1 -1 -1 1 1	1 1 1 1 1 -1 -1 1 1	$ \begin{array}{c} 2 \\ -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ 1 \end{array} $			$     \begin{array}{r}       4 \\       -4 \\       6 \\       -6 \\       1 \\       -1 \\       3 \\       -3 \\       -11 \\       11     \end{array} $	$\begin{array}{c} \cdot \\ \cdot \\ -6 \\ -6 \\ 1 \\ -13 \\ -13 \\ 13 \\ -11 \\ 11 \end{array}$	$ \begin{array}{r} 6 \\ -6 \\ 3 \\ -3 \\ 4 \\ -4 \\ -2 \\ -2 \\ -2 \\ 2 \end{array} $	$     \begin{array}{r}       -2 \\       2 \\       3 \\       -3 \\       4 \\       -4 \\       -6 \\       6 \\       -2 \\       2     \end{array} $	$     \begin{array}{r}       -2 \\       -2 \\       4 \\       4 \\       -1 \\       -1 \\       5 \\       3 \\       3     \end{array} $	$ \begin{array}{r} -2 \\ -2 \\ 4 \\ 4 \\ -1 \\ -1 \\ 5 \\ 3 \\ 3 \end{array} $	$ \begin{array}{c} 4 \\ 4 \\ 1 \\ -4 \\ -4 \\ 2 \\ 2 \\ 6 \\ 6 \end{array} $	$     \begin{array}{r}       -2 \\       -2 \\       -2 \\       -2 \\       5 \\       5 \\       5 \\       3 \\       3     \end{array} $	-5 -5 -1 1 1	$ \begin{array}{c}     -3 \\     -2 \\     -2 \\     -1 \\     -2 \\     -2 \\   \end{array} $	$ \begin{array}{c}     -2 \\     2 \\     1 \\     -1 \\     -5 \\     -3 \\     3 \end{array} $
X.17 X.18 X.19 X.20 X.21 X.22 X.23 X.24 X.25 X.26	1 1 1 -2 -2	-3 $-2$	-1	8	$ \begin{array}{c} -4 \\ 3 \\ 2 \\ 3 \\ 6 \end{array} $	2 2 2	-10 · 2 2 · -6 -6 -6		2 -2 -2 -2 -2 -2	-2 2 2 2 2 2 2 2			2 -1 -1 -1 -1 -1	-5 -5 -5 -5	3 -3 -3 -3 -3	-12 12	$ \begin{array}{c} 4 \\ -4 \\ -7 \\ 7 \\ \vdots \end{array} $	$ \begin{array}{c} -1 \\ -1 \\ 12 \\ 12 \\ 1 \\ 1 \end{array} $	$ \begin{array}{c} -1 \\ -1 \\ -12 \\ -12 \\ 1 \\ 1 \end{array} $	2 2	5 5 5 	-3 3 -3 3	-3 3	3 -3 -3 3
X.27 X.28 X.29 X.30 X.31 X.32 X.33 X.34 X.35 X.36	2 2 2 	3	-1 :	-3 -3 1 1 3 3	-3 -3 1 1 3 3		$   \begin{array}{r}     -55 \\     57 \\     75 \\     -55 \\     -2 \\     -2   \end{array} $	$ \begin{array}{c}     5 \\     -5 \\     -7 \\     -7 \\     -5 \\     -5 \\     -5 \\     -2 \\     2 \end{array} $	$\begin{bmatrix} -2 \\ 1 \\ -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ -2 \\ 2 \end{bmatrix}$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c} -1 \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -2 \end{array} $	$ \begin{array}{c} 1 \\ -1 \\ 1 \\ -1 \\ -1 \\ -1 \\ 1 \\ 2 \\ -2 \end{array} $		$ \begin{array}{c} -1 \\ 1 \\ -11 \\ 11 \\ 11 \\ -11 \\ 9 \\ -9 \\ -4 \\ 4 \end{array} $	-5 5 1 -1 19 -19 -9 -9	-8 8 -2 -2 -2 -2	$ \begin{array}{c} -4 \\ 4 \\ -2 \\ 2 \\ -2 \\ -4 \\ 4 \end{array} $	$     \begin{array}{r}     5 \\     -4 \\     -4   \end{array} $	$ \begin{array}{r} -1 \\ -1 \\ -5 \\ -5 \\ 3 \\ 3 \\ 5 \\ -4 \\ -4 \end{array} $	2 2 -8 -8	$ \begin{array}{r} -7 \\ -7 \\ -7 \\ 1 \\ 1 \\ -3 \\ -3 \\ -1 \\ -1 \\ -4 \\ -4 \\ -4 \end{array} $	1 -1 -5 5 1 -1 3 -3	$ \begin{array}{c} -2 \\ 2 \\ 4 \\ -4 \\ -4 \\ \vdots \\ \vdots \end{array} $	3 -3 1 -1 -5 5 1 -1
X.37 X.38 X.39 X.40 X.41 X.42 X.43 X.44 X.45		$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{c} -1 \\ 1 \\ 1 \\ 1 \end{array}$	$ \begin{array}{c} 1 \\ 4 \\ 1 \\ 4 \\ 3 \\ -6 \\ -3 \\ 3 \\ -3 \end{array} $	$ \begin{array}{c} 1 \\ -2 \\ 1 \\ -2 \end{array} $ $ \begin{array}{c} \cdot \\ -3 \\ -3 \end{array} $	1 1 1								:	$\begin{array}{c} 5\\ 9\\ -5\\ -9\\ -17\\ -12\\ 12\\ -14\\ 17\\ 14\end{array}$	$ \begin{array}{r} -7 \\ -3 \\ 7 \\ 3 \\ 1 \\ 6 \\ -6 \\ 4 \\ -1 \\ -4 \end{array} $	$ \begin{array}{r} -7 \\ -3 \\ 7 \\ 3 \\ -6 \\ -4 \\ 3 \\ 4 \end{array} $	$     \begin{array}{c}       1 \\       7 \\       1 \\       7 \\       7 \\       2 \\       2 \\       4 \\       7 \\       4     \end{array} $	$     \begin{array}{c}       1 \\       7 \\       1 \\       7 \\       2 \\       2 \\       4 \\       7 \\       4     \end{array} $	$ \begin{array}{c} -5 \\ 1 \\ -5 \\ 2 \\ 2 \\ 4 \\ -5 \\ 4 \end{array} $	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c} -7 \\ 3 \\ 7 \\ -3 \\ 1 \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ &$	$ \begin{array}{r} -1 \\ -6 \\ 1 \\ 6 \\ 1 \\ -3 \\ 3 \\ 4 \\ -1 \\ -4 \end{array} $	$ \begin{array}{c} -3 \\ 1 \\ 3 \\ -1 \\ -1 \\ -4 \\ 4 \\ 2 \\ 1 \\ -2 \end{array} $
X.47 X.48 X.49 X.50 X.51 X.52 X.53	1 1	-1	-1 -1	6 -3 -3 -3 -3 4	-2		6 4	6 -6	-2 -2 4	-2 -2 -4 ·	•	-2 2		-15	21	6 -6	-6 6	12	-12 -12	6 6	-6 -6	-6 6	3 -3	
X.55 X.56 X.57 X.58 X.60 X.61 X.62 X.63	22 2			$ \begin{array}{c} 4 \\ -4 \\ -4 \end{array} $ $ \begin{array}{c} -3 \\ -3 \end{array} $	-2 2 2 	-1 -1 -1	-6 -6 -5 -5	-5 5	-2 -2 -2 -1 -1	2 2 2 2 	: : : : : : : :	-1 1		15	-21	-6	-6	5 5 1 1	5 5 1	-8 -7 -7 -8 -8	-5 -1 -1 -5 -5	-3	-8 -8 -8	-5 -7 -7
X.65 X.66 X.67 X.68 X.70 X.71 X.72 X.73 X.74 X.75	3 3 -4 -4 -4 -1		1	$\begin{array}{c} 10 \\ -3 \\ -3 \end{array}$	:	-1	-14 -5 -5 5 6	-5 -5 -5 6	-1 -1 -1 1 1 -2		1			$ \begin{array}{c}                                     $	$ \begin{array}{c}                                     $	6 -6 -7 -7 -6 -6	-6 6 -13 13 6 -6	$ \begin{array}{c} 4 \\ -3 \\ -3 \end{array} $ $ \begin{array}{c} 2 \\ 2 \\ -6 \\ -6 \\ -3 \\ -3 \end{array} $	$ \begin{array}{c} 4 \\ -3 \\ -3 \end{array} $ $ \begin{array}{c} 2 \\ 2 \\ -6 \\ -3 \\ -3 \\ -3 \end{array} $	5 5 	6 -3 -3	3 -3 -2 2 -6 6 3 -3 -3	-3 3 4 -4 -4	-3 -3 -2 -2 -2 -1 1 -3
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2P 3P 5P 7P 11P 13P	$\begin{array}{c} 12_{12} \\ 6_{12} \\ 4g \\ 12_{12} \\ 12_{12} \\ 12_{12} \\ 12_{12} \end{array}$	$ \begin{array}{c} 12_{13} \\ 6_{17} \\ 4h \\ 12_{13} \\ 12_{13} \\ 12_{13} \\ 12_{13} \end{array} $	$\begin{array}{c} 12_{14} \\ 6_{12} \\ 4e \\ 12_{14} \\ 12_{14} \\ 12_{14} \\ 12_{14} \end{array}$	$\begin{array}{r} 12_{15} \\ 6_{17} \\ 4i \\ 12_{15} \\ 12_{15} \\ 12_{15} \\ 12_{15} \\ 12_{15} \end{array}$	$\begin{array}{r} 12_{16} \\ 6_3 \\ 4a \\ 12_{16} \\ 12_{16} \\ 12_{16} \\ 12_{16} \end{array}$	$\begin{array}{r} 12_{17} \\ 6_{19} \\ 4i \\ 12_{17} \\ 12_{17} \\ 12_{17} \\ 12_{17} \end{array}$	$\begin{array}{c} 12_{18} \\ 6_{19} \\ 4h \\ 12_{18} \\ 12_{18} \\ 12_{18} \\ 12_{18} \end{array}$	$\begin{array}{r} 12_{19} \\ 6_{11} \\ 4g \\ 12_{19} \\ 12_{19} \\ 12_{19} \\ 12_{19} \end{array}$	$\begin{array}{c} 12_{20} \\ 6_{10} \\ 4f \\ 12_{20} \\ 12_{20} \\ 12_{20} \\ 12_{20} \end{array}$	$\begin{array}{r} 12_{21} \\ 6_{12} \\ 4k \\ 12_{21} \\ 12_{21} \\ 12_{21} \\ 12_{21} \end{array}$	$\begin{array}{r} 12_{22} \\ 6_{23} \\ 4g \\ 12_{22} \\ 12_{22} \\ 12_{22} \\ 12_{22} \end{array}$	$ \begin{array}{r} 12_{23} \\ 6_{23} \\ 4e \\ 12_{23} \\ 12_{23} \\ 12_{23} \\ 12_{23} \end{array} $	$\begin{array}{r} 12_{24} \\ 6_{28} \\ 4i \\ 12_{24} \\ 12_{24} \\ 12_{24} \\ 12_{24} \end{array}$	$\begin{array}{r} 12_{25} \\ 6_{28} \\ 4h \\ 12_{25} \\ 12_{25} \\ 12_{25} \\ 12_{25} \end{array}$	$\begin{array}{r} 12_{26} \\ 6_{22} \\ 4j \\ 12_{26} \\ 12_{26} \\ 12_{26} \\ 12_{26} \end{array}$	$\begin{array}{r} 12_{27} \\ 6_{28} \\ 4l \\ 12_{28} \\ 12_{27} \\ 12_{28} \\ 12_{27} \end{array}$	$\begin{array}{r} 12_{28} \\ 6_{28} \\ 4l \\ 12_{27} \\ 12_{28} \\ 12_{27} \\ 12_{28} \end{array}$	13a 13a 13a 13a 13a 13a	14a	$     \begin{array}{r}       14b \\       7a \\       14b \\       14b \\       2c \\       14b \\       14b \\     \end{array} $
X.1 X.2 X.3 X.4 X.5 X.6 X.7 X.8 X.9 X.10 X.11 X.12 X.13 X.14 X.15 X.16 X.17 X.18	1 -11 -11 -11 -11 -11 -11 -11 -11 -11 -	1 1 2 2 2 -2 -1 -1 -1 2 -2 -2 -2 -3 3 3	1 -3 -3 -2 2 -2 -2 -3 -3 -5 5 3 3 -3	1 1 2 2 2 2 2 2 -1 -1 2 2 2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2	-1 -1 -1 -1 -2 2 3 -3 -2 2	1 1 1 -1 -1 2 2 2 2 2 2 -1 -1 -2 -2 -2 -2	$     \begin{array}{c}       1 \\       -1 \\       -1    \end{array} $	1 -1 -1 -1 1 -2 2 3 -3 -3 -2 -2 -2 -2 -2 -2	1 1 1 1 	1 -11 -11 -2 2 2 2 -1 -11 -11 -11 -11 -1	1 -1 2 -2 -1 -1 -1 -2 -2 -21 -2 -2	1 1 	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	1 1 -1 -1 1 1 -1 -1 -1 2 2 2 1 1	1 -1 -2 2 -1 1 -2 2 -1 1 -2 2	-1 -1 -1 C C -1 1	-1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1	1 1 1	$\begin{array}{c} 1 \\ 1 \\ 1 \\ -2 \\ -2 \\ 2 \\ 2 \\ \vdots \\ -2 \\ 1 \end{array}$	1 -1 1 -1
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X.51 X.52 X.53 X.54 X.55 X.56 X.57 X.60 X.61 X.62	-1 -1 -3 -3 -3	· · · · · · · · · · · · · · · · · · ·	-1 -1 -1 	-2 -2 -2 -2 -2		-2 -2 -2 -2 -2 -3 -3	2 2 2 2 2 2 3 -3	-2 -2 -2 -3				2 2 2	-2 -2 -2 -2 -2	2 2 2 2 2	-			1 1 1 -1 -1 1 1	2 2 2 -1 -1 -1	-1 -1 -1 -1 
$X.63 \\ X.64 \\ X.65$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	-4 -4	3	4 4 -2 3 3 3	3 -3 -2 -2	-2 -2 -2 -2 -1 -1	2 2 -2 -2 -1 -1	2 -2 -1 1 2 -2	-22 22 2 1 1	1	-1 1 2 -2 -2 2 -2	-2 -1 -1 -2 -2 -2 -2 -2	1 1 1 -2 -1 -1 -1	-1 -1 -2 -1 -1	1 -11 -1 -1 -1		C C C	1		-1 -1 -1 -1 -2 -2 -1 -1
X.66 X.67 X.68 X.69 X.70 X.71 X.72 X.73 X.74 X.75 X.76 X.77 X.78 X.80 X.81 X.82 X.83 X.84	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	-4 -22 3 3 3 2 2 2	$\begin{array}{c} 4 \\ -1 \\ -1 \\ -2 \\ -2 \\ -2 \\ -3 \\ -3 \\ -1 \\ -3 \\ 1 \\ -3 \\ 2 \\ 2 \end{array}$	$ \begin{array}{c} -2 \\ 3 \\ 3 \\ 3 \\ 2 \\ 2 \\ 2 \\ -1 \\ -1 \\ -2 \\ -6 \\ . \end{array} $	3 -3 -2 -2 -2 	-2 -2 -1 -1 -1 -2 -2 -2 1	2 -2 -1 -1 -1 -2 -2 -2 1	$\begin{array}{c} \cdot \\ -2 \\ -2 \\ -1 \\ 1 \\ 2 \\ -2 \\ -1 \\ 2 \\ -2 \\ 1 \\ -2 \\ 2 \\ \end{array}$	-2 2 2 2 1 1 1	-4 1 1 2 2 -2 -2 -2 1 1 1 1 -1 -3 -3 -1	$\begin{array}{c} \cdot \\ -1 \\ 1 \\ \cdot \\ -2 \\ -2 \\ -2 \\ -2 \\ -1 \\ 1 \\ -2 \\ -2 $	-2 -2 -2 -2 -2 -2 -2 -2 2 2 2 2	1 1 1 1 1	1 1 1 1 1	1 -1 -1	1 -1 -1 -1	1 -1 -1 -1	1	-2 -2 -1 -1 -1	-2 1 -1

2 3 5 7 11 13		3 1 1		5 5	44	3 4	3 3	3 3	3 3	3	4 2	4 2	2 3	3	2 3	2 2	2 2	3 1 1	3 1	3	1		2 1	2 i
2F 3F 5F 7F 11F 13F	14c 7a 14c 14c 2b	15a 5a 3a 15a 15a 15a	8 f 16a 16a 16a	$16b \\ 16b \\ 16b$	18a 9a 6 <sub>2</sub> 18a 18a 18a	$^{9b}_{62}_{18b}_{18b}$	$ 9a $ $ 6_{14} $ $ 18f $ $ 18c $	$^{66}_{18e}_{18d}$	9a 6 <sub>6</sub> 18d 18e	9a 6 <sub>14</sub> 18c 18f	$\frac{6_{10}}{18g}$ $\frac{18g}{18g}$	9a 6 <sub>11</sub> 18h 18h 18h 18h	$^{6_{14}}_{18i}_{18i}$	$^{69}_{18j}_{18j}$	9b 66 18k 18k 18k 18k	$     \begin{array}{r}       18l \\       9c \\       6_{24} \\       18m \\       18l \\       18m \\       18l \\     \end{array} $	$^{625}_{18l}_{18m}$	$     \begin{array}{r}       10a \\       20a \\       4a \\       20a \\       20a     \end{array} $	10c $20b$ $4e$ $20b$ $20b$	10d 20d 4d 20d 20d	21a 7a 21a 3a 21a	26	11a $22b$ $22b$ $22b$ $2a$	$\begin{array}{c} 22c \\ 22c \\ 22a \end{array}$
5 F F F F F F F F F F F F F F F F F F F	-1111111111111111111	3 a a a a a a a a a a a a a a a a a a a			188a 188a 188a 333332 22 21 -11 333555 -88 -22 -21 -16666633 -33 -33 -33 -666-33 -33 -33 -33	188b 188b 11 1 1 1 1 1 1 1 1 1 1 1 1 1 1	$\begin{array}{c} 18J_{C} \\ 18J_{C} \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1$	188e 18de 11	$\begin{array}{c} 188e \\ \hline 1 \\ 1 \\ -1 \\ -1 \\ \end{array}$	$\begin{array}{c} 186 \\ \hline 1 \\ 1 \\ -1 \\ 1 \\ \hline 1 \\ -1 \\ -1 \\ \hline 0 \\ D \\ D \\ -2 \\ -2 \\ -2 \\ 2 \\ 1 \\ -1 \\ -1 \\ -1 \\$	18gg 18gg 18gg 18gg 18gg 18gg 18gg 18gg	$\begin{array}{c} 18h \\ 18h \\ 1 \\ 1 \\ -1 \\ -1 \\ 2 \\ 2 \\ 2 \\ -1 \\ -1$	$ \begin{array}{c} 18i \\ 18i \\ \hline 1 \\ 1 \\ -1 \\ -2 \\ 2 \\ 1 \\ -1 \\ -1 \\ -2 \\ 2 \\ -1 \\ 1 \\ -1 \\ -$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	188k 11	18m	188m 181 181 181 181 181 181 181 181 181	$ \begin{array}{c} 20a \\ 20a \\ 1 \\ -1 \\ -1 \\ 1 \end{array} $	206 20b 1 -1 -1 -1 -1 -1	1 -1 -1 -1 -1 -1 -1 -1 -1 -1	21a	22cc 11 11 -11 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1	22ab 22bb 22bb 22bb 22bb 22bb 22bb 22bb	$   \begin{array}{r}     2b \\     22a \\     \hline     1 \\     -1 \\   \end{array} $
X.68 X.68 X.70 X.71 X.72 X.74 X.75 X.76 X.77 X.78 X.80 X.81 X.82 X.83 X.84		1 1 1 1 1 1 1	-1 -1 -1	-i	-3 -3 4 4 	-2	2	1 1	1		1	1 1 1	-1 1 2 -2 -2	1 1 1		-111	-i 1 1 	-1 1 -1 -1 	1 1 -1 -1 -1	-1 1	1 1 1 -1 -1 -1			

Character table of  $A_1(Fi'_{24})$  (continued)

2 3 5 7 11 13	5 1	5 1	5 1	5 1	4 1	4 1	4 1	4 1	1	. 1	3 1 1	2 1 1	2 1 1		3 2	2 2	2 1	2 1	2 1	1
7P	$     \begin{array}{r}       8b \\       24a \\       24a \\       24a   \end{array} $	$12_{6}$ $8c$ $24b$ $24b$ $24b$	$     \begin{array}{r}       8d \\       24c \\       24c \\       24c     \end{array} $	$     \begin{array}{r}       24d \\       125 \\       8e \\       24d \\       24d \\       24d \\       24d \\       24d     \end{array} $	$     \begin{array}{r}       24e \\       \hline       12_{13} \\       8g \\       24g \\       24g \\       24e \\       24e     \end{array} $	$ \begin{array}{r} 24f \\ 12_{20} \\ 8a \\ 24h \\ 24h \\ 24f \\ 24f \end{array} $	$     \begin{array}{r}     24g \\     12_{13} \\     8g \\     24e \\     24e \\     24g \\     24g   \end{array} $	$12_{20} \\ 8a \\ 24f \\ 24f \\ 24h$	13a 26a 26a 26a 26a	14a 28a 28a 4a 28a	$^{10a}_{61}_{30a}_{30a}$	$15a$ $10e$ $6_4$ $30b$ $30b$	15a 10b 65 30c 30c	$18g$ $12_{3}$ $36a$ $36a$ $36a$	$18g$ $12_3$ $36b$ $36b$ $36b$	$18b$ $12_4$ $36c$ $36c$ $36c$	14b $42c$ $67$ $42c$	$21a$ $14a$ $42b$ $6_1$ $42b$	$     \begin{array}{r}       21a \\       14b \\       42a \\       68 \\       42a     \end{array} $	30a 20a 12 <sub>1</sub> 60a 60a 60a
X.1 X.2 X.3 X.4 X.5	$ \begin{array}{c} 1 \\ 1 \\ -1 \\ -1 \end{array} $	$-1 \\ -1$	1	1	$-1 \\ -1 \\ \cdot \\ \cdot \\ \cdot$	$ \begin{array}{c} 1 \\ -1 \\ -1 \\ 1 \end{array} $	-1 -1	$ \begin{array}{c} 1 \\ -1 \\ -1 \\ 1 \end{array} $	-1	$     \begin{array}{ccc}                                   $	-2	$\begin{array}{c} 1 \\ 1 \\ -2 \\ -2 \\ \end{array}$		$ \begin{array}{c} 1 \\ -1 \\ -1 \\ 1 \end{array} $	$ \begin{array}{c} 1 \\ -1 \\ -1 \\ 1 \end{array} $		$-1$ $-1$ $-1$ $\bar{C}$	1 1 1 1	-1 -1 -1 -1 -1	$-1 \\ -2 \\ -2 \\ \cdot$
X.6 X.7 X.8 X.9 X.10 X.11	-2 -2 -2		$ \begin{array}{r} -2 \\ -2 \\ -2 \\ -2 \\ -1 \end{array} $	$-2 \\ 2 \\ -1$	-1 1		-1 1	1 -1	-1		-2 1 1 1 1 -1	$ \begin{array}{c} -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ 1 \end{array} $	1 -1 -1 -1 -1	:	i	-1 -1	C -1 1	$ \begin{array}{c} 1 \\ -1 \\ -1 \end{array} $	$ \bar{C} $ $ -1 $ $ 1 $ $ \cdot $ $ \cdot $	-1
X.12 X.13 X.14 X.15 X.16 X.17 X.18	$ \begin{array}{c} 1 \\ -1 \\ -1 \\ 1 \end{array} $	$     \begin{array}{r}       -1 \\       1 \\       -1 \\       -1 \\       1     \end{array} $	$-1 \\ -1 \\ -1 \\ 1$	$ \begin{array}{c} 1 \\ 1 \\ -1 \\ -1 \\ 1 \end{array} $	-1 -1		-1 -1		-1 -1		-1 $-1$ $-1$ $2$	1 2 2 1 1	1 -1 -1	$-1 \\ -1 \\ 1 \\ 1$	-1 -1 1 1 -1	-1	-1	-1	-1 -1	$ \begin{array}{c} -1 \\ -2 \\ 2 \\ -1 \\ 1 \end{array} $
X.19 X.20 X.21 X.22 X.23 X.24	1		3 1	-1			-1 -1	1 -1		1	2 2					-1 -1 1	1	1	Î	
X.25 X.26 X.27 X.28 X.29 X.30 X.31 X.32 X.33 X.34 X.35	$ \begin{array}{c} -1 \\ -1 \\ -1 \\ 1 \\ 1 \\ -1 \\ -1 \\ 1 \end{array} $	$-1 \\ -1 \\ -1 \\ 3$	$ \begin{array}{c} 1 \\ -1 \\ -1 \\ 1 \\ 1 \\ -1 \end{array} $	$ \begin{array}{c} 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ 1 \end{array} $		-					-1 -1 -1 1 -1 -1 1 1	-1 -1 1 -1 -1 -1 -1 -1	-1 -1 -1 -1 1 -1 -1 -1	$ \begin{array}{c} 1 \\ -1 \\ 1 \\ -1 \end{array} $	-1 -1 -1 -1 -1 -1	-1 1 1 -1 -1 1	C	1	C C	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
X.36 X.37 X.38 X.39 X.40 X.41 X.42 X.43 X.44 X.45 X.46 X.47	-1 -1 -1 -1 -1 1 :	$ \begin{array}{cccc} -1 & & & & \\ 1 & & & & \\ -1 & & & & \\ -2 & & & & \\ -2 & & & & \\ & & & & \\ \end{array} $	-1 -1 -1 -1	$ \begin{array}{c} 1 \\ -1 \\ 1 \\ -2 \\ 2 \\ -1 \\ \vdots \\ 2 \end{array} $	-1 1 -1 1 -1 1 1	-1 -1 -1 1 -1 -1 -1	-1 i -1 -1 -1 -1 i	1 1 1 -1 -1 1 1			i	1	-1		-i i i -1 -1	1 -1 i				-1
X.48 X.49 X.50 X.51 X.52 X.53 X.54 X.55 X.56		-i	-i -1	-2	-1		-1		-1 -1 -1 -1	-1 1	-2			3 3 -3 -3 -3	-3 -3 3 3		C C C C -1 1	-1 -1 -1 -1 -1 -1		
X.57 X.58 X.59 X.60 X.61 X.62 X.63 X.64	-1 -1 -1 -1 -1	-1			-I	-1 1	-İ I	-1 1	-1 -1	-1 1		1 1	i -1		-1 -1			1 1 1		: -1
X.65 X.66 X.67 X.68 X.69 X.70	:		i 1		-i -i 1	-1 1	-1 1	-1	1		$-\dot{2}$					-1 1	Ĉ	-1 $-1$ $-1$ $2$	C i	
X.71 X.73 X.74 X.75 X.76 X.77 X.78 X.79 X.80 X.81 X.82 X.83 X.84	-1 -1 -1 -1 -1 -1 -1 -1	-1 $1$ $-1$ $1$ $-1$ $-1$ $1$ $-1$	-1 $1$ $1$ $-1$ $-1$	$     \begin{array}{r}       -1 \\       -1 \\       1 \\       -1 \\       -1 \\       1    \end{array} $		-i	i -1	-1 -1 -1		-1	-1	-1 -1 1 1 	-1 -1 -1 1				-1 1 1 -1 -1	1 1 1 -1 -1 -1	-1 1 1 -1 -1	-1

Character table of  $A_1(\mathrm{Fi}_{24}')$  (continued)

2 3 5 7 11 13	19 9 2 1 1 1	19 9 2 1 1 1	17 6 1 1 1	14 6 2 1	19 4 1	19 4 1	14 4 1	17 3	10 7 1 1	9 9	8 7	5 7	11 4 1 1	14 4 :	14 1 1
${2P}$	$\frac{1a}{1a}$	$\frac{2a}{1a}$	$\frac{2b}{1a}$	$\frac{2c}{1a}$	$\frac{2d}{1a}$	$\frac{2e}{1a}$	$\frac{2f}{1a}$	$\frac{2g}{1a}$	$\frac{3a}{3a}$	3 <i>b</i> 3 <i>b</i>	$\frac{3c}{3c}$	$\frac{3d}{3d}$	$\frac{4a}{2a}$	$\frac{4b}{2e}$	$\frac{4c}{2e}$
$^{3P}_{5P}$	$1a \\ 1a$	$\frac{2a}{2a}$	$\frac{2b}{2b}$	$\frac{2c}{2c}$	$\frac{2d}{2d}$	$\frac{2e}{2e}$	$\frac{2f}{2f}$	$\frac{2g}{2g}$	$\frac{1a}{3a}$	$\frac{1a}{3b}$	$\frac{1a}{3c}$	$\frac{1a}{3d}$	$\frac{4a}{4a}$	$\frac{4b}{4b}$	$\frac{4c}{4c}$
7P $11P$	1a	$ \begin{array}{c} 2a \\ 2a \\ 2a \end{array} $	$\frac{2b}{2b}$	$\frac{2c}{2c}$	$\frac{2d}{2d}$	$\frac{2e}{2e}$	$\frac{2f}{2f}$	2q	$\frac{3a}{3a}$	$\frac{3b}{3b}$	$\frac{3c}{3c}$	$\frac{3d}{3d}$	4a	$\frac{4b}{4b}$	4c
13P	$1a \\ 1a$	2a	2b	2c	2d	2e	2f	$\frac{2g}{2g}$	3a	$\frac{3b}{3b}$	$\frac{3c}{3c}$	$\frac{3a}{3d}$	$\frac{4a}{4a}$	4b	$\frac{4c}{4c}$
X.85 X.86	577368 577368	577368 577368	$-21384 \\ -21384$	$-5832 \\ 5832$	216 216	$\frac{216}{216}$	$-216 \\ -216$	$-72 \\ -72$	729 729	:	:	:	$-216 \\ -216$	$-216 \\ -216$	-56
$X.87 \\ X.88$	579150 579150	579150 579150	$-6930 \\ -6930$	$-5850 \\ 5850$	$\frac{1230}{1230}$	$\frac{1230}{1230}$	$-90 \\ 90$	$\frac{174}{174}$	$-405 \\ -405$	$\frac{324}{324}$	81 81	:	$-210 \\ 210$	$-102 \\ -102$	$-90 \\ 90$
$X.89 \\ X.90$	600600 600600	600600 600600	$21560 \\ 21560$	$-7000 \\ 7000$	920 920	920 920	$^{-120}_{120}$	$-136 \\ -136$	$\frac{525}{525}$	$-96 \\ -96$	$\frac{12}{12}$	$-42 \\ -42$	$-280 \\ 280$	$-248 \\ 248$	$^{40}_{-40}$
X.91 X.92	640640 675675	$-640640 \\ 675675$	10395	4725	$-1152 \\ -165$	$1152 \\ -165$	405	411	812	$-232 \\ 135$	$128 \\ -54$	$-16 \\ 54$	525	21	-75
X.93	675675	675675	10395	-4725	-165	-165	-405	411		135	-54	54	-525	-21	75
$X.94 \\ X.95$	$720720 \\ 720720$	$720720 \\ 720720$	$-33264 \\ -33264$	$-1680 \\ 1680$	$\frac{1104}{1104}$	$\frac{1104}{1104}$		$-112 \\ -112$	$\frac{1386}{1386}$	$-18 \\ -18$		$^{-18}_{-18}$	$-336 \\ -336$	$^{48}_{-48}$	$^{-16}_{16}$
$X.96 \\ X.97$	800800 800800	$-800800 \\ 800800$	12320		$\frac{1120}{2080}$	$-1120 \\ 2080$		32	$3220 \\ -560$	$-128 \\ -614$	$-74 \\ -20$	$\frac{34}{34}$		:	:
$X.98 \\ X.99$	800800 800800	800800	$-12320 \\ -12320$	$-5600 \\ 5600$	800 800	800 800	$^{-160}_{160}$	$-416 \\ -416$	$-245 \\ -245$	520 520	16 16	34 34	$^{280}_{-280}$	$^{64}_{-64}$	
X.100	800800 852930	800800	-36960	2430	$-480 \\ 1026$	$-480 \\ 1026$	-594	160	1960 729	196	-56	-20	594	-162	46
$X.101 \\ X.102$	852930	852930	$-29646 \\ -29646$	-2430	1026	1026	-594	18	729				-594	162	-46
$X.103 \\ X.104$	873600 938223	-873600 $938223$	-2673	9477	$-640 \\ 1647$	$\frac{640}{1647}$	$-2\dot{7}$	$20\dot{7}$	$\frac{1680}{-729}$	420	-84	-66	$-2\dot{7}$	$-2\dot{7}$	5
$X.105 \\ X.106$	938223 960960	938223 $-960960$	-2673	-9477	$\frac{1647}{3392}$	-3392	27	207	$-729 \\ -672$	-348	138	30	27	27	-5
$X.107 \\ X.108$	972972 972972	972972	$-24948 \\ -24948$	$9828 \\ -9828$	1836 1836	1836 1836		$-180 \\ -180$		$-243 \\ -243$			$-252 \\ 252$	$-252 \\ 252$	$^{36}_{-36}$
X.109	1029600	-1029600	24340		1440	-1440			-720	$\frac{252}{252}$	90	9		202	
X.111	$\frac{1029600}{1164800}$	-1029600 $1164800$			-2560	$-1440 \\ -2560$		512	-720 $560$	344	90 128	20	:		
X 113	$\begin{array}{c} 1201200 \\ 1201200 \end{array}$	$\begin{array}{c} 1201200 \\ 1201200 \end{array}$	$-30800 \\ 18480$	2800	-2000	-2000	240	$\frac{176}{560}$	$\frac{420}{420}$	$-30 \\ -30$	$-48 \\ 114$	$-30^{51}$	:	-80	-80 ·
$X.114 \\ X.115$	$1201200 \\ 1360800$	1201200 1360800	$-30800 \\ 30240$	-2800	$\frac{560}{1440}$	$\frac{560}{1440}$	-240	$\frac{176}{288}$	420	$-30 \\ 486$	-48	51		80	80
$X.116 \\ X.117$	1201200 1360800 1360800 1372800	1360800 1372800	$\frac{30240}{49280}$	-6400	$\frac{1440}{640}$	$\frac{1440}{640}$		$\frac{288}{128}$	1560	$^{486}_{-312}$	$-24^{\circ}$	12		-256	
X.118	$1372800 \\ 1372800$	-1372800 $1372800$	49280	6400	1920 640	$-1920 \\ 640$		128	1920 1560	-312	$156 \\ -24$	$-42 \\ 12$		256	
X.120	1441792	1441792	49280	-8192	040	040	:	120	-512	640	64	-8	:	200	:
X.122	$\frac{1441792}{1441792}$	$\substack{1441792 \\ -1441792}$		8192	:	:	:	:	$-512 \\ -512$	$\frac{640}{640}$	$\frac{64}{64}$	$^{-8}_{-8}$	:	:	:
$X.123 \\ X.124$	$\frac{1441792}{1791153}$	$-1441792 \\ 1791153$	-5103	5103	-251i	-2511	$-81^{\circ}$	81	-512	640	64	-8	567	$-8\dot{1}$	-8i
X.125	1791153 1830400	$1791153 \\ -1830400$	-5103	-5103	-2511	$-2511 \\ -2560$	81	81	880	1096	16	16	-567	81	81
X.127	1876446 1876446	1876446 1876446	37422 37422		$-1890 \\ -1890$	-1890	$^{270}_{-270}$		729 729				$378 \\ -378$	$-162 \\ 162$	$-110 \\ -110$
X.129	1965600 2027025	1965600	30240		-480	-480		-480		-270	108	54		93	
X.131	2027025	2027025 2027025	$-24255 \\ -24255$	-7875 $7875$	$-1455 \\ -1455$	$-1455 \\ -1455$	$^{45}_{-45}$	33 33		$-324 \\ -324$	81 81		$-105 \\ -105$	-93	$^{45}_{-45}$
	$2050048 \\ 2050048$	$2050048 \\ 2050048$		$-10752 \\ -10752$	$\frac{2048}{2048}$	$\frac{2048}{2048}$	$-512 \\ -512$	:	-1232		$-80 \\ -80$	$^{-8}_{-8}$	:	:	:
X 135	$2316600 \\ 2316600$	2316600 2316600		$-1800 \\ 1800$	$-840 \\ -840$	$-840 \\ -840$	$-360 \\ 360$	$\frac{24}{24}$	$\frac{405}{405}$	$-162 \\ -162$			$^{120}_{-120}$	$^{24}_{-24}$	$^{120}_{-120}$
X.136 Y.137	$2402400 \\ 2402400$	$-2402400 \\ 2402400$	12320	5600	$3360 \\ -160$	$-3360 \\ -160$	-480	160	$-420 \\ -735$	-384	$-78 \\ 48$	$\frac{48}{-6}$	-280	64	
X.138	2402400 $2402400$ $2555904$	2402400 2402400 2555904	12320 32768	-5600 $4096$	-160	-160	-480	160	-735 $-384$	-384	48	$-6 \\ -24$	$\frac{-280}{280}$ $512$	-64	
X.140	2555904	2555904	-32768	-4096					-384	192 192	-96	-24	512	:	:
$X.141 \\ X.142$	2555904 2555904 2594592	2555904 $2555904$	$-32768 \\ 32768$	$^{4096}_{-4096}$				:	$-384 \\ -384$	$\frac{192}{192}$	-96		$-512 \\ -512$	:	:
$X.143 \\ X.144$	$2594592 \\ 2729376$	$\substack{-2594592 \\ 2729376}$	7776	7776	-864	$-2400 \\ -864$	-864	-288	$2268 \\ -729$	-648	-162		216	:	$-6\dot{4}$
X.145	2729376 3326400	2729376 -3326400	7776	-7776	$-864 \\ -4800$	-864 $4800$		-288	-729	216	108	54	-216		64
X.147	4392960	-4392960	:	:	-2048	2048				-480	-48	-48		:	
X.149	4804800	$-4717440 \\ -4804800$	:	:	-3520	$-2688 \\ 3520$			1680	-648 $-120$	132	42	:	:	:
X.150	5111808	-5111808							-768	384	-192	-48			

2 3 5	13 3	$\begin{array}{c} 11 \\ 2 \\ 1 \end{array}$	13 1	11 2	$^{11}_{2}$	11 2	12 1	11 1	10 1	5 1 2	$^{10}_{\ 7}_{\ 1}$	9	8 7	8 5 1	8 5 1	8 6	8	8 4	5 7	9 4
$\begin{array}{c} 7 \\ 11 \\ 13 \end{array}$	:	:	:	:	:	:	:	:	:	:	1	:	:	:	:	:	1	1	:	: :
2P 3P 5P	$\begin{array}{c} 4d \\ 2e \\ 4d \\ 4d \end{array}$	$\begin{array}{c} 4e \\ 2d \\ 4e \\ 4e \end{array}$	$\begin{array}{c} 4f \\ 2e \\ 4f \\ 4f \end{array}$	$\frac{4g}{2d}$	$\begin{array}{c} 4h \\ 2g \\ 4h \\ 4h \end{array}$	$\begin{array}{c} 4i \\ 2g \\ 4i \\ 4i \end{array}$	$\frac{4j}{2e}$	$\begin{array}{c} 4k \\ 2d \\ 4k \\ 4k \end{array}$	$\begin{array}{c} 4l \\ 2g \\ 4l \\ 4l \end{array}$	5a 5a 1a	6 <sub>1</sub> 3a 2a	6 <sub>2</sub> 3b 2a	6 <sub>3</sub> 3 <sub>c</sub> 2 <sub>a</sub>	6 <sub>4</sub> 3a 2b	6 <sub>5</sub> 3a 2c	6 <sub>6</sub> 3b 2b	$\frac{67}{3a}$	68 3a 2c	$\begin{array}{r} 6_9 \\ 3d \\ 2a \\ 6_9 \end{array}$	3b 2e
$\begin{array}{c} 3P \\ 7P \\ 11P \\ 13P \end{array}$	$\begin{array}{c} 4d \\ 4d \\ 4d \\ 4d \end{array}$	4e $4e$ $4e$	4f $4f$ $4f$	$\begin{array}{c} 4g \\ 4g \\ 4g \\ 4q \end{array}$	4h $4h$ $4h$	4i $4i$ $4i$ $4i$	$\begin{array}{c} 4j \\ 4j \\ 4j \\ 4i \end{array}$	4k $4k$ $4k$	4l 4l 4l	5a 5a 5a	$   \begin{array}{c}     6_1 \\     6_1 \\     6_1 \\     6_1   \end{array} $	$6_2 \\ 6_2 \\ 6_2 \\ 6_2$	63 63 63	$\begin{array}{c} 6_4 \\ 6_4 \\ 6_4 \\ 6_4 \end{array}$	$\begin{array}{c} 6_5 \\ 6_5 \\ 6_5 \\ 6_5 \end{array}$	6 <sub>6</sub> 6 <sub>6</sub> 6 <sub>6</sub>	$     \begin{array}{r}       6_8 \\       6_7 \\       6_8 \\       6_7     \end{array} $	$     \begin{array}{r}       6_{7} \\       6_{8} \\       6_{7} \\       6_{8}     \end{array} $	69	$     \begin{array}{r}       6_{10} \\       6_{10} \\       6_{10} \\       6_{10}     \end{array} $
X.85 X.86 X.87	$72 \cdot$	$-24 \\ -24$	24 24 30	$-24 \\ 24 \\ 14$	6	6 -	$-24 \\ 24$	$     \begin{array}{r}       -8 \\       -8 \\       -10     \end{array} $	6	-7 -7	729 $729$ $-405$	324	81	81 81	$-81 \\     81 \\     -45$	36	$-\frac{27}{27}$	$\begin{array}{r} 27 \\ -27 \\ 27 \\ 27 \end{array}$		-12
$X.88 \\ X.89 \\ X.90$	-18 - 72 - 72	$^{-10}_{40}_{40}$	$\frac{30}{24}$	$^{-14}_{24}$	6	6	$^{10}_{-8}$	$-10 \\ -8 \\ -8$	-6 :	:	$^{-405}_{525}$ $^{525}_{525}$	$^{324}_{-96}$ $^{-96}$	$\frac{12}{12}$	$\begin{array}{c} -45\\ 5\\ 5\end{array}$	$^{45}_{65}$ $^{-65}$	$^{36}_{-40}$	$^{-27}_{-7}$		$-42 \\ -42$	-16
$X.91 \\ X.92 \\ X.93$	75 -	-45 $-45$	11 11	$-\frac{1}{3}$	$-5 \\ -5$	$-5 \\ -5$	11	3	$-\frac{1}{3}$	-10	-812	232 135 135	$-128 \\ -54 \\ -54$			27 27			16 54 54	72 15 15
X.94 X.95 X.96	-80 -80 -96		16 16	-16 16	8 32	$-8 \\ 32$	16 -16	16 16	:	$-5 \\ -5 \\ .$	1386 $1386$ $-3220$ $-560$	$-18 \\ -18 \\ 128 \\ -614$	$   \begin{array}{r}     36 \\     36 \\     74 \\     -20   \end{array} $	$-54 \\ -54 \\ -80$	66 -66	-54 $-54$ $-46$	$-42 \\ 42 \\ \cdot$		$     \begin{array}{r}       -18 \\       -18 \\       -34 \\       \hline       34     \end{array} $	6 6 32 10
$X.97 \\ X.98 \\ X.99 \\ X.100$	-96 32	40 40	32 -32	$^{24}_{-24}$			:	8 8	:	:	-245 $-245$ $1960$	520 520 196	16 16 -56	$-35 \\ -35 \\ 120$	$-25 \\ -25$	$-8 \\ -8 \\ -132$	$-\overset{\cdot}{7}$	$-\overset{\cdot}{7}$	$     \begin{array}{r}       34 \\       34 \\       -20     \end{array} $	8 8 -12
$X.101 \\ X.102 \\ X.103$	$-54 \\ -54$	6 6	18 18	6	-18 - 18 - 32	$^{-18}_{32}$	-6 -6	$-\frac{1}{2}$	-6 6	5 5	729 729 -1680	-420	84	-81 -81	-81 81		$-27 \\ 27 \\ .$	$^{-27}_{27}$	66	28
$X.104 \\ X.105 \\ X.106$	63 63	15 15		$^{21}_{-21}$	$-9 \\ -9 \\ 16$	$^{-9}_{-9}$	$^{21}_{-21}$	$-1 \\ -1$	-3 3	$-2 \\ -2 \\ 10$	$-729 \\ -729 \\ 672$		-138	-81 -81	-81 81	:	$-{27 \atop -27}$	$-{}^{27}_{-27}$	-30	28
$X.107 \\ X.108 \\ X.109$	60 - 60 -		-20 - 20 - 20 - 10 - 10 - 10 - 10 - 10 -		$^{-12}_{-8}$	-12 - 12 - 8	$^{-12}_{12}$	12 12	$-12 \\ -12 \\ .$	$-3 \\ -3 \\ .$	720	-243 $-243$ $-252$	-90			81 81	$\dot{ar{J}}$	<u>.</u> <u>J</u>	-9	$-27 \\ -27 \\ 36$
	-16		-16	:	-8 16	8 16 -	-16		-16	:	720 560 420	$-252 \\ 344 \\ -30$		$-20^{\circ}$	100	8 34	$\frac{J}{28}$	$J$ $2\dot{8}$	$-9 \\ 20 \\ 51$	$^{36}_{-40}$
$X.114 \\ X.115$	-16 $-16$ $-96$ $-96$		-16 $-16$ $-32$ $-32$		-16 16	-16 16	16	-16 :	16	:	420 420	$-30 \\ -30 \\ 486 \\ 486$	-48 -48	$^{-60}_{-20}$	-100	$     \begin{array}{r}       66 \\       34 \\       -54 \\       -54    \end{array} $	-28	-28	-30 51	$ \begin{array}{r} 34 \\ 2 \\ -18 \\ -18 \end{array} $
X.110 X.117 X.118 X.119	-90		-32		32	-32					$-1560 \\ -1920 \\ 1560$	-312	-24 $-156$ $-24$	-40 $-40$	80 -80	32 32	-8	-8	12 42 12	$-8 \\ -12 \\ -8$
$X.120 \\ X.121 \\ X.122$										$-8 \\ -8 \\ -8$	$-512 \\ -512 \\ 512$	640 640 -640	64 64 -64		$-128 \\ 128$		$_{-64}^{64}$	$-64$ $\bar{K}$	-8 -8 8	
$X.123 \\ X.124 \\ X.125$	81 81	9	33 33	-9 9	9	9 -	-33 33	9	15 -15	$-\frac{8}{3}$	512	-640 :	-64				$\bar{K}$	K	8	
$X.126 \\ X.127 \\ X.128$			-18 -18	-30 30	18 18	18 18	$_{-6}^{\dot{6}}$	10 10	$-\frac{\dot{6}}{6}$	$_{-4}^{\dot{4}}$	$-880 \\ 729 \\ 729$	-1096 :		$-81 \\ -81$	$^{81}_{-81}$		$-{}^{27}_{-27}$	$-27 \\ -27$		-40 :
$X.129 \\ X.130 \\ X.131$	-96 9 9		$^{32}_{-15}$	$^{17}_{-17}$	-32 - 3 - 3 - 3	$-32 \\ -3 \\ -3$	-11 11	$\begin{array}{c} -3 \\ -3 \end{array}$	$-9^{\circ}$			-270 $-324$ $-324$	81	$^{-90}_{-90}$	90 -90	$-54 \\ -36 \\ -36$	:	:	54	$\frac{12}{12}$
X.132 $X.133$ $X.134$ $X.135$	-24 $-24$	40 40	$-\frac{.}{8}$		:	:	-24 24	-8 -8	:		$-1232 \\ -1232 \\ 405 \\ 405$	-224 $-224$ $-162$ $-162$	$^{-80}_{-80}$	45 45	$     \begin{array}{r}     -48 \\     48 \\     -45 \\     45     \end{array} $	18 18	27 -27	$\begin{array}{c} \cdot \\ \cdot \\ 27 \\ -27 \end{array}$	-8 -8	32 32 6 6
X.136 X.137 X.138		-40 -40	-8	-24 $-24$	24	-24		-8 -8			420 $-735$ $-735$	-264 $-384$ $-384$	78 48 48	35 35	-25 25	8 8	$-\frac{7}{7}$	$-\frac{7}{7}$		$ \begin{array}{r}     24 \\     -16 \\     -16 \end{array} $
$X.139 \\ X.140 \\ X.141$	:	:		:						4 4 4	$-384 \\ -384 \\ -384$	192 192 192	$-96 \\ -96 \\ -96$	$^{-64}_{64}$	$^{64}_{-64}$	$^{-64}_{64}$	$^{64}_{-64}$	$^{64}_{-64}$	$-24 \\ -24 \\ -24$	
$X.142 \\ X.143 \\ X.144$	:	-24	:	24	-24	$\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}}{\overset{\cdot}{\overset{\cdot}}{\overset{\cdot}{\overset{\cdot}}{\overset{\cdot}}{\overset{\cdot}{\overset{\cdot}}{\overset{\cdot}}{\overset{\cdot}}{\overset{\cdot}}{\overset{\cdot}}}}}}}$	:	8	:	$-8 \\ 1 \\ 1$	$     \begin{array}{r}       -384 \\       -2268 \\       -729 \\     \end{array} $	192 648	$^{-96}_{162}$	-64 81	-64 81	-64 :	-64 $27$	27	-24 :	-24
X.145 X.146 X.147		-24 :		-24	-16	16	:	8	:	10	-729 912	480	$-108 \\ 48$	81	-81 :	:	-27 :	-27 :		32
$X.148 \\ X.149 \\ X.150$	:	:	:	:	16	-16	:	:		-10 8	$^{2268}_{-1680}$ $^{768}$	648 $120$ $-384$	$-132\\192$		:	:	:	:	$-42 \\ 48$	-24 -8

2 3	9 4	$^{10}_{3}$	$^{10}_{3}$	5 6	5 5	5 5	8	8	$\frac{8}{3}$	8	$\frac{8}{3}$	$\frac{7}{3}$	7 3	$_{4}^{5}$	$\frac{5}{4}$	$\frac{5}{4}$	5 4	5 3	5 3	5 3	5 3	3 1	8
7 11 13		:	:	:	:	:	:	:	:	:	:	:	:									i	:
2P 3P 5P 7P 11P 13P	$6_{11}$ $3b$ $2d$ $6_{11}$ $6_{11}$ $6_{11}$ $6_{11}$		$6_{13}$ $3a$ $2e$ $6_{13}$ $6_{13}$ $6_{13}$ $6_{13}$	$6_{14}$	$6_{15}$ $3c$ $2c$ $6_{15}$ $6_{15}$ $6_{15}$ $6_{15}$	$ \begin{array}{c} 6_{16} \\ 3c \\ 2b \\ 6_{16} \\ 6_{16} \\ 6_{16} \\ 6_{16} \end{array} $	$6_{17}$	$6_{18}$ $3c$ $2g$ $6_{18}$ $6_{18}$ $6_{18}$ $6_{18}$	$6_{19}$ $3b$ $2g$ $6_{19}$ $6_{19}$ $6_{19}$ $6_{19}$	$6_{20}$ $3a$ $2f$ $6_{20}$ $6_{20}$ $6_{20}$ $6_{20}$	$6_{21}$ $3a$ $2g$ $6_{21}$ $6_{21}$ $6_{21}$ $6_{21}$	$\begin{array}{c} 6_{22} \\ 3c \\ 2e \\ 6_{22} \\ 6_{22} \\ 6_{22} \\ 6_{22} \end{array}$	$6_{23}$ $3c$ $2d$ $6_{23}$ $6_{23}$ $6_{23}$ $6_{23}$	624 $3d$ $2f$ $625$ $624$ $625$ $624$	$624 \\ 625 \\ 624$	$6_{26}$ $3d$ $2c$ $6_{27}$ $6_{26}$ $6_{27}$ $6_{26}$	$626 \\ 627 \\ 626$	$\begin{array}{c} 2g \\ 628 \\ 628 \\ 628 \end{array}$	629 $629$ $629$	$6_{30}$ $3c$ $2g$ $6_{30}$ $6_{30}$ $6_{30}$	$631 \\ 631 \\ 631$	7a 7a 7a 7a 1a 7a 7a	$_{8a}^{8a}$
X.85 X.86 X.87 X.88 X.89 X.90 X.91	-12 $-12$ $-16$	$9 \\ -21 \\ -21 \\ -19 \\ -19$	$   \begin{array}{r}     9 \\     -21 \\     -21 \\     -19   \end{array} $	-18 18 20 -20	9 -9 -16 16	9 9	9 9 -4 -4	9 9 -4 -4	-12 -12 -12 8 8	-9 9 3 -3 -15 15	9 9 3 3 5 5	-3 -3 -4 -4	-3 -3 -4 -4	-6 -6	-6 -6	-2 -2	2 -2	2 2	· · · · 2 2	-3 -3 2 2	-3 3	$\begin{array}{c} 1 \\ -2 \\ -2 \\ \end{array}$	-6 6
X.92 X.93 X.94 X.95 X.96 X.97	$     \begin{array}{r}       15 \\       15 \\       6 \\       6 \\       -32 \\       10     \end{array} $	-6 -6 28 16	-6 -6 -28 16	27 -27 -6 6	12 -12	-28	$ \begin{array}{r} -6 \\ -6 \\ -4 \\ -4 \\ 6 \\ -4 \end{array} $	$ \begin{array}{r} -6 \\ -6 \\ -4 \\ -4 \\ -6 \\ -4 \end{array} $	3 3 2 2 2	-6 6	2 2 -16	-6 -6	-6 -6 -2 4	6 -6	6 -6	-6 6	-6 6	6 6 2 2 -6 2	6 6 2 2 6 2	-4 -4 -4			-1 -1
X.98 X.99 X.100 X.101 X.102 X.103 X.104		$ \begin{array}{c} 11 \\ 11 \\ -24 \\ 9 \\ 9 \\ -16 \\ -9 \end{array} $	9 9 16 -9	16 -16	-2 2	-8 -8 12	-8 -8 -8	-8 -8 -8	-8 -8 -20	$     \begin{array}{r}       -7 \\       7 \\       -9 \\       9 \\       -9 \\    \end{array} $	13 13 -8 -9 -9	-4 -4 : : 4	-4 -4	-2 -2	$-\frac{2}{2}$	-2 2	-2 2	-2 -2 4	-2 -2 4 -6	4 4 4	-2 -2		-6 6 -3
X.105 X.106 X.107 X.108 X.109 X.110	-27 $-27$ $-36$ $-36$	-48 -48	-9 -32 48 48	27 -27 -27			-6 -6 -6	6 6 6	9 9	9	-9	-2 6 6	-6 -6					6 -3 -3	-6 3			$-1$ $\vdots$ $-2$ $-2$	$ \begin{array}{c} 3 \\ -4 \\ 4 \\ \vdots \\ \cdot \end{array} $
X.111 X.112 X.113 X.114 X.115 X.116 X.117	$\frac{2}{34}$	$ \begin{array}{r} -16 \\ -28 \\ 4 \\ -28 \\ \vdots \\ -8 \end{array} $	-16 -28 4 -28	-8 8	10 -10	-28 -2 -6 -2 -	-16 8 2 8	-16 8 2 8	8 2 2 2 18 18 -16	-12 12	-16 -4 20 -4	8 -4 10 -4	8 -4 10 -4	-3 3	-3 3	i -i 8	i -i	$ \begin{array}{c} -4 \\ -1 \\ 2 \\ -1 \\ \vdots \\ -4 \end{array} $	$     \begin{array}{r}     -4 \\     -1 \\     2 \\     -1 \\     \vdots \\     -4   \end{array} $	-4 2 2 2	6 -6	· · · ·	
X.118 X.119 X.120 X.121 X.122 X.123	12 -8	-8 :	-8 : :	28 16 -16	-8 16 -16	-4 : :	12 8	-12 8	-16 : :		8	12 -8	-12 -8			$\begin{array}{c} -8 \\ -8 \\ 8 \\ A \\ \bar{A} \end{array}$	-8 -8 -8 Ā A	6 -4	-6 -4	-4		2 2 2 2 2 2	
X.124 X.125 X.126 X.127 X.128 X.129 X.130	40 -30 12	16 9 9	-16 9 9	-18			12 9	12 9	-6 12		-9 -9 -9	8 12 -3	-8 -12 -3					6	6		-3	$ \begin{array}{c}     -2 \\     -2 \\     -2 \\     -2 \end{array} $	$ \begin{array}{c}     -3 \\     -6 \\     6 \\     -3 \end{array} $
X.131 X.132 X.133 X.134 X.135 X.136 X.137		$ \begin{array}{c} -16 \\ -16 \\ 21 \\ 21 \\ -12 \\ 17 \end{array} $		$ \begin{array}{r} 18 \\ -48 \\ 48 \\ -18 \\ 18 \\ -16 \end{array} $	$     \begin{array}{r}       -9 \\       -12 \\       12 \\       -18 \\       18 \\                         $	-9 18 18	9	9 -18 -8	12 -6 -6 -8	$ \begin{array}{r}     6 \\     -16 \\     16 \\     3 \\     -3 \\     \vdots \\     15 \end{array} $	6 -3 -3 -5	-3 8 8 -18 -4	-3 8 8	-8 -8	-8 -8					3 -6 -6	$ \begin{array}{r}     3 \\     -4 \\     4 \\     6 \\     -6 \end{array} $	-i -1 -1	3
X.138 X.139 X.140 X.141 X.142 X.143	-16 -16	17	17	16 -8 8 -8 8	$     \begin{array}{r}       -2 \\       -8 \\       8 \\       -8 \\       8     \end{array} $	8 8 -8 -8 8	-8 -8	-8 -8	-8 -8	-15	-5	-4 -4	-4 -4	-6	-6	$     \begin{array}{r}       -2 \\       -8 \\       8 \\       -8 \\       8     \end{array} $	$     \begin{array}{r}       2 \\       -2 \\       -8 \\       8 \\       -8 \\       8     \end{array} $	-2	-2	4	-6	1 1 1 1	•
X.144 $X.145$ $X.146$ $X.147$ $X.148$ $X.149$ $X.150$			-9 -9 -16 -12 16				12 -12	-12 -12 12		9 -9	9 9	-12 8 4	12 -8 -4					6 -6	-6 -6			$     \begin{array}{c}       -1 \\       -1 \\       \hline       -2 \\       \vdots \\       2     \end{array} $	

2 3 5	8	8	8	8	8	6 1	6	4	3 4	2 3	5 1 2	$\frac{4}{1}$	5 i	5 1	3 1 1	4 1	2	6 3 1	8 3	5 4	5 4	8 2	8 2	6 3	7 2
7 11 13		:		:	:	:	:	:	:	:	:	:	:	:	:	:	i	:	:	:	:	:	:	:	:
2P	$\frac{8b}{4d}$	$\frac{8c}{4d}$	$\frac{8d}{4d}$	$\frac{8e}{4d}$	8 f 4 f	$\frac{8g}{4h}$	$\frac{8h}{4i}$	$\frac{9a}{9a}$	9b 9b	9c 9c	$\frac{10a}{5a}$	$\frac{10b}{5a}$	$\frac{10c}{5a}$	$\frac{10d}{5a}$	$\frac{10e}{5a}$	$\frac{10f}{5a}$	$\frac{11a}{11a}$	$\frac{12_1}{6_1}$	$6_{13}$	$\frac{12_3}{6_{10}}$	$\frac{12_4}{6_2}$	$\frac{12_5}{6_{10}}$	$\frac{12_{6}}{6_{13}}$	$\frac{127}{610}$	$\frac{12_8}{6_{13}}$
$\begin{array}{c} 3P \\ 5P \end{array}$	8b 8b	8c		8e		8g	8h $8h$	3b	3b 9b	$\frac{3d}{9c}$	10a	10b	10c	10d	10e	10f	$\frac{11a}{11a}$	$\frac{4a}{124}$	$\frac{4b}{12a}$	$6_{10} \\ 4b \\ 12a$	$\frac{4a}{124}$	$6_{13} \\ 4d \\ 12_{5} \\ 12_{5} \\ 12_{5}$	4d	6 <sub>10</sub> 4d	$\frac{4d}{12}$
7P 11P	8b 8b	8c	8d	8e	8 f	8a	8h	9a	9b	9c	10a	$10b \\ 10b$	10c	10d $10d$	10e	10f $10f$	11a	$12_{1}^{1}$ $12_{1}^{1}$	$\frac{12_{2}}{12_{2}}$	$\frac{123}{123}$	$124 \\ 124 \\ 124$	125	12 <sub>6</sub>	$\frac{127}{127}$	12 <sub>8</sub>
$\frac{13P}{X.85}$	$\frac{8b}{-4}$	$\frac{8c}{4}$	$\frac{8d}{-4}$	8e	$\overset{\circ}{8}f$	$\overset{\circ}{8g}$	8h	9a	9b	9c	$\frac{10a}{-7}$	$\frac{10b}{-7}$	$\frac{10c}{10c}$	$\frac{10d}{10d}$	10e 1	10f	11a	$\frac{12_{1}^{1}}{-9}$	$\frac{12_{2}^{2}}{-9}$	$12_{3}^{3}$	$12_{4}^{4}$	$12_{5}^{1}$ $12_{5}^{1}$	126	$12_{7}^{7}$	128 =3
X.86 X.87	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	2	$-\frac{1}{2}$	2		•		$-\dot{7}$	7	1	1	1	-1	:	9 15	93	_6	6	$-\frac{3}{3}$	$-\frac{3}{3}$	:	$-\frac{3}{3}$
X.88 X.89	2 4	$-\frac{5}{4}$	2 4	-2	2	2	2	3	3						:			-1555	$-\frac{3}{1}$	$\frac{6}{4}$	-6	-3	$-3^{\circ}$	:	3 -3 -3
$X.90 \\ X.91$	4	-4	4			i	i	3	$-\overset{\circ}{4}$	· 2	10		_i	2				$-\tilde{5}$	-1	-4	$-\frac{8}{8}$	$-3 \\ -12$	$-\frac{3}{12}$		-3
$X.92 \\ X.93$	3	$-\frac{3}{3}$	3	$-\frac{3}{3}$		$-\frac{3}{3}$	$-1 \\ -1$													-3	-3			3	:
$X.94 \\ X.95$					:			:		:	$-5 \\ -5$	$-\frac{5}{5}$	$-1 \\ -1$	$-1 \\ -1$	$\frac{1}{1}$	$-\frac{1}{1}$		$^{6}_{-6}$	$^{-6}_{6}$	-6 6	$-\frac{6}{6}$	$-2 \\ -2$	$-2 \\ -2$	10 10	$^{-2}_{-2}$
$X.96 \\ X.97$		:	:	:	:			$-2 \\ -2$	$-2 \\ -2$	$-2 \\ -2$	:	:	:	:	:	:	:	:		:	:	12	-12	$-\dot{6}$	:
$X.98 \\ X.99$	4	$^{-4}_{4}$		$-4 \\ -4$	:		:	$-5 \\ -5$	1	1 1		:	:	:	:		:	$-\frac{5}{5}$	$-7 \\ -7$	$^{-8}_{8}$	$^{-8}_{8}$	3	3	:	$-3 \\ -3$
$X.100 \\ X.101$	$-\dot{2}$	$\dot{2}$	$\dot{2}$	$-\dot{2}$	$-\dot{2}$	:	:	-8 ·	4	-2	5	5	i	i	-i	i	i	9	9			$-\frac{8}{3}$	$-\frac{8}{3}$	-4	8 3 3
$X.102 \\ X.103$	-2 :	-2	2	2	-2			$-\dot{6}$	:	:	5	-5	1	1	-1	-1	$\frac{1}{2}$	-9	-9	:	:	-3	-3	:	
$X.104 \\ X.105$	$-5 \\ -5$	$-1^{1}$	$-5 \\ -5$	$-1 \\ -1 \\$	$-1 \\ -1$	$-\frac{3}{3}$	$-1 \\ -1$			:	$-2 \\ -2$	$-\frac{2}{2}$	2 2 2	2 2	$\frac{2}{2}$	$-\frac{2}{2}$	:	$-9 \\ 9$	$-9 \\ 9$	:	:	3	3	:	3
X.106 X.107	:	:		:	4	:	:	-6	-6	:	$^{-10}_{-3}$	3	1	$-\frac{2}{1}$	-3	-1	:	:	:	$-\dot{9}$	$-\dot{9}$	:	:	$-\frac{1}{3}$	:
X.108 X.109	:	:		:	4	:	:	:	:	:	-3	-3	1	1	-3	1	:	:		9	9	:	:	-3 ·	:
$X.110 \\ X.111 \\ X.112$	÷	÷		:	:	:	:	2 3	$\dot{2}$	$\dot{2}$	:				:		-i	:		-8		_i		2	
X.112 X.113 X.114					:			$-\frac{3}{3}$		:	:	:	:	:	:	:	:	:	4	-0	:	-4 $-4$	$-4 \\ -4 \\ -4$	2 2	$-4 \\ -4 \\ -4$
X.114 X.115 X.116	:	:	:	:	:	•	:		:		:	:	:	:	:		i 1	:	-4		:	-4	-4	$-\frac{2}{6}$	-4
X.117 X.118								$-\dot{3}$	6			:		:	:	:	·	:	-16	$-\dot{4}$	:	·	÷		:
X.119 X.120								$-\frac{1}{4}$	_?	-2	-8	. 8							16	4		·			:
$X.121 \\ X.122$	i	i				i	•	4	$-\frac{5}{2}$	-2	$-\frac{8}{8}$	-8			:									:	:
$X.123 \\ X.124$	-3	3	-3	3	-3	3	i	4	$-\frac{1}{2}$	$-\frac{1}{2}$	8	3	-1	-1	-3	-1	i								:
$X.125 \\ X.126$	-3	-3	-3	-3	-3	-3	1	$-\dot{2}$	$-\dot{2}$	$\dot{4}$	3	-3	-1	-1	-3	1	1	:		:	:	:		:	:
$X.127 \\ X.128$	$-2 \\ -2$	$-\frac{2}{2}$	$\frac{2}{2}$	$-\frac{2}{2}$	2 2		:	:	:	:	$-4 \\ -4$	$^{4}_{-4}$	:	:	$\frac{2}{2}$		:	$-9 \\ 9$	$-9 \\ 9$	:	:	$-3 \\ -3$	$-3 \\ -3$	:	3 3
$X.129 \\ X.130$	i		$-\dot{3}$	$-\dot{7}$	i	-i	- i	:	:	:	:	:			:	:	-1	:	:	$-\dot{6}$	6	:	:	-6	$\dot{6}$
$X.131 \\ X.132$	1	3	-3	7	1	1	-1	$-\dot{2}$	4	i	$-\dot{2}$	2	$-\dot{2}$	$-\dot{2}$	:	2	:	:		6	-6	:	:	:	6
X.133 X.134	4	$-\dot{4}$	4	$-\frac{1}{4}$	:	:	:	-2	4	1	-2	-2	-2	-2	:	-2		15	3	6	$-6^{\circ}$	$-\frac{1}{3}$	$-\overset{\cdot}{3}$	$-\dot{6}$	$-\dot{3}$
X.135 X.136	4	4	4	4	:	:	:		6	:	:	:	:	:	:	:		-15	-3	-6	6	$-3 \\ -12$	$-3 \\ 12$	-6	-3 :
X.137 X.138	$\frac{4}{4}$	$-4 \\ 4$	$-4 \\ -4$	$-4^{-4}$	- :	:	:	3	3	:	:	:	:	:		:	:	-5	$-11 \\ 11$	$^{-8}_{8}$	$-\frac{8}{8}$	5 5	5 5	8 8	$-1 \\ -1$
X.139 X.140	÷	÷		:	:		:		$-3 \\ -3 \\ -3$		4	-4 4			$-2 \\ 2 \\ 2 \\ -2$		$-1 \\ -1 \\ -1$	$-16 \\ -16$			8	:	:	:	:
$X.141 \\ X.142 \\ X.143$	:	:			:	:	:	:	$-3 \\ -3$	:	4 4 8	$-4 \\ 4$		:	$-\frac{2}{2}$	:	$-1 \\ -1$	16 16			$^{-8}_{-8}$	-12	12	:	:
X.145 X.144 X.145	$-\frac{i}{4}$	4 _4	4	$-\frac{i}{4}$			:	:	:	:	1	$-1^{i}$	i 1	1 1	1 1	1 -1	i 1	-9 9	-9 9	:	:	3 3	3 3	:	$-\frac{3}{2}$
X.146 X.147	-4	-4					:	6	6	:	-10	-1	· 2	- <u>i</u>		-1				:	:			:	-3
X.148 X.149	:	:					:	_6		:	10	:	$-\frac{2}{2}$	$\frac{-2}{2}$	:		$\dot{2}$	:				12	$-12^{\dot{2}}$	:	:
X.149 X.150									$-\dot{6}$		$-\dot{8}$						$-\dot{2}$					:	:		

2 3 5 7	7 2	5 3	8	6 2	6 2	6 2	$\frac{6}{2}$	$\frac{4}{3}$	5 2	5 2	5 2	6 1	6		$\frac{4}{2}$	$\frac{4}{2}$	$\frac{4}{2}$	5 1	$_{1}^{4}$	$_{1}^{4}$
7 11 13								:		:	:							:	:	:
2 <i>P</i> 3 <i>P</i>	$6_{13} \\ 4b$	$6_{22} \\ 4b$	$6_{13} \\ 4c$		$6_{17} \\ 4h$	$6_{12} \\ 4e$	$6_{17} \\ 4i$	$\frac{12_{16}}{6_3}$	$6_{19} 4i$	$6_{19} \\ 4h$	$\frac{6_{11}}{4g}$	$6_{10} \\ 4f$	$6_{12} \\ 4k$	$6_{23} \\ 4g$	$6_{23} \\ 4e$	$6_{28} \\ 4i$	$6_{28} \\ 4h$	$6_{22} \\ 4j$	$\frac{6_{27}}{6_{28}}$	$\frac{12_{28}}{6_{28}}$
5P 7P 11P 13P	129 $129$ $129$ $129$	$12_{10}$ $12_{10}$ $12_{10}$ $12_{10}$	$12_{11}$ $12_{11}$ $12_{11}$ $12_{11}$	$12_{12}$ $12_{12}$ $12_{12}$ $12_{12}$	$12_{13}$ $12_{13}$ $12_{13}$ $12_{13}$	$12_{14}$ $12_{14}$ $12_{14}$ $12_{14}$	12 <sub>15</sub> 12 <sub>15</sub> 12 <sub>15</sub> 12 <sub>15</sub>	12 <sub>16</sub> 12 <sub>16</sub> 12 <sub>16</sub>	$12_{17}$ $12_{17}$ $12_{17}$ $12_{17}$	12 <sub>18</sub> 12 <sub>18</sub> 12 <sub>18</sub> 12 <sub>18</sub>	$12_{19}$ $12_{19}$ $12_{19}$ $12_{19}$	$12_{20}$ $12_{20}$ $12_{20}$	$12_{21}$ $12_{21}$ $12_{21}$ $12_{21}$	$12_{22} \\ 12_{22} \\ 12_{22} \\ 12_{22} \\ 12_{22}$	$12_{23}$ $12_{23}$ $12_{23}$ $12_{23}$	$12_{24}$ $12_{24}$ $12_{24}$ $12_{24}$	12 <sub>25</sub> 12 <sub>25</sub> 12 <sub>25</sub> 12 <sub>25</sub>	1226 $1226$ $1226$ $1226$	$12_{28}$ $12_{27}$ $12_{28}$ $12_{27}$	$12_{27} \\ 12_{28} \\ 12_{27} \\ 12_{29}$
X.85 X.86 X.87	3	:	- 1 1	-3 -1	:	-3 -3 -1	-3				2		1 1 1 -1		-1 -1			-1		:
X.88 X.89 X.90	$     \begin{array}{r}       -3 \\       3 \\       -3 \\       1 \\       -1     \end{array} $	$     \begin{array}{r}       3 \\       -3 \\       -2 \\       2     \end{array} $	$     \begin{array}{r}       3 \\       -3 \\       1 \\       -1     \end{array} $	$-{1 \atop 3}$	-3 -3	$-1 \\ 1 \\ 1$	-3 :	$     \begin{array}{r}       -3 \\       3 \\       2 \\       -2     \end{array} $		:	-2 :	:	$-1 \\ 1 \\ 1 \\ 1$		$     \begin{array}{r}       -1 \\       -2 \\       -2     \end{array} $	:	:	$-\frac{1}{2}$	:	
X.91 X.92 X.93 X.94	6	6 -6 -6	· · 2	2	$\begin{array}{c} -2 \\ -2 \end{array}$	2	$\begin{array}{c} -2 \\ -2 \end{array}$	$_{-6}^{\dot{6}}$	1 1	1 1	$-\frac{3}{2}$	$     \begin{array}{r}       -1 \\       -1 \\       -2     \end{array} $	-2		2	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$ $-2$	:	
X.95 X.96 X.97	-6 ·	-6 -	-2 :	-2 :	2 -4	2	$-\frac{1}{2}$		4 2	$-\frac{1}{2}$	$-\frac{2}{2}$	$-\frac{2}{2}$	$-\frac{1}{2}$	$-\frac{2}{2}$	2	$-\frac{1}{2}$	2 2	- <u>2</u>		
$X.98 \\ X.99 \\ X.100$	$-\frac{1}{1}$	$\begin{array}{c} 4 \\ -4 \\ \end{array}$	-9 9	-3 3	:	1	:	$-\frac{2}{2}$		:		4	$-1 \\ -1$		$-2 \\ -2 \\ .$	:		:	:	
$X.101 \\ X.102 \\ X.103 \\ X.104$	$\begin{array}{c} 3 \\ -3 \\ \vdots \\ 3 \end{array}$	:	$-1 \\ -1 \\ -1$	-3 -3	4	3 3 3	-4	:	2	$-\dot{2}$	:		1 1 -1		:	2	$-\dot{2}$	:	:	
X.104 X.105 X.106 X.107	-3 ·		1	-3 -3	$-\dot{2}$	3	2		2 3 3	-2 3 3	3	i	-1			2	-2			
$X.108 \\ X.109 \\ X.110$					$-\frac{1}{2}$		2 2		3 2 2	$     \begin{array}{r}       3 \\       -2 \\       -2     \end{array} $	-3 ·	1				$-1 \\ -1$	1 1		Ċ C	C Ĉ
$X.111 \\ X.112 \\ X.113$	4	$-\frac{1}{2}$	4		$\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}$	4	$\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}$	:	$-\frac{1}{2}$	$-\frac{1}{2}$	:	2 2 2 2	-4		$-\dot{2}$	1 2	1 2	2	-i	-i
$X.114 \\ X.115 \\ X.116 \\ X.117$	-4 8	2	-4 :	:	4	:	4	:	-2 :	-2 :	:	$-\frac{2}{-2}$		:	:	1	1	-2 :		1
X.1118 X.1119 X.120	-8	−8			-4	:	4	:	-2 :		:				:	-2 :		:	:	
$X.121 \\ X.122 \\ X.123$		:			:	:	:	:	:	:	:			:	:	:	:	:	:	
$X.124 \\ X.125 \\ X.126 \\ X.127$	-3	:	-1	_3 -3	:		:	:	•	•	•	:	i			:	:	:	:	:
$X.128 \\ X.129 \\ X.130$	$-3 \\ -6 \\ -6$	-3	1	$-3 \\ 3 \\ 2$	-3	$\begin{array}{c} 3 \\ 3 \\ -2 \end{array}$	-3	3	$-\frac{1}{2}$	$-\frac{1}{2}$		2	1	-i	i	$-\frac{1}{2}$	$-\frac{1}{2}$	i	:	:
$X.131 \\ X.132 \\ X.133 \\ Y.134$	6	3		-2	-3 :	-2	-3 :	-3 :		:	-2			1	1	:	:	-1 :	:	
$X.134 \\ X.135 \\ X.136 \\ X.137$	$-3 \\ -5 \\ -5$	4	-3 $-3$	$-1 \\ 1 \\ 3$	6	1 1 -1	-6	2		:	$-\frac{2}{2}$	$-2 \\ -2 \\ .$	1 1		$-2 \\ -2 \\ \vdots \\ 2$	:	:	:	:	
$X.138 \\ X.139 \\ X.140$	5	$-\frac{1}{4}$	3	-3 :		-1 :	:	$     \begin{array}{r}       -2 \\       -4 \\       -4    \end{array} $					1		2 2	:			:	:
X.141 X.142 X.143		:	:		$-\dot{6}$		6	4 4	:	:	:	:		:	:	:	:	:	:	:
$X.144 \\ X.145 \\ X.146 \\ X.147$	-3 3	:	$-1 \\ 1 \\ \cdot$	-3 3	$-\dot{4}$	-3 -3	4	:	4	$-\dot{4}$	:	:	-1 -1		:	$-\frac{1}{2}$	2	:	:	:
X.148 X.149 X.150	:	:			4	:	$-\overset{\cdot}{4}$	:	$-\overset{\cdot}{4}$	4	:				:	2	$-\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}{\overset{\cdot}$	:	:	

2 3 5	1	3 1 i	2 1 i	2 i	3 1 1	5	5 ·	4 4	3 4	3 3	3 3	3 3	3 3	4 2	4 2	2 3	2 3	2 3	2 2	2 2	3 1 1	3 i	3 1	2 1 i
11 13	1 13a	:	:	:	15a	160	: 16b	18a	186	18c	18d	: 18e	18 <i>f</i>	180	: 18h	18i	18 <i>i</i>	: 18k	181	18m	200	206	206	
$\begin{array}{c} 3P \\ 5P \\ 7P \end{array}$	13a 13a 13a 13a	7a $14a$ $14a$ $2a$	7a $14b$ $14b$ $2c$	7a $14c$ $14c$ $2b$	15a $5a$ $3a$ $15a$	8f 16a 16a 16a	8f $16b$ $16b$ $16b$	$     \begin{array}{r}       9a \\       62 \\       18a \\       18a   \end{array} $	$^{9b}_{62}_{18b}$	9a 6 <sub>14</sub> 18f 18c	9a 6 <sub>6</sub> 18e 18d	$     \begin{array}{r}       9a \\       66 \\       18d \\       18e     \end{array} $	9a 6 <sub>14</sub> 18c 18 f	$     \begin{array}{r}       9a \\       6_{10} \\       18g \\       18a     \end{array} $	9a 6 <sub>11</sub> 18h 18h	$   \begin{array}{r}     9b \\     6_{14} \\     18i \\     18i   \end{array} $	9c $69$ $18j$ $18i$	$^{9b}_{66}_{18k}_{18k}$	$   \begin{array}{r}     9c \\     6_{24} \\     18m \\     18l   \end{array} $	9c $625$ $18l$ $18m$	10a 20a 4a 20a 20a	10c $20b$ $4e$ $20b$ $20b$ $20b$	10d $20c$ $4c$ $20c$ $20c$	$7a \\ 21a \\ 3a \\ 21a$
3P 5P 5P 7P 11P 11P 12N 12N 12N 12N 12N 12N 12N 12N	$13a \\ 13a \\ 13a$	14a $14a$ $2a$ $14a$ $14a$ $14a$	$^{14b}_{14b}_{2c}$	$^{14c}_{14c}_{2b}$	$5a \\ 3a \\ 15a \\ 15a \\ 15a \\ -1$	$16a \\ 16a \\ 16a$	$16b \\ 16b \\ 16b$	$^{62}_{18a}_{18a}$	$^{62}_{18b}$	$^{6_{14}}_{18f}_{18c}$	$^{66}_{18e}_{18d}$	$^{66}_{18d}_{18e}$	$^{6_{14}}_{18c}_{18f}$	$^{6_{10}}_{18g}_{18a}$	$^{6_{11}}_{18h}_{18h}$	$^{6_{14}}_{18i}_{18i}$	$^{69}_{18j}_{18i}$	$^{66}_{18k}$ $^{18k}$	$^{624}_{18m}_{18l}_{18m}$	$^{625}_{18l}_{18m}$	$^{20a}_{4a}_{20a}_{20a}$	$^{20b}_{\ 4e}_{\ 20b}_{\ 20b}$	20c $4c$ $20c$ $20c$ $20c$ $20c$	$7a \\ 21a \\ 3a \\ 21a$
X.130 X.131 X.132 X.133 X.134 X.135 X.136 X.138 X.141 X.142 X.144 X.144 X.145 X.144 X.144 X.145 X.149 X.149 X.140	-11	-1 -1 -1 1 1 1 1 -1 -1 -1	-1 1 1 -1 1 -1 1 -1 1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	-2 -2 -2 -2 -1 -1 -1 -2 -2 -2 -2 -2 -1 -1 -1 -2 -2 -2 -2 -1 -1 -1 -1 -2 -2 -2 -2 -1 -1 -1 -1 -2 -2 -2 -2 -1 -1 -1 -1 -2 -2 -2 -2 -1 -1 -1 -1 -2 -2 -2 -2 -1 -1 -1 -1 -2 -2 -2 -2 -1 -1 -1 -1 -1 -2 -2 -2 -2 -1 -1 -1 -1 -2 -2 -2 -2 -1 -1 -1 -1 -2 -2 -2 -2 -1 -1 -1 -1 -2 -2 -2 -2 -1 -1 -1 -1 -2 -2 -2 -2 -1 -1 -1 -1 -2 -2 -2 -2 -1 -1 -1 -1 -2 -2 -2 -2 -1 -1 -1 -1 -2 -2 -2 -2 -2 -1 -1 -1 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2			-2 -2 -2	-6 3 3 -3 -3 -3 -3 -3 -3 -6	-11 -22 -22 -22 	-1 -1 -1 -2 -2 -2 -2 -2	-1122-22	-111-222-22	2 2 2 2 2 2 3 -11 -11 	2 2 2 2 2 2 3 -1 -1 -1 -1 	-1 -1 1 1 -1 -1 -1 -1		-11-11-11-1-1-1-1-1-1-1-1-1-1-1-1-1-1-1-	-111	-111	2 2 2 2 2 2 1 -1			-1 -1 -1 1 1 1 1 -1 -1 -2

Character table of  $A_1(Fi'_{24})$  (continued)

2 3 5	2	2	2	5 1	5 1	5 1	5 1	4 1	4 1	4 1	4 1	1	2	3 1 1	$\begin{array}{c} 2 \\ 1 \\ 1 \end{array}$	$\begin{array}{c} 2 \\ 1 \\ 1 \end{array}$	3 2	3 2	2 2	2 1	2 1 i	2 1 i	$\begin{smallmatrix}2\\1\\1\end{smallmatrix}$
11 13	1 22a	i 22b	$\frac{22c}{}$	24a	24b	24c	24d	24e	24 <i>f</i>	24g			1 28a	30a						:	:	:	60a
2P 3P 5P 7P	$\frac{22a}{22a}$	$\frac{22b}{22b}$	11a $22c$ $22c$ $22a$	$\frac{8b}{24a}$	$\frac{8c}{24b}$	$12_{6} \\ 8d \\ 24c \\ 24c$	$\frac{8e}{24d}$	$\frac{0}{24a}$	8a	12.10	$12_{20} \\ 8a \\ 24f \\ 24f$	13a 26a 26a	14a 28a 28a	15a 10a 6 <sub>1</sub>	15a 10e 64	15a 10b 65	$18g$ $12_{3}$ $36a$	$18g$ $12_{3}$ $36b$	18b 12 <sub>4</sub> 36c	$   \begin{array}{c}     21a \\     14b \\     42c   \end{array} $	21a $14a$ $42b$	$\begin{array}{c} 21a \\ 14b \\ 42a \end{array}$	30a 20a 12 <sub>1</sub>
11P 13P X.85	2b	2a		24a	24b	24c	24d	$24g \\ 24e \\ 24e$	24h $24f$ $24f$	$24e \\ 24g \\ 24g$	24f 24h 24h	26a	$\frac{28a}{28a}$	30a $30a$	306	30c	36a	366	36c	42c	426	42a	60a
X.86 X.87 X.88				$-1 \\ -1 \\ -1 \\ -1$	$-1 \\ -1 \\ 1$	$-1 \\ -1 \\ -1$	$-1 \\ -1 \\ 1$	i -1		1		-1 :	1			1				$-1 \\ -1 \\ 1$	1 1 1	$-\frac{1}{1}$	
X.89 X.90 X.91 X.92			:	1 1	$-\frac{1}{1}$	1	$-\frac{1}{1}$		i	:	1		:	$-\dot{2}$	:		$-1 \\ -1 \\ .$	$-1 \\ -1 \\ .$	$-1 \\ 1 \\ .$	:	:		:
$X.93 \\ X.94 \\ X.95$			:		:		:	:	-1 :	:	-1 :			1 1	1 1	1 -1	:						1 -1
X.96 X.97 X.98 X.99		:		1 1	-i	-1 -1	1 -1	:	:	:	:			:	:	:	1	i -1	1 -1	:	:		:
$X.100 \\ X.101 \\ X.102$	$-1 \\ -1 \\ -1$	1 1		1	-i	-i -1	-1 -1						-i	$-1 \\ -1 \\ -1$	$-1 \\ -1 \\ -1$	-1 1		-1		1 -1	1 1	1 -1	$-\frac{1}{1}$
X.103 X.104 X.105 X.106		-2 :		1 1		1 1	$-\overset{i}{{1}}$	:	:		:		-1	1 1 2	$-1 \\ -1$	$-\frac{1}{1}$	:			$-1 \\ 1$	$-1 \\ -1$	$-\frac{1}{1}$	${\stackrel{\dot{1}}{\scriptstyle -1}}$
X.107 X.108 X.109							:		-1 1		-1 1									Ċ	-1	Ċ	
X.110 X.111 X.112 X.113	-i	-i	-i	:	:	:	:	•	:	•				:	:	:	i	i		C :	-1 :	<i>C</i>	:
$X.114 \\ X.115 \\ X.116$	1 1	1 1	1 1				:				$-\overset{\cdot}{\overset{I}{I}}$	$-1 \\ -1 \\ -1$					-i	-i					
$X.117 \\ X.118 \\ X.119 \\ X.120$		•	:		:	:	:	:	:	:	:	i	:	-2		2	-1 i	-1 i		1 -1	$     \begin{array}{r}       -1 \\       -2 \\       -1 \\       -1     \end{array} $	1 -i 1	:
X.121 X.122 X.123			:		:	:	:					1 -1 -1		$-\frac{2}{2}$		-2				-1 C C	$-1 \\ 1 \\ 1 \\ 1$	$-\frac{1}{\bar{C}}$ $C$	
$X.124 \\ X.125 \\ X.126$	1 1	1 1						:	:		:		:				:				2	:	
X.127 X.128 X.129 X.130	1	-i	1	$-\frac{1}{1}$	$-1 \\ -1 \\ \vdots$	-1 -1	-1 1 2	-i		-i				$-1 \\ -1 \\ \vdots$	$-1 \\ -1 \\ \vdots$	$-\frac{1}{1}$				-1 1	1 1	-1 1	$-\frac{1}{1}$
X.131 X.132 X.133 X.134				-2 1		i	-2	1	:	1	•		i	$-\frac{1}{2}$	:	$-\overset{\cdot}{\overset{\cdot}{2}}$	:	•	:	-i	-i	-i	:
$X.135 \\ X.136 \\ X.137$				1 1	1	1 -i	- i i	:					-1				i	1	-i	1	-1	1	:
$X.138 \\ X.139 \\ X.140 \\ X.141$	-111111111111111111111111111111111111		. 1	1	1	-1 :	-1 :	•	:	•			1 −1	1 1 1 1	$\begin{array}{c} 1 \\ -1 \\ -1 \end{array}$	-i 1 -1	-1 :	-1 :	$     \begin{array}{r}       1 \\       -1 \\       -1 \\       1     \end{array} $	1 -1 1	1 1 1 1	-1 $1$ $1$	$-\frac{1}{1}$
$X.142 \\ X.143 \\ X.144$	$-1 \\ -1$	-i i	$-1 \\ -1$	-i	i	1	-i						$-1 \\ -i$	$\begin{array}{c} 1 \\ 2 \\ 1 \end{array}$	1 1	1 i			1	$-1 \\ -1 \\ -1$	1 -1	$-1 \\ -1$	i i
$X.145 \\ X.146 \\ X.147 \\ X.148$	-1 :	1	-1 :	-1 :	-1 :	1	1	:	:	:		1	1	$\begin{array}{c} 1 \\ . \\ 2 \\ -2 \end{array}$	1	-1 :	:		:	1	$-1 \\ \dot{2}$	1	-1 :
X.149 X.150	:					:	:	:	:	:	:			$-\frac{1}{2}$	:		:		:	:	$-2^{-\frac{1}{2}}$		:

where  $A=16\zeta(3)+8,\ B=6\zeta(3)+3,\ C=-2\zeta(3)-1,\ D=4\zeta(3)+2,\ E=-2\zeta(11)^9-2\zeta(11)^5-2\zeta(11)^4-2\zeta(11)^3-2\zeta(11)-1,\ F=18\zeta(3)+9,\ G=112\zeta(3)+56,\ H=12\zeta(3)+6,\ I=4\zeta(12)_4\zeta(12)_3+2\zeta(12)_4,\ J=96\zeta(3)+48,\ K=-128\zeta(3)-64.$ 

#### References

- John Cannon and Catherine Playoust. An Introduction to Magma. School of Mathematics and Statistics, University of Sydney, 1993.
- [2] John J. Cannon, Derek F. Holt. Automorphism group computation and isomorphism testing in finite groups. *J. Symbolic Computat.*, 35:241-267, 2003.
- [3] R.W. Carter. Simple groups of Lie type. John Wiley and Sons, London, 1972.
- [4] J.H. Conway, R.T. Curtis, S.P. Norton, R.A. Parker, and R.A. Wilson. Atlas of finite groups. Clarendon Press, Oxford, 1985.
- [5] B. Fischer. Finite groups generated by 3-transpositions. Inventiones Math., 13:232-246, 1971.
- [6] B. Fischer. Finite groups generated by 3-transpositions II. Lecture Notes, University of Warwick, Coventry, 1979.
- [7] J. Hall, L. H. Soicher Presentaions of some 3-transposition groups. Comm. Algebra, 23:232-246, 1995.
- [8] Holt, D. F., Cohomology and group extensions in Magma, in W. Bosma, J. Cannon (eds), Discovering mathematics with Magma, Springer, Berlin 2006, pp. 221–141.
- [9] G. James. The modular characters of the Mathieu groups. J. Algebra, 27:57-111, 1973.
- [10] H. Kim, G. O. Michler. Simultaneous constructions of the sporadic groups Co<sub>2</sub> and Fi<sub>22</sub>. in (L. -C. Kappe, A. Magidin, R. F. Morse, eds.) Computational Group Theory and the Theory of Groups, Contemporary Mathematics 470, 141–234, Amer. Math. Soc, Providence, RI., (2008).
- [11] H. Kim. Representation theoretic existence proof for Fischer's sporadic group Fi<sub>23</sub>. Senior Thesis, Dept. Math. Cornell University, 2008. See also arXiv: 0904.0639v1.
- [12] G. O. Michler. Theory of finite simple groups. Cambridge University Press, Cambridge, 2006.
- [13] G. O. Michler. Constructing finite simple groups from irreducible subgroups of GL<sub>n</sub>(2), in (L. -C. Kappe, A. Magidin, R. F. Morse, eds.) Computational Group Theory and the Theory of Groups, Contemporary Mathematics 470, 235–262, Amer. Math. Soc, Providence, RI., (2008).
- [14] G. O. Michler. Theory of finite simple groups II. Cambridge University Press, Cambridge (to appear in 2009).
- [15] Ch. Praeger, L. Soicher. Low rank representations and graphs for sporadic groups. Cambridge University Press, Cambridge, 1997

Department of Mathematics, Yale University, New Haven, CT. 06511, USA

DEPARTMENT OF MATHEMATICS, CORNELL UNIVERSITY, ITHACA, N.Y. 14853, USA